SCHOLAR Study Guide

SQA Higher Physics
Unit 2: Electricity and Electronics

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SCHOLAR Study Guide Unit 2: SQA Higher Physics

1. SQA Higher Physics

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Electric fields

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1.1 Introduction

You may have noticed crackling and perhaps seen sparks when taking off a nylon shirt or blouse in the dark. You will have seen lightning. You will certainly have used electrical appliances of all kinds. All of these depend on a fundamental property of matter - charge.

In this Topic we look at how charge can only be described by its properties and in particular how charges exert forces on each other - the electric force. One convenient way we have of describing the forces that charges exert on each other is by introducing the concept of an electric field.

You will have already met the term ’potential difference’. This term is explained in this Topic in terms of the work done in moving a charge in an electric field. We also look briefly at some applications of charges and fields.

1.2 Charge, forces and fields

Learning Objective

To describe some properties and effects of electric charge.
To describe what is meant by an electric field.

1.2.1 Electric charge

Thales, an ancient Greek experimenter, noticed that when he rubbed amber with cloth, the amber attracted small pieces of straw - it exerted a force on the straw. This effect was described about 2000 years later as being due to a ’charge’ of electricity. The word ’electricity’ comes from the Greek word *elektron* meaning amber. The idea of charge originated because it was first thought that electricity was like a fluid that could be poured. We still sometimes say ’charge your glasses’ meaning fill them up with drink.

We now know that charge is a fundamental property of matter. The magnitude of the charge carried by one electron or one proton is known as the **fundamental unit of charge *e***. This means that charge is quantised, or comes in multiples of this fundamental charge. Experiments have shown that there are only two types of charge. More than 200 years ago these two types were called ’positive’ and ’negative’ by the American physicist Benjamin Franklin.

An object can be charged by adding negatively-charged particles such as electrons to it, in which case it becomes negatively charged, or by removing electrons from it, making it positively charged. Further experiments have shown that a negatively-charged object attracts a positively-charged object and that objects that have similar charges repel each other.

1.2.2 Electric field

We have just noted that charged objects attract or repel each other. In other words, charges exert attractive or repulsive (repelling) *forces* on each other. A convenient way of describing this electric force is to use the concept of the electric field. An **electric field** is the region around a charged object where the charge exerts a force on other charges.

It is usual to represent an electric field by using lines of force. The strength of the electric field at any point is shown by the separation of the lines of force. The direction is shown by arrowheads on the lines, pointing the way a **positive** charge would experience a force in the field.

Lines of force

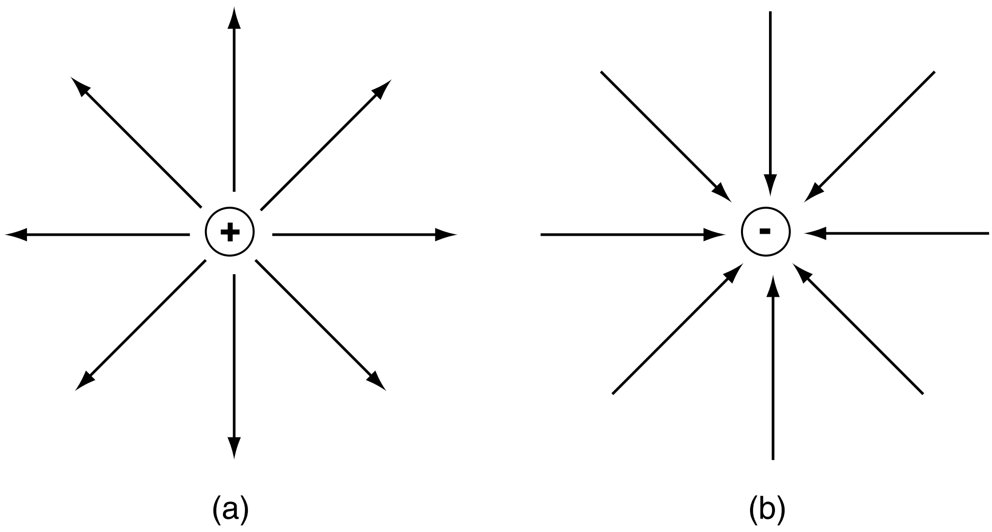
In the nineteenth century, Michael Faraday first used the idea of lines of force to map out an electric field. This simulation allows you to repeat his experiment.

15 min

The electric field lines around a point charge are radial.

The electric field around an isolated point charge is shown in Figure 1.1. The actual number of lines drawn is not significant - only their relative closeness gives an indication of the strength of the electric field. This shape of field is called a radial field.

Figure 1.1: Electric field around (a) an isolated positive charge, and (b) an isolated negative charge



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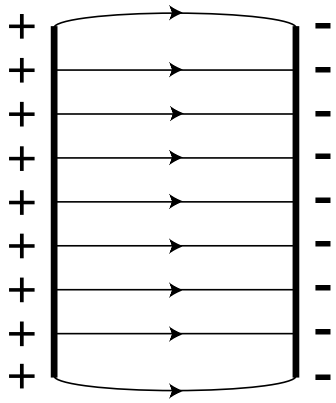
You might recognise that this shape of field is similar to the gravitational field around the Earth. The gravitational field strength *g* is a measure of the gravitational force of attraction on a mass. It is measured in newtons per kilogram. Near the Earth's surface *g* has the approximate value 10 N kg^{-1}. It decreases as the distance from the Earth's surface increases. In a similar way, electric field strength is a measure of the electrical force on a charge. The units of electric field strength are newtons per coulomb (N C^{-1}). An equivalent unit, the volt per metre (V m^{-1}) is also frequently used.

A radial field is a non-uniform field. Figure 1.2 shows the field between a pair of charged metal plates, which is a uniform field. For this course, all calculations on electric fields will be on uniform fields.

Figure 1.2: Electric field between two charged metal plates



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A **conductor** is a material through which electric charge can flow. This is because there are free electric charges in a conductor. If an electric field is applied to a conductor, the charges experience a force, and this force causes the free electric charges in the conductor to move.

15 min

Electric field patterns

This simulation allows you to examine the electric field lines for various combinations of 'point' charge and charged metal plates.

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15 min

Quiz 1 Charges, forces and fields

Multiple choice quiz.

First try the questions. If you get a question wrong or do not understand a question, there are 'Hints' at the the back of the book . The hints are in the same order as the questions. If you read the hint and still do not understand then ask your tutor.

Useful data:

fundamental charge *e*	1.60 x 10^{-19} C

Q1: How many protons are needed to carry a charge of 1 C?

a) 1.6 x 10^{-19}
b) 8.85 x 10^{-12}
c) 1
d) 6.25 x 10^{18}
e) 1.6 x 10^{19}

. . . .

Q2: Two metal spheres are hanging from nylon threads. When the spheres are brought close together they repel each other. Which one of the following statements could be true?

a) One sphere is negatively charged, the other is positively charged.
b) One sphere is uncharged, the other is positively charged.
c) One sphere is uncharged, the other is negatively charged.
d) Both spheres are uncharged.
e) Both spheres are positively charged.

. . . .

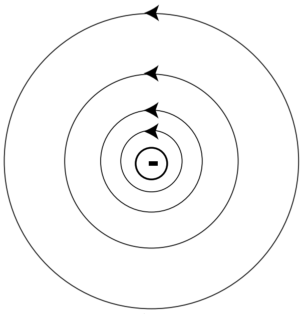
Q3: When a positive charge is placed in an electric field it

a) experiences a force.
b) loses its charge.
c) moves in a circle.
d) becomes a negative charge.
e) doubles its charge.

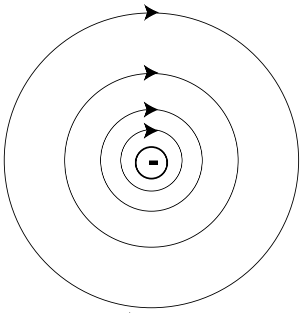
. . . .

Q4: Which of the following shows the electric field round an isolated negative charge?

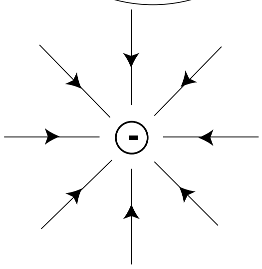
a)



b)



c)



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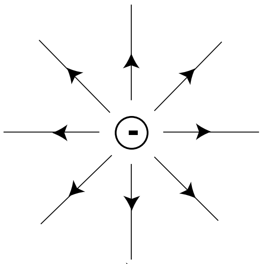
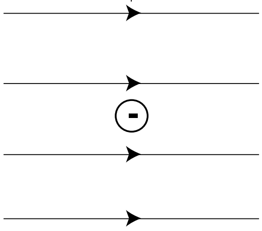


e)



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Q5: What is the unit of electric field strength?

a) N kg^{-1}
b) N C^{-1}
c) C N^{-1}
d) C kg^{-1}
e) kg N^{-1}

. . . .

1.3 Work done and potential difference

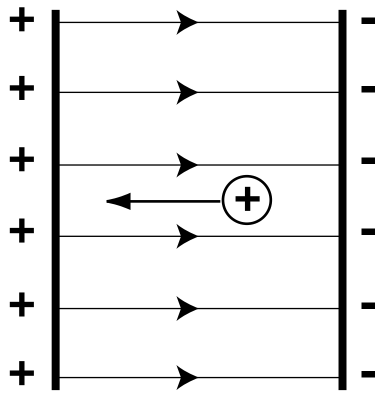
Learning Objective

To state that work is done when a charge is moved in an electric field.
To state what is meant by potential difference.
To state and use the relationship between work done in joules, charge in coulombs and potential difference in volts.
To describe the motion of charged particles in electric fields.

1.3.1 Potential difference and the volt

Consider a positively-charged particle placed in the uniform electric field set up between two parallel metal plates as shown in Figure 1.3.

Figure 1.3: A charge being moved in an electric field



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Work would need to be done to move a positive charge to the left in this field. This is because the electric field exerts a force to the right on the charge.

This work done is stored in the electric field as electrical potential energy. When the charge has been moved to the left-hand plate, the electric field strength is increased. The greater the electrical potential energy, the greater is the potential difference between the metal plates.

The **potential difference** between the two plates is a measure of the work done in moving the charge between the two plates. This is used to define the unit of potential difference, the volt, as follows:
If one joule of work *W* is done in moving one coulomb of charge *Q* between two points, the potential difference *V* between the two points is one volt.

$$V = \frac{W}{Q} \tag{1.1}$$

....

So

1 volt = 1 joule per coulomb

If a charged particle is placed in an electric field then the field will do work on the particle in moving it. In this case, the particle gains an amount of energy equal to the amount of work done by the field.

We have already compared the electric field around a charge with the gravitational field around the Earth. We can take this comparison further.

At the top of a mountain there is a greater gravitational potential than at the bottom. This means that work has to be done and energy has to be used up in moving a mass up a mountain. This work is done against the gravitational field and is stored in the mass as gravitational potential energy. If the mass is allowed to fall down the hill, the potential energy of the mass decreases. The difference in heights between the top of the hill and the bottom (*not* the actual heights relative to some base line such as sea level) determines the amount of gravitational potential energy.

In a similar way, it is the potential difference between the plates in Figure 1.3 that determines the amount of energy needed to move the charge from one plate to the other, not the voltage of either plate relative to, for example, earth potential.

Example
A proton is accelerated in a uniform electric field set up by a potential difference of 500 V.
Calculate the energy gained by the proton.

We know that the charge on a proton is 1.6 x 10^{-19} C, so using $V = {}^{W}/_{Q}$,

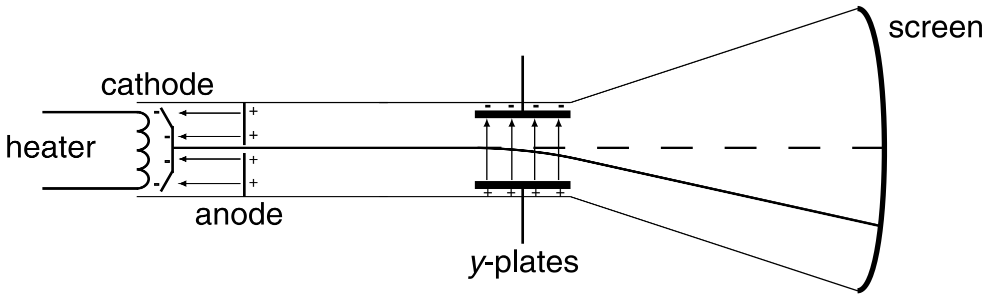
$$W = QV$$
$$\therefore W = 1.6 \text{ x } 10^{-19} \text{ x } 500$$
$$\therefore W = 8 \text{ x } 10^{-17} \text{ J}$$

. . . .

1.3.2 The cathode ray tube

In a cathode ray tube, such as is shown in Figure 1.4, electrons ('cathode rays') are freed from the heated cathode. These electrons are accelerated while in the electric field set up between the cathode and the anode, gaining kinetic energy. Some electrons pass through a hole in the anode. From the anode to the *y*-plates, the electrons travel in a straight line at constant speed, obeying Newton's first law of motion.

Figure 1.4: The cathode ray tube



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A second electric field is set up between the *y*-plates, this time at right angles to the initial direction of motion of the electrons. This electric field supplies a force to the electrons at right angles to their original direction. The resulting path of the electrons is a parabola. The motion of the electrons while between the *y*-plates is similar to the motion of a projectile thrown horizontally in a gravitational field.

When they leave the region of the *y*-plates, the electrons again travel in a straight line with constant speed (now in a different direction), eventually hitting the screen as shown. The point on the screen where the electrons hit is determined by the strength of the electric field between the *y*-plates. This electric field strength is in turn determined by the potential difference between the *y*-plates. So the deflection of the electron beam can be used to measure a potential difference.

Example

The potential between the cathode and the anode of a cathode ray tube (see Figure 1.4) is 200 V.

Assuming that the electrons are given off from the heated cathode with zero velocity and that all of the electrical energy given to the electrons is transformed to kinetic energy, calculate

1. the electrical energy gained by an electron between the cathode and the anode;
2. the horizontal velocity of an electron just as it leaves the anode.

[The mass of an electron is 9.11 x 10^{-31} kg]

1. The electrical energy gained by an electron is equal to the work done by the electric field between the cathode and the anode, so

$$\begin{aligned} W &= QV \\ \therefore W &= 1.6 \text{ x } 10^{-19} \text{ x } 200 \\ \therefore W &= 3.2 \text{ x } 10^{-17} \text{ J} \end{aligned}$$

2. If all of this energy is transformed to kinetic energy, then

$$\begin{aligned} E_k &= \tfrac{1}{2}mv^2 \\ \therefore 3.2 \text{ x } 10^{-17} &= \tfrac{1}{2} \text{ x } 9.11 \text{ x } 10^{-31} \text{ x } v^2 \\ \therefore v^2 &= \frac{2 \text{ x } 3.2 \text{ x } 10^{-17}}{9.11 \text{ x } 10^{-31}} \\ \therefore v &= 8.4 \text{ x } 10^6 \text{ m s}^{-1} \end{aligned}$$

. . . .

The cathode ray tube

10 min

This simulation allows you to see the path of electrons in the electric field set up between the cathode and the anode in a cathode ray tube, and calculate the kinetic energy gained by an electron. It also allows the path of the electrons to be changed by applying a potential difference between the y-plates.

15 min

Quiz 2 Work done and potential difference

Multiple choice quiz.

First try the questions. If you get a question wrong or do not understand a question, there are 'Hints' at the the back of the book . The hints are in the same order as the questions. If you read the hint and still do not understand then ask your tutor.

Useful data:

fundamental charge *e*	1.60×10^{-19} C
mass of electron	9.11×10^{-31} kg

Q6: The potential difference between two points is

a) a measure of the work done in moving one coulomb of charge between the two points.
b) a measure of the electrical force on a charge of one coulomb.
c) the gravitational potential energy between the points measured in joules.
d) equal to the fundamental unit of charge, in coulombs.
e) the region between the two points where charges exert forces, in newtons.

. . . .

Q7: Which of the following is equivalent to the volt?

a) joule per kilogram
b) newton per kilogram
c) joule per newton
d) coulomb per kilogram
e) joule per coulomb

. . . .

Q8: How much energy is needed to move a charge of 5×10^{-3} C through a potential difference of 8 kV?

a) 4.0×10^{7} J
b) 1.6×10^{6} J
c) 40 J
d) 4.0×10^{-5} J
e) 6.3×10^{-7} J

. . . .

Q9: A uniform electric field uses up 1.9×10^{-16} J of work in moving an oil drop containing 40 excess electrons between two points in the field.
What is the potential difference between the points?

a) 4.8×10^{3} V
b) 30 V
c) 1.2 V
d) 3.4×10^{-2} V

e) 1.2×10^{-3} V

....

Q10: An electron is accelerated from rest by a potential difference of 50 V.
What is the final velocity of the electron?

a) 8.0×10^{-18} m s^{-1}
b) 4.2×10^{-6} m s^{-1}
c) 2.4×10^{-5} m s^{-1}
d) 4.2×10^{6} m s^{-1}
e) 5.7×10^{6} m s^{-1}

....

1.4 Summary

By the end of this Topic you should be able to:

- describe some of the properties and effects of electric charge;
- describe what is meant by an electric field;
- state that an electric charge experiences a force in an electric field;
- state that work is done when a charge is moved in an electric field;
- state what is meant by potential difference;
- state and use the relationship between work done in joules, charge in coulombs and potential difference in volts.

1.5 Assessment

Online assessments

Two online test are available. Each test should take you no more than 20 minutes to complete. Both tests have questions taken from all parts of the Topic.

40 min

Topic 2

Resistors in circuits

Contents

2.1 Introduction

In your previous studies of electricity, you will have met charge, voltage, current and resistance. In this Topic, we will look more deeply into all of these quantities and build up a better understanding of how they are related.

The electric circuit will also be familiar to you. An electric circuit in its simplest form consists of a source of energy (perhaps a battery), a component to transform the electrical energy into some other form of energy, and a closed path of conductors. In this Topic, we will look again at the electric circuit, and in particular at the source of energy used in a circuit.

The final part of the work in this Topic will consist of a detailed look at different circuits containing resistances. We will consider conservation of energy and of charge in closed circuits and use these to derive the expressions for total resistance in series and parallel circuits. Finally we will look at the Wheatstone bridge circuit, originally developed to measure resistance accurately, but now a circuit often used in combination with op-amps for monitoring and control applications.

2.2 e.m.f. and internal resistance

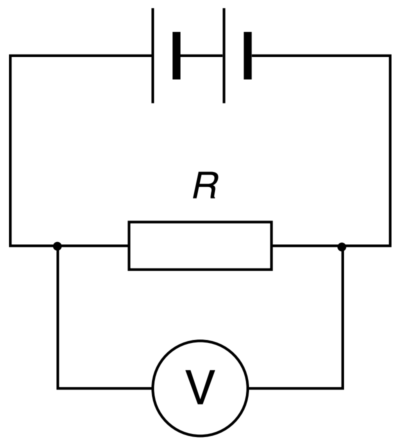
Learning Objective

To explain what is meant by the e.m.f. of a source.
To describe an electrical source in terms of e.m.f. and internal resistance.
To carry out calculations using the relationships involving the e.m.f., the terminal p.d. and the internal resistance of a source.

2.2.1 Potential difference, current and resistance

It is worth spending a short time considering what is meant by **potential difference** and resistance, before considering electrical sources in more detail.

Figure 2.1: Resistor in a circuit



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In the circuit shown in Figure 2.1, the e.m.f. of the cell is needed to drive a current *I* through the resistor - we say that the resistor has the property of **resistance**.

A potential difference *V* is set up across the resistor *R*, and this potential difference can be measured by placing a voltmeter across the resistor.The resistance *R* of a conductor is defined as the ratio of the potential difference across the conductor to the current through it.

$$R = \frac{V}{I} \tag{2.1}$$

. . . .

When the voltage *V* is measured in volts, and the current *I* is measured in amperes, then the resistance *R* is measured in ohms (Ω). If the ratio $^V/_I$ is constant for all values of *V* (as is the case with most metallic conductors) then the conductor obeys Ohm's law and we say that that the conductor is an ohmic conductor.

Ohmic conductors

10 min

This is a paper-based activity to give you practice in using the relationship $R = {}^V/_I$.

The following set of results was obtained by measuring the current through a conductor for various values of potential difference applied across it.

Potential difference (V)	1.0	2.3	3.7	4.8	7.3	8.9
Current (A)	0.18	0.41	0.66	0.86	1.3	1.6

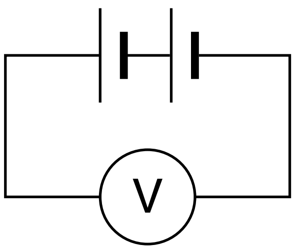
By considering all of these results, show that this conductor is ohmic.

The relationship $R = {}^V/_I$ can be used to show whether a conductor obeys Ohm's law.

2.2.2 Sources and circuits

We have already seen that an electrical source, such as a cell, a battery or even the mains supply or a thermocouple, supplies the energy to the charges in an electrical circuit. The term used for this quantity is **electromotive force**, or e.m.f. for short. Electromotive force (*E*), like potential difference, is measured in volts.

Figure 2.2: Measuring the e.m.f. of an electrical source



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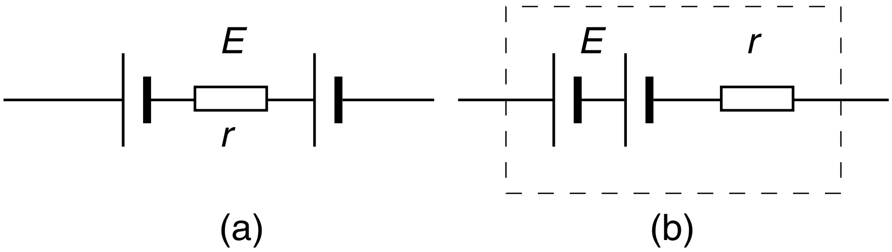
A voltmeter placed across the terminals of an electrical source, as in Figure 2.2, measures the e.m.f. of the source. This is because under these conditions the voltmeter is measuring the open circuit potential difference across the terminals of the source (the terminal potential difference or t.p.d. of the source). The open circuit t.p.d. of a source is equal to the e.m.f. of the source, as we will see shortly.

Consider a battery sending a current round an electrical circuit. This current is a flow of electrical charges around the circuit, both externally to the battery (in the external circuit, often called the **load resistor**, or more simply, the 'load') but also internally in the battery itself. If a 'perfect' source were to exist then it would supply a constant e.m.f. between its terminals no matter what current is taken from it. Such an ideal source does not exist (just like a frictionless surface does not exist). All practical sources of electrical energy present an opposition to the movement of the charges *through the source itself* - they have an **internal resistance**, given the symbol *r*.

We now have to think of a source of electrical energy as a supplier of e.m.f. *E* in series with an internal resistance *r*. Although the internal resistance appears between the plates of the source as in Figure 2.3(a), it is often more convenient to separate the source of e.m.f. from the internal resistance, and picture the source as in Figure 2.3(b).

Figure 2.3: Source of electrical energy showing the internal resistance



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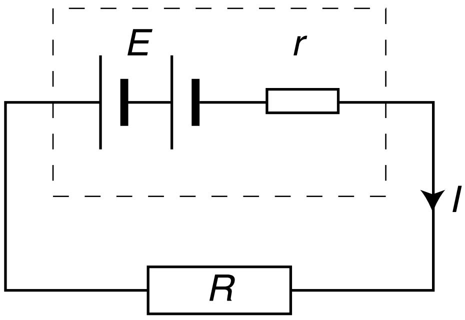
The internal resistance of a source obeys Ohm's law and so *r* is constant and is independent of the current *I* through it. The e.m.f. of a source is equal to the sum of all the potential differences across all of the resistors in the circuit, *including the potential difference across the internal resistance, r*. Consider the circuit shown in Figure 2.4.

Figure 2.4: Source connected to an external resistor



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In this circuit, the e.m.f. of the source *E* is equal to the potential difference *V* across the external resistor of resistance *R*, plus the potential difference across the internal resistance *r*.

$$\begin{aligned} E =& IR + Ir \\ E =& V + Ir \\ E - V =& Ir \\ r =& \frac{E - V}{I} \end{aligned} \tag{2.2}$$

. . . .

In Equation 2.2, the term *V* is the potential difference that appears at the terminals of the source. For this reason it is called the **terminal potential difference (t.p.d.)**. The term *Ir* represents the potential difference that is 'lost' across the internal resistance of the source, and never appears in the external circuit. This term is often called the '**lost volts**'. It is worth noting that both *E* and *r* are properties of the source and are constant (at least in the short term, if the source is not abused). On the other hand both the terminal potential difference and the lost volts depend on the current taken from the source, and so are not constant.

Example

A cell has an e.m.f. of 1.5 V. Its terminal potential difference falls to 1.2 V when driving a current through an external resistor of resistance 5.0 Ω.

Calculate the current in the circuit, the lost volts and the internal resistance of the cell.

Applying Ohm's law to the external resistor:

$$\begin{aligned} V =& IR \\ \text{so } I =& \frac{V}{R} \\ I =& \frac{1.2}{5.0} \\ I =& 0.24 \text{ A} \end{aligned}$$

Re-arranging Equation 2.2:

$$\begin{aligned} Ir =& E - V \\ Ir =& 1.5 - 1.2 \\ \text{lost volts } =& 0.3 \text{ V} \end{aligned}$$

Using Equation 2.2:

$$\begin{aligned} r =& \frac{E - V}{I} \\ r =& \frac{0.3}{0.24} \\ r =& 1.25\ \Omega \end{aligned}$$

. . . .

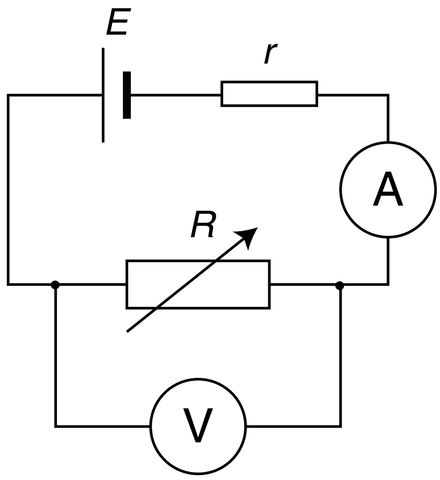
25 min

Measuring the e.m.f. and internal resistance of a source

A paper-based graph-plotting and data analysis exercise.

The circuit shown in Figure 2.5 was used to find the e.m.f. E and the internal resistance r of a cell.

Figure 2.5: Circuit to find e.m.f. and internal resistance



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The following results were obtained.

Voltage V (V)	Current I (A)
1.110	1.110
1.246	0.831
1.368	0.507
1.446	0.371
1.485	0.265
1.524	0.186

Plot a graph of current against voltage, and from it calculate:
1. the e.m.f. E of the cell,
2. the internal resistance r of the cell,
3. the maximum current that the cell is capable of delivering (the '**short-circuit current**').

The e.m.f. and internal resistance of a source can both be calculated from a graph of voltage against current.

. . . .

Quiz 1 e.m.f. and internal resistance

15 min

Multiple choice quiz.

First try the questions. If you get a question wrong or do not understand a question, there are 'Hints' at the the back of the book . The hints are in the same order as the questions. If you read the hint and still do not understand then ask your tutor.

Q1: Which of the following terms is equivalent to internal resistance?

a) $\frac{V}{I}$
b) $\frac{E}{I}$
c) $\frac{E-V}{I}$
d) IR
e) Ir

. . . .

Q2: Which of the following terms is known as the 'lost volts'?

a) E
b) V
c) IR
d) Ir
e) $\frac{E-V}{I}$

. . . .

Q3: A cell has an e.m.f. of 1.54 V. When it is in series with a 1.00 Ω resistor, the reading on a voltmeter across the cell is 1.40 V.
What is the internal resistance of the cell?

a) 0.10 Ω
b) 0.14 Ω
c) 0.71 Ω
d) 1.40 Ω
e) 1.54 Ω

. . . .

Q4: A cell has an e.m.f. of 1.52 V.
What is its internal resistance if its short-circuit current is 2.50 A?

a) 4.02 Ω
b) 3.80 Ω
c) 1.64 Ω
d) 0.980 Ω
e) 0.608 Ω

. . . .

Q5: A battery of e.m.f. 9.0 V and internal resistance 3.0 Ω is connected to a resistance of 15 Ω.
What is the potential difference across the terminals of the battery?

a) 0.5 V
b) 1.5 V
c) 7.5 V
d) 9.0 V
e) 15 V

. . . .

2.3 Conservation of energy and charge

Learning Objective

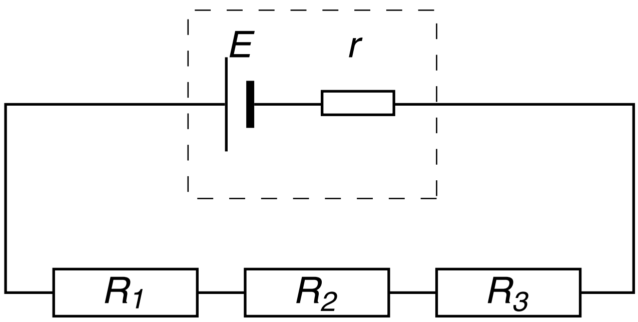
To explain what is meant by conservation of energy and of charge.
To consider conservation of energy and charge in a closed circuit.
To derive and use the expression for the total resistance of any number of resistors in series.
To derive and use the expression for the total resistance of any number of resistors in parallel.

All of the evidence that is available to us at the present time suggests that certain quantities are conserved - that is they can neither be created nor destroyed. Two quantities that are conserved that concern us at present are energy and charge.

2.3.1 Resistors in series

Consider a circuit consisting of a cell that has an e.m.f. of E and an internal resistance of r, connected to three resistors R_1, R_2 and R_3 as shown in Figure 2.6.

Figure 2.6: Series circuit



. . . .

Voltage is energy per unit charge and energy and charge are both conserved in this circuit. The energy per charge supplied by the cell is equal to the total energy dissipated by the resistors in the circuit. So e.m.f. *E* (the energy per charge supplied to the charges) must equal the sum of all the potential differences across all the resistors (the energy per charge transformed by the resistances) in the circuit.

The potential difference across a resistor *R* carrying a current *I* is *IR*, so:

$$\begin{aligned} E &= Ir + IR_1 + IR_2 + IR_3 \\ E &= I\left(r + R_1 + R_2 + R_3\right) \\ E &= IR \end{aligned}$$

where *R* is the equivalent resistance in the circuit.

$$\begin{aligned} R &= r + R_1 + R_2 + R_3 \\ R &= \text{the sum of all the individual resistances} \end{aligned}$$

In general, for *n* resistors connected in series

$$R = R_1 + R_2 + \; .... \; R_n \tag{2.3}$$

. . . .

Example

Four resistors with resistances 1.8 Ω, 2.2 Ω, 2.7 Ω and 3.3 Ω are connected in series across a supply that has an e.m.f. of 5 V and negligible internal resistance. Calculate the equivalent resistance and the current in the circuit.

The equivalent resistance of resistors in series is given by the sum of all the individual resistances, so

$$\begin{aligned} R &= 1.8 + 2.2 + 2.7 + 3.3 \\ R &= 10\ \Omega \end{aligned}$$

The current in the circuit is given by $I = {}^{E}/_{R}$, so

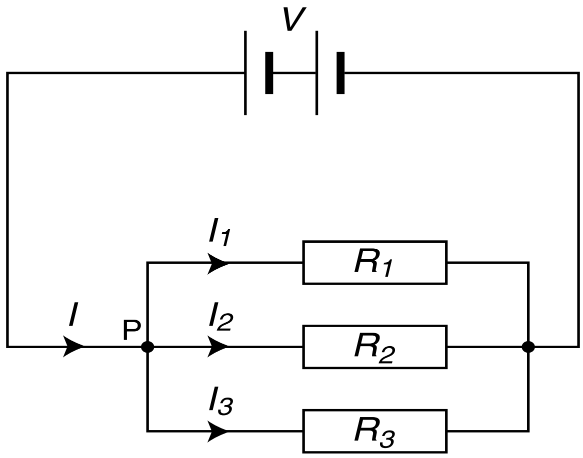
$$\begin{aligned} I &= \frac{E}{R} \\ I &= \frac{5}{10} \\ I &= 0.5\ \text{A} \end{aligned}$$

. . . .

2.3.2 Resistors in parallel

Consider a circuit consisting of three resistors in parallel, connected across a supply with a terminal potential difference of *V* volts, as shown in Figure 2.7. For this analysis, we are not concerned with the e.m.f. or the internal resistance of the source, only its t.p.d. *V*. The load resistance does not change, so *V* is constant.

Figure 2.7: Parallel circuit



. . . .

Since charge is conserved in this circuit, the rate at which charge flows into junction P must equal the rate at which charge flows out of the same junction. The rate of flow of charge is current, so

$$I = I_1 + I_2 + I_3$$

The three resistors are connected across the supply and therefore the potential difference *V* appears across all three resistors so, using $I = {}^{V}/_{R}$, we have

$$\begin{aligned} \frac{V}{R} &= \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \\ \frac{V}{R} &= V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \\ \frac{1}{R} &= \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \end{aligned}$$

Where *R* is the equivalent resistance in the circuit.

In general, for *n* resistors connected in parallel

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \ \dots\ + \frac{1}{R_n} \tag{2.4}$$

. . . .

Example

Two resistors with resistances 27 Ω and 33 Ω are connected in parallel across a cell that has an e.m.f. of 1.5 V and negligible internal resistance. Calculate the equivalent resistance and the current taken from the cell.

The equivalent resistance of resistors in parallel is given by the relationship ${}^{1}/_{R} = {}^{1}/_{R_1} + {}^{1}/_{R_2} + {}^{1}/_{R_3}$, so

$$\begin{aligned}\frac{1}{R} &= \frac{1}{27} + \frac{1}{33} \\ \frac{1}{R} &= 0.067 \\ \therefore R &= 15\ \Omega\end{aligned}$$

The current in the circuit is given by $I = {}^{E}/_{R}$, so

$$\begin{aligned}I &= \frac{E}{R} \\ I &= \frac{1.5}{15} \\ I &= 0.1 \text{ A (100 mA)}\end{aligned}$$

. . . .

In the example, you will see that the total equivalent resistance *R* is 15 Ω which is less than either individual resistance. This is always the case when resistors are connected in parallel - the total resistance is always less that each of the individual resistances.

There are a couple of points that make it easier to calculate the equivalent resistance *for two resistors only* connected in parallel.

Firstly, if two resistors *with the same resistance* are connected in parallel, then the equivalent resistance is half either value. So the equivalent resistance of two 10 Ω resistors connected in parallel is 5 Ω. You might like to confirm this for any other values that you want to choose.

Also for two resistors only connected in parallel, although the resistances can be different in this case, the relationship can be reduced to a simpler expression:

$$R = \frac{R_1 \times R_2}{R_1 + R_2}$$

This is sometimes called the M.A.D. way of calculating the equivalent resistance of two resistors connected in parallel, because you Multiply the values, Add the values and Divide one by the other. You might like to confirm that this relationship is correct, by starting with the expression derived for resistances in parallel.

Resistors in series and parallel

In this on-line activity, you have to calculate the total resistance in a circuit containing two resistors connected in either series or parallel. Using this value of total resistance, you then have to calculate the value of one of the resistors, given the value of the other one.

20 min

When resistors are connected in series, the total resistance *R* is given by the relationship

$$R = R_1 + R_2 + R_3$$

When resistors are connected in parallel, the total resistance *R* is given by the relationship

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

The relationship linking current, voltage and resistance is

$$R = \frac{V}{I}$$

---

15 min

Quiz 2 Resistors in series and parallel

Multiple choice quiz.

First try the questions. If you get a question wrong or do not understand a question, there are 'Hints' at the the back of the book . The hints are in the same order as the questions. If you read the hint and still do not understand then ask your tutor.

Multiple choice quiz.

Q6: Which expression is used to calculate the resistance *R* of three resistors connected in series?

a) $R = R_1 + R_2 + R_3$
b) $R = R_1 \times R_2 \times R_3$
c) $R = \frac{R_1 \times R_2 \times R_3}{R_1 + R_2 + R_3}$
d) $R = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$
e) $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

....

Q7: Which expression is used to calculate the resistance *R* of three resistors connected in parallel?

a) $R = R_1 + R_2 + R_3$
b) $R = R_1 \times R_2 \times R_3$
c) $R = \frac{R_1 \times R_2 \times R_3}{R_1 + R_2 + R_3}$
d) $R = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$
e) $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

....

Q8: Three resistors with values 180 Ω, 330 Ω and 390 Ω are connected in series, and the combination is connected across a 9.0 V battery.
Calculate the current taken from the battery.

a) 810 A

b) 100 mA
c) 10 mA
d) 0.35 mA
e) 0.39 μA

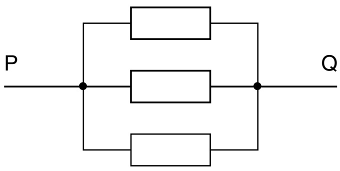
. . . .

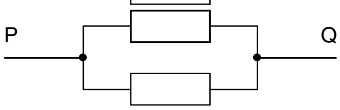
Q9: Three resistors with values 12.0 Ω, 82.0 Ω and 100 Ω are connected in parallel. What is the equivalent resistance?

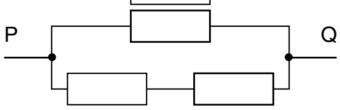
a) 0.106 Ω
b) 9.48 Ω
c) 200 Ω
d) 492 Ω
e) 98 400 Ω

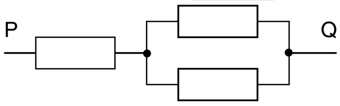
. . . .

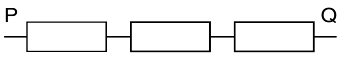
Q10: Three 10 Ω resistors are available.
How should some or all of them be connected to give an equivalent resistance between points P and Q of 15 Ω?

a)


b)


c)


d)


e)


. . . .

2.4 The Wheatstone bridge

Learning Objective

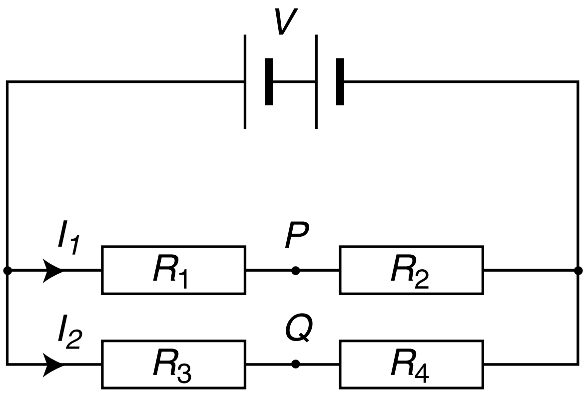
To describe what a Wheatstone bridge circuit is.
To state the relationship between the resistors in a balanced Wheatstone bridge circuit.
To carry out calculations involving the resistances in a balanced Wheatstone bridge circuit.
To state the relationship between the out-of-balance p.d. and the change of resistance as the resistance of one resistor in an initially balanced Wheatstone bridge circuit is varied by a small amount.

The **Wheatstone bridge circuit** consists of a series/parallel arrangement of resistors. Although it bears his name, Sir Charles Wheatstone did not actually invent this circuit. Wheatstone was a railway engineer in the nineteenth century who was concerned with electrical signalling on the railways. He used the bridge circuit as a means of sending messages. In one application, the Wheatstone bridge circuit is used to measure resistance. This is the application that we will first consider.

2.4.1 The balanced Wheatstone bridge circuit

In its usual arrangement, the Wheatstone bridge circuit consists of four resistors, connected to a source as shown in Figure 2.8.

Figure 2.8: The Wheatstone bridge circuit



. . . .

Consider firstly the potential divider consisting of R_1 and R_2. Using the relationship between voltage, current and resistance, we see that

$$\begin{aligned} V_{R_1} &= I_1 R_1 \\ V_{R_2} &= I_1 R_2 \\ V &= V_{R_1} + V_{R_2} \end{aligned}$$

In a similar way, by considering R_3 and R_4, we see that

$$\begin{aligned} V_{R_3} &= I_2 R_3 \\ V_{R_4} &= I_2 R_4 \\ V &= V_{R_3} + V_{R_4} \end{aligned}$$

By a suitable choice of resistor values, we can arrange that $V_{R_1} = V_{R_3}$ and $V_{R_2} = V_{R_4}$. For this condition, we have

$$I_1R_1 = I_2R_3$$
$$\text{so } \frac{I_2}{I_1} = \frac{R_1}{R_3}$$
$$\text{and } I_1R_2 = I_2R_4$$
$$\text{so } \frac{I_2}{I_1} = \frac{R_2}{R_4}$$
$$\therefore \frac{R_1}{R_3} = \frac{R_2}{R_4}$$

In this situation, points P and Q in Figure 2.8 are at the same potential, so V_{PQ} is zero. A voltmeter connected across these points would read zero potential - it would show null deflection. In this condition, the Wheatstone bridge circuit is said to be balanced.

The balanced Wheatstone bridge is used to measure the value of an unknown resistor (say R_1) as follows. Two of the resistors (say R_3 and R_4) are fixed-value resistors that have known resistances. Usually these values are equal or close to each other. The final resistor (R_2) is a variable resistor, often a resistance box, where the value of the resistance can be altered, but is known. When the bridge circuit is balanced, and this condition is recognised by a sensitive centre-zero galvanometer connected across P and Q showing null deflection, the value of the unknown resistor R_1 is calculated from

$$R_1 = R_3\left(\frac{R_2}{R_4}\right) \tag{2.5}$$

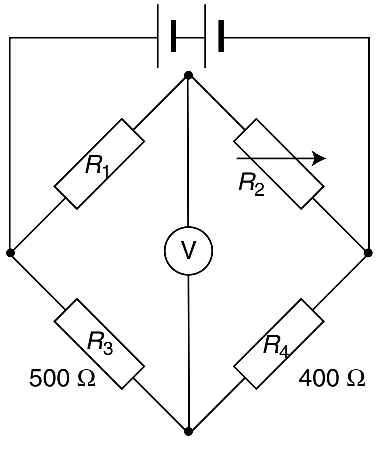
....

The benefit of this method of measuring resistance is that the internal resistance of the meter used does not affect the reading, since the meter is read when the potential difference across it is zero, and hence there is no current in the meter branch when the bridge circuit is balanced. An accurate meter is not needed, only a sensitive one with a correctly marked zero.

Example

The Wheatstone bridge circuit shown in Figure 2.9 is balanced when R_2 is set to 384 Ω.

Figure 2.9: Wheatstone bridge



....

Calculate the resistance of R_1.

The normal way of representing a Wheatstone bridge circuit is shown in Figure 2.9. You should be able to see that electrically this is similar to Figure 2.8.

At balance:

$$R_1 = R_2 \left(\frac{R_3}{R_4}\right)$$
$$\therefore R_1 = 384 \left(\frac{500}{400}\right)$$
$$\therefore R_1 = 480\ \Omega$$

....

2.4.2 The out-of-balance Wheatstone bridge

The Wheatstone bridge circuit is frequently used in the out-of-balance condition. In these applications, the Wheatstone bridge is initially balanced, with at least one of the four arms consisting of a component that has a resistance that changes as a physical quantity changes. Devices such as a thermistor (resistance changes as temperature changes), a light dependent resistor - LDR (resistance changes as light intensity changes) and a strain gauge (resistance changes as it is stretched) are all suitable.

It is found that the out-of-balance potential difference, and so also the reading on the galvanometer, is directly proportional to the change of resistance of one arm, as long as the change is small. By 'small' we usually mean about 5% of the value of the resistance at balance. The potential difference produced in most practical applications is very small, of the order of a few microvolts in some cases. This small potential difference is used as an input to an amplifier circuit. This allows the physical changes that cause the change in resistance, and hence the potential difference, to be monitored and perhaps acted upon.

Cracks in the brickwork

30 min

This is an on-line activity showing how the out-of balance Wheatstone bridge circuit is used to monitor an active crack in a building.

The out-of-balance potential difference across an initially-balanced Wheatstone bridge circuit is proportional to the change in resistance in one arm.

Quiz 3 The Wheatstone bridge

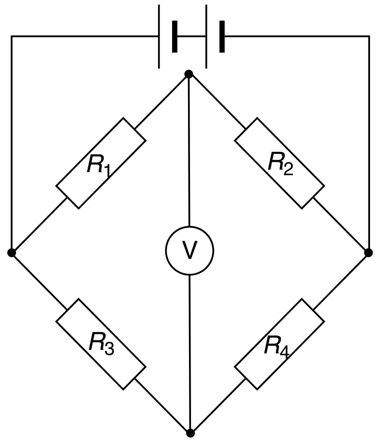
15 min

Multiple choice quiz.

First try the questions. If you get a question wrong or do not understand a question, there are 'Hints' at the back of the book . The hints are in the same order as the questions. If you read the hint and still do not understand then ask your tutor.

The Wheatstone bridge circuit shown in Figure 2.10 is used in the first three questions in this quiz.

Figure 2.10: Wheatstone bridge circuit



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Q11: The Wheatstone bridge circuit shown in Figure 2.10 is balanced.
What is the relationship between the resistances?

a) $R_1 + R_2 = R_3 + R_4$
b) $R_1 - R_2 = R_3 - R_4$
c) $R_1 \times R_2 = R_3 \times R_4$
d) $\frac{R_1}{R_2} = \frac{R_3}{R_4}$
e) $R_1 = \frac{R_2}{R_3 \times R_4}$

....

Q12: The Wheatstone bridge circuit shown in Figure 2.10 is balanced when R_2 = 60 Ω, R_3 = 30 Ω and R_4 = 90 Ω.
What is the value of R_1?

a) 20 Ω
b) 30 Ω
c) 60 Ω
d) 90 Ω
e) 180 Ω

....

Q13: The Wheatstone bridge circuit shown in Figure 2.10 is balanced when R_1 = 600 Ω and R_2 = 400 Ω.
The resistances of R_3 and R_4 could be

a) R_3 80 Ω; R_4 120 Ω
b) R_3 120 Ω; R_4 80 Ω
c) R_3 400 Ω; R_4 400 Ω
d) R_3 400 Ω; R_4 600 Ω
e) R_3 600 Ω; R_4 600 Ω

. . . .

Q14: The resistance R in one arm of an initially-balanced Wheatstone bridge circuit is altered by a small amount ΔR.
It is found that the out-of-balance potential difference is

a) proportional to R
b) proportional to R^2
c) proportional to ΔR
d) proportional to $(\Delta R)^2$
e) unchanged

. . . .

Q15: A Wheatstone bridge circuit is initially balanced with R_1 = R_2 = R_3 = R_4 = 1000 Ω.
When R_1 is altered to 1010 Ω, the out-of-balance potential difference is 50 mV.
What is the out-of-balance potential difference when R_1 is 1005 Ω?

a) 5 mV
b) 10 mV
c) 25 mV
d) 50 mV
e) 1000 mV

. . . .

. . . .

2.5 Summary

By the end of this Topic you should be able to:

- explain what is meant by the e.m.f. of a source;
- describe an electrical source in terms of e.m.f. and internal resistance;
- carry out calculations using the relationships involving the e.m.f., the terminal p.d. and the internal resistance of a source;
- explain what is meant by conservation of energy and of charge;
- derive and use the expression for the total resistance of any number of resistors in series by considering conservation of energy and charge in a closed circuit;
- derive and use the expression for the total resistance of any number of resistors in parallel by considering conservation of charge in a closed circuit;

- describe what a Wheatstone bridge circuit is;
- state the relationship between the resistors in a balanced Wheatstone bridge circuit;
- carry out calculations involving the resistances in a balanced Wheatstone bridge circuit;
- state the relationship between the out-of-balance p.d. and the change of resistance as the resistance of one resistor in an initially balanced Wheatstone bridge circuit is varied by a small amount.

2.6 Assessment

Online assessments

Three online tests are available. Each test should take you no more than 20 minutes to complete. Test one contains questions on e.m.f. and internal resistance. The questions in test two cover resistors in series and in parallel and the Wheatstone bridge. Test three contains questions taken from all parts of the Topic.

Topic 3

Alternating current and voltage

Contents

3.1 Introduction

The electricity supply from a battery is **d.c.** This means that when the battery is being used, it supplies a constant e.m.f. so the current is always in the same direction - d.c. stands for 'direct current'.

The electricity supply to our homes, schools and factories from the National Grid is an **a.c.** supply. This means that the current from the supply constantly changes direction - a.c. stands for 'alternating current'. In Great Britain, the voltage of the supply is described as '230 V, 50 Hz'. In other countries the values may be different, but virtually all countries use a.c. for their public electricity supply. There are two main reasons why a.c. is used. Firstly that is the form of electricity generated by commercial generators. Secondly transformers only work on a.c. supplies and transformers are essential to step voltages up and down for the transmission of electrical energy. Less energy is dissipated as heat in the cables when a high voltage and so a smaller current is used to transmit electrical energy.

In this Topic we will look at what we mean by the frequency of an a.c. supply, and how we can measure it. We will also consider what is meant by the voltage of an a.c. supply, and how we can attach a number (e.g. 230) to what is a constantly changing value. Finally, we will look at resistors in a.c. circuits.

3.2 The frequency of an a.c. supply

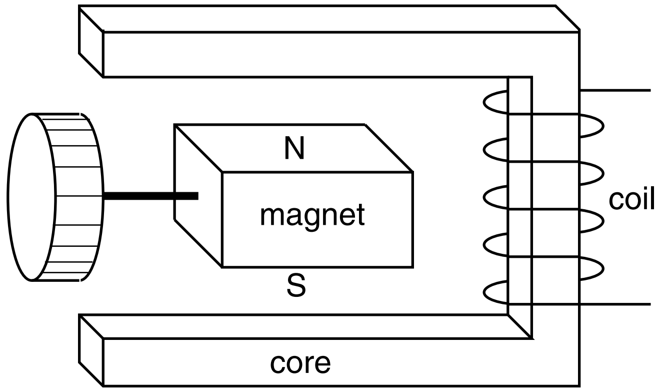
Learning Objective

To describe what is meant by the frequency of an a.c. supply.
To describe how to measure the frequency of a low-voltage a.c. supply using an oscilloscope.
To relate the period of a waveform to its frequency.

3.2.1 a.c. waveforms

When a coil of wire is rotated at a constant rate in a magnetic field (as in an a.c. generator - an '**alternator**') or when a magnet is rotated at a constant rate near to a coil of wire (as in Figure 3.1), then a constant e.m.f. is induced in the coil.

Figure 3.1: A bicycle dynamo



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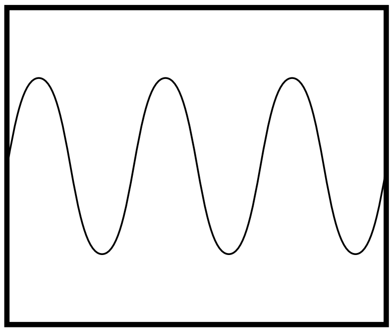
The waveform of the e.m.f. generated is shown in Figure 3.2.

Figure 3.2: e.m.f. generated by a bicycle dynamo



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It can be seen from this waveform that the e.m.f. generated is constantly changing in amplitude - it is an a.c. waveform. This e.m.f. is a sine wave - the most common form of a.c. supply. For the rest of this Topic, we will only consider waveforms that are sine waves, but remember the term 'a.c.' applies to all changing waveforms, not just sine waves.

If the magnet of the bicycle dynamo is made to rotate at a faster rate, then the amplitude of the e.m.f. generated increases. This is because the rate at which the magnetic field lines cut the coil is increased. There is another change that happens when the magnet rotates faster - the **frequency** of the a.c. waveform generated increases. In Figure 3.2, there are three complete cycles shown. This corresponds to three complete revolutions of the magnet in the dynamo. If these three revolutions take one second, we say that the frequency *f* of the a.c. generated is 3 cycles per second, or 3 hertz (Hz).

If there are three waves made per second in Figure 3.2 then it follows that it takes $^1/_3$ of a second to make one complete wave. The **period** *T* of the wave is $^1/_3$ s.

$$T = \frac{1}{f} \tag{3.1}$$

....

3.2.2 Measuring the frequency of an alternating supply

The frequency of an alternating supply can be measured using an oscilloscope. There are two ways to do this. If a low-voltage supply of known frequency is available, then an uncalibrated oscilloscope can be used. The more usual way to measure frequency is to use a calibrated oscilloscope, and calculate the frequency from the settings of the controls of the oscilloscope.

With the first method, the known-frequency supply is connected to the oscilloscope, and the time-base control of the oscilloscope is adjusted to give a suitable trace on the screen. The most suitable trace would be one where there is one complete wave seen on the screen. *Without adjusting the time-base setting of the oscilloscope*, the known-frequency supply is removed and replaced by the supply with the unknown frequency. This unknown frequency can be calculated by multiplying the number of waves now seen on the screen by the known frequency. This method is often known as the 'method of substitution'.

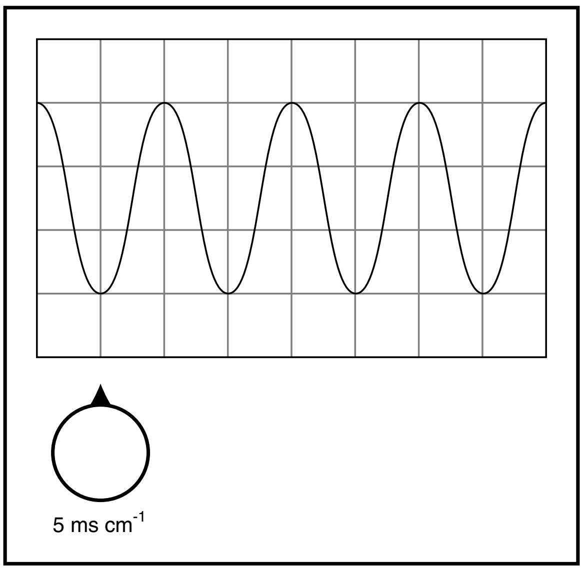
The more direct method of measuring the frequency of a low-voltage a.c. supply is to use the time-base setting of a calibrated oscilloscope. The time-base of an oscilloscope is usually calibrated in seconds (or milli- or microseconds) per centimetre. This tells us how long it takes the spot to travel one centimetre across the screen, from left to right.

To use a calibrated oscilloscope, the supply is connected to the input of the oscilloscope and the time-base adjusted to give a suitable number of waves on the screen. Using the time-base setting, along with the number of waves on the screen and the width of the screen, the time for one wave to be made is calculated. The frequency can then be calculated by using Equation 3.1.

Example

A low-voltage a.c. supply is connected to an oscilloscope and four complete waves are produced when the time-base of the oscilloscope is set to 5 ms cm^{-1}, as shown in Figure 3.3. Each square on the oscilloscope screen has a 1 cm side.

Figure 3.3: a.c. waveform seen on oscilloscope screen.



. . . .

Calculate the frequency of the supply.

4 waves take up 8 cm across the screen
so 1 wave takes up 2 cm across the screen
The time-base is set to 5 ms cm^{-1}
so 1 wave is made in 5 $\times$ 2 ms = 10 ms

$$T = \frac{1}{f}$$
$$\text{so } f = \frac{1}{T}$$
$$\therefore f = \frac{1}{10 \times 10^{-3}}$$
$$\therefore f = 100 \text{ Hz}$$

. . . .

Measuring frequency using an oscilloscope

20 min

In this on-line activity, you have to calculate the frequency of the a.c. voltage obtained from a signal generator. To do this, you need to read the time-base setting of the oscilloscope and use the relationship between the period of a wave and its frequency.

Full instructions are given in the simulation.

By using the time-base setting on an oscilloscope, and applying the relationship $T = {}^{1}/_{f}$, the frequency of a low-voltage a.c. supply can be calculated.

Quiz 1 Frequency of a.c.

15 min

Multiple choice quiz.

First try the questions. If you get a question wrong or do not understand a question, there are 'Hints' at the the back of the book . The hints are in the same order as the questions. If you read the hint and still do not understand then ask your tutor.

Q1: Why is a.c. used for all mains electricity supplies?
(i) Transformers only work on a.c.
(ii) a.c. electricity can be stored but d.c. electricity cannot.
(iii) Commercial generators generate a.c.

a) (i) only
b) (ii) only
c) (iii) only
d) (i) and (ii) only
e) (i) and (iii) only

. . . .

Q2: What is the correct relationship between the period (*T*) and the frequency (*f*) of a wave?

a) $T = f$
b) $T = \frac{1}{f}$
c) $T = \frac{1}{f^2}$
d) $T = f^2$
e) $T = \sqrt{f}$

. . . .

Q3: A signal generator produces an alternating voltage of frequency 50 Hz.
What is the period of the wave?

a) 0.4 ms
b) 2.5 ms
c) 7.1 ms
d) 20 ms
e) 50 ms

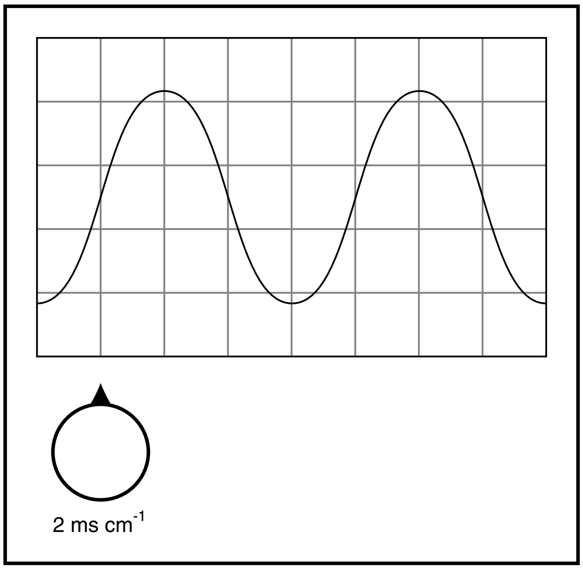
. . . .

Q4: A wave with a period of 25 ms is generated.
What is the frequency of this wave?

a) 5.0 Hz
b) 6.3 Hz
c) 25 Hz
d) 40 Hz
e) 625 Hz

. . . .

Q5: An a.c. supply was connected to an oscilloscope and the following trace was obtained when the time-base of the oscilloscope was set to 2 ms cm^{-1}.



Each square on the screen has a 1 cm side.
What is the frequency of the supply?

a) 125 Hz
b) 160 Hz
c) 250 Hz
d) 500 Hz
e) 1000 Hz

. . . .

3.3 Peak and r.m.s. values of voltage and current

Learning Objective

To state what the abbreviation r.m.s. stands for and to explain what is meant by an r.m.s. value.
To state the relationship between peak and r.m.s. values for a sinusoidally-varying voltage and current.
To describe an experiment using an oscilloscope to measure voltage across lamps with d.c. and a.c. sources to compare peak and r.m.s. values.
To carry out calculations involving peak and r.m.s. values of voltage and current.

We have just shown that an alternating voltage is constantly changing so how can we describe a supply of electricity as, for example, a 230 V a.c. supply? Since the volt is defined as one joule per coulomb, we use this to define what we mean by the value of an a.c. voltage. We would expect a 12 V car headlamp to produce the same quantity of light (and heat) whether it is operated from a 12 V d.c. car battery or from a 12 V a.c. supply obtained from a transformer connected to the mains supply. In other words, we would expect a 12 V supply to transform 12 joules of energy for every coulomb of charge that flows through the headlamp irrespective of whether it is a d.c. or an a.c. supply.

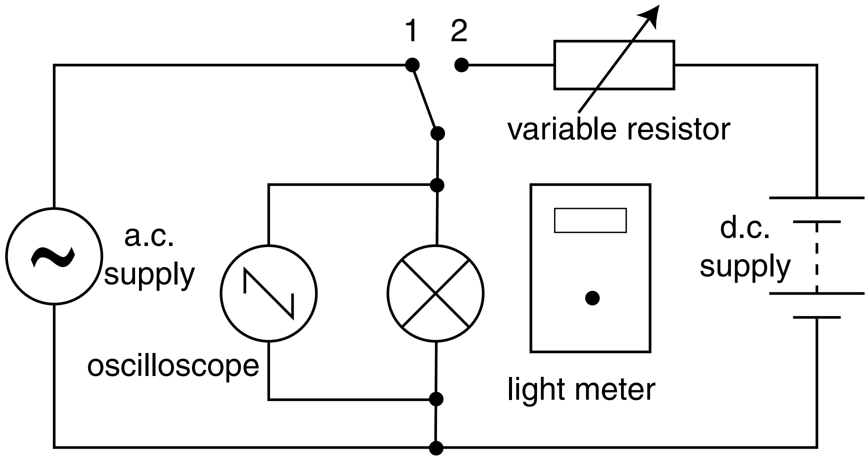
We compare the steady value of a d.c. voltage with the 'effective voltage' of an a.c. supply that transforms the same energy in a resistor. The average of a sine wave over any whole number of cycles is zero. We use a value called the r.m.s. voltage (r.m.s. is an abbreviation for 'root mean square') for this comparison. This is because, as you will see later in this Topic, the energy transformed in a resistor depends on the square of the voltage.

12 V d.c. = 12 V r.m.s. a.c.

3.3.1 Comparing the energy transformed by a.c. and d.c. supplies

The apparatus shown in Figure 3.4 allows us to compare the energy transformed in a lamp by a d.c. supply with the energy transformed in the same lamp by an a.c. supply.

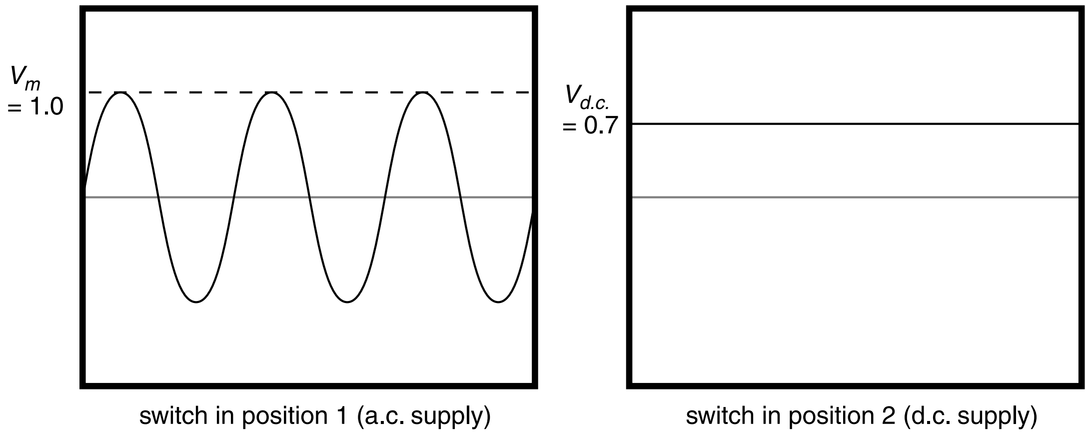
Figure 3.4: Apparatus to compare a.c. and d.c. supplies



. . . .

With the switch in position 1, the reading on the light meter (placed beside the lamp), is noted. *Without changing the relative positions of the lamp and the light meter*, the switch is changed to position 2. The variable resistor is adjusted until the lamp is as bright as before, using the reading on the light meter to check this. The traces seen on the oscilloscope screen when the lamp is lit equally brightly (and therefore transforming the same amount of energy) are shown in Figure 3.5.

Figure 3.5: Comparison of a.c.and d.c. supplies



. . . .

We can see that the same amount of energy is transformed in the lamp by a d.c. supply that has a value approximately 0.7 times the peak value of an a.c. supply. This means that the effective or r.m.s. value of an alternating voltage ($V_{r.m.s.}$) is approximately 0.7 times the peak value. (There is no commonly accepted symbol for peak voltage. You may see it written as V_{peak}, V_p, $V_{max.}$ or V_m. In this Topic, we have used V_m.) The optional activity shows the exact relationship, which is:

$$V_{r.m.s.} = \frac{V_m}{\sqrt{2}}$$
$$V_m = \sqrt{2} \times V_{r.m.s.} \tag{3.2}$$

. . . .

Since the current through a resistor is proportional to the potential difference across it, the relationship in Equation 3.2 also holds for current.

$$I_{r.m.s.} = \frac{I_m}{\sqrt{2}}$$
$$I_m = \sqrt{2} \times I_{r.m.s.} \qquad (3.3)$$

. . . .

Examples

1.

(a) Calculate the peak voltage of the 230 V a.c. mains supply.

(b) Remembering that the peak value of an alternating waveform is half the peak-to-peak value, calculate the voltage swing of the 230 V a.c.mains supply.

(a)

$$V_{r.m.s.} = \frac{V_m}{\sqrt{2}}$$
$$\therefore V_m = \sqrt{2} \times V_{r.m.s.}$$
$$\therefore V_m = \sqrt{2} \times 230$$
$$\therefore V_m = 325 \text{ V}$$

(b)

$$V_{peak-to-peak} = 2 \times V_m$$
$$\therefore V_{peak-to-peak} = 2 \times 325$$
$$\therefore V_{peak-to-peak} = 650 \text{ V}$$

So the voltage at the live wire of a 230 V a.c. mains supply swings through 650 V.

. . . .

2.

Calculate the peak value of an alternating current in a 0.25 W, 68 Ω resistor so that it is not overloaded.

$$P = I_{r.m.s.}^2 R$$
$$\therefore I_{r.m.s.} = \sqrt{\frac{P}{R}}$$
$$\therefore I_{r.m.s.} = \sqrt{\frac{0.25}{68}}$$
$$\therefore I_{r.m.s.} = 61 \text{ mA}$$

$$I_m = \sqrt{2} \times I_{r.m.s.}$$
$$\therefore I_m = \sqrt{2} \times 61$$
$$\therefore I_m = 86 \text{ mA}$$

. . . .

15 min

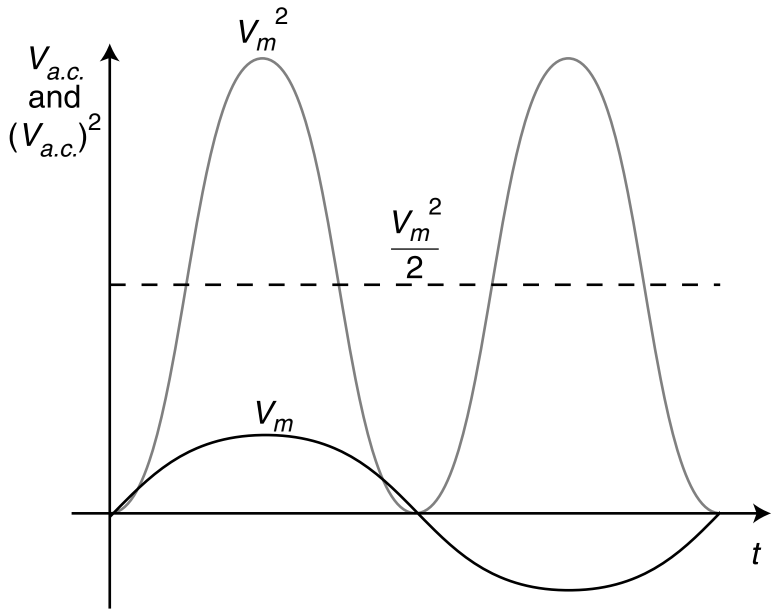
r.m.s. values of voltage and current

This optional activity takes you through the mathematics that relate the r.m.s. value of a sinusoidally-varying voltage to the peak value.

This is a reading exercise. It may be helpful, once you have read it through a couple of times, to sketch the graph and write down the main points of the mathematics without looking at the notes.

To compare a steady voltage ($V_{d.c.}$) with a sinusoidally-varying alternating voltage ($V_{a.c.}$) that transforms energy at the same rate in a resistor, consider Figure 3.6.

Figure 3.6: Alternating voltage over one complete cycle



....

When the a.c. voltage graph is squared, it is seen to be a sine graph of twice the frequency. Taken over a whole number of complete waves, the mean or average of the (voltage)^2 graph is given by the line through the centre of the graph, with a value of $V_m^2/2$. This is known as the *mean square voltage*.

If this alternating voltage is applied across a resistor, then energy is transferred at an average rate given by:

$$power = \frac{\frac{1}{2}V_m^2}{R}$$

A steady voltage of $V_{d.c.}$ applied across the same resistor would transfer energy at an average rate given by:

$$power = \frac{V_{d.c.}^2}{R}$$

so we have:

$$\frac{V_{d.c.}^2}{R} = \frac{\frac{1}{2}V_m^2}{R}$$
$$\therefore V_{d.c.}^2 = \frac{1}{2}V_m^2$$
$$\therefore V_{d.c.} = \frac{V_m}{\sqrt{2}}$$

The steady voltage ($V_{d.c.}$) that transfers energy at the same rate is the *root mean square* or r.m.s. voltage ($V_{r.m.s.}$).

$$V_{r.m.s.} = \frac{V_m}{\sqrt{2}} \approx 0.7 \times V_m \quad (3.4)$$

....

Similarly:

$$I_{r.m.s.} = \frac{I_m}{\sqrt{2}} \approx 0.7 \times I_m \quad (3.5)$$

....

The r.m.s. and peak values of voltage and current are given by the relationships

r.m.s. value = 0.7 x peak value

....

3.4 Resistors in a.c. circuits

Learning Objective

To describe what is meant by impedance.
To state the relationship between impedance, r.m.s. potential difference and r.m.s. current in a resistive a.c. circuit.
To state the relationship between r.m.s. current and frequency in a resistive a.c. circuit.

The resistance of a resistor is defined as the ratio of the potential difference across it to the current through it. This relationship holds whether the circuit has a d.c. or an a.c. supply.

With a d.c. supply, it is only resistors that offer any opposition to the steady flow of charges. However, with an a.c. supply, there are other components that also offer an opposition to the steady flow of charges. For this reason, the opposition to current in an a.c. circuit is called **impedance**. The impedance of a resistor is the same as its resistance, and is also measured in Ω.

$$impedance = \frac{r.m.s.\ potential\ difference}{r.m.s.\ current}$$

The ratio ${}^{V_{r.m.s.}}/_{I_{r.m.s.}}$ remains constant for a resistor no matter what the frequency of the a.c. supply is, so the alternating current in a resistor does not change with frequency. There are other components that are used in circuits for which this is not the case. You will meet these components, capacitors and inductors, in other Topics or Courses.

20 min

The resistor and a.c.

There are two parts to this on-line activity. In the first part, you have to deduce the relationship between r.m.s. potential difference, r.m.s. current and resistance. In the second part, you have to observe the effect on the alternating current through a resistor as the frequency of the supply is altered.

Full instructions are given in the simulation.

Make sure you drag the resistor into the circuit above the voltmeter at the top of the circuit.

The resistance of a resistor to a.c. (its impedance) is given by ${}^{V_{r.m.s.}}/_{I_{r.m.s.}}$.
The alternating current in a resistor is independent of frequency.

---

15 min

Quiz 2 Voltage and frequency

Multiple choice quiz.

First try the questions. If you get a question wrong or do not understand a question, there are 'Hints' at the the back of the book . The hints are in the same order as the questions. If you read the hint and still do not understand then ask your tutor.

Q6: What is the correct relationship between the peak voltage (V_m) and the r.m.s. voltage ($V_{r.m.s.}$) of an a.c. source obtained from the mains?

a) $V_m = 2V_{r.m.s.}$
b) $V_m = \sqrt{2} \times V_{r.m.s.}$
c) $V_m = V_{r.m.s.}$
d) $V_m = \frac{V_{r.m.s.}}{2}$
e) $V_m = \frac{V_{r.m.s.}}{\sqrt{2}}$

....

Q7: The r.m.s. voltage of an a.c. supply is 10 V.
What is the peak voltage?

a) 5.0 V
b) 7.1 V
c) 10 V
d) 14 V

e) 20 V

. . . .

Q8: The insulation of a certain capacitor breaks down when a voltage greater than 16 V d.c. is applied across it.
What is the greatest r.m.s. voltage that can appear across this capacitor when it is connected into an a.c. circuit?

a) 8.0 V
b) 11 V
c) 16 V
d) 23 V
e) 32 V

. . . .

Q9: The peak value of an alternating current in a lamp of resistance 556 Ω is 0.6 A.
What is the power of the lamp?

a) 100 W
b) 200 W
c) 337 W
d) 400 W
e) 800 W

. . . .

Q10: A resistor of resistance 27 Ω is connected across a variable-frequency, fixed-voltage supply. With the frequency of the supply set at 40 Hz, the current in the resistor is recorded as 130 mA.
What will the current be when the frequency of the supply is reduced to 20 Hz?

a) 20 mA
b) 27 mA
c) 40 mA
d) 64 mA
e) 130 mA

. . . .

3.5 Summary

By the end of this Topic you should be able to:

- describe what is meant by the frequency of an a.c. supply;
- describe how to measure the frequency of a low-voltage a.c. supply using an oscilloscope;
- relate the period of a waveform to its frequency;

- state what the abbreviation r.m.s. stands for and explain what is meant by an r.m.s. value;
- state the relationship between peak and r.m.s. values for a sinusoidally-varying voltage and current;
- describe an experiment using an oscilloscope to measure voltage across lamps with d.c. and a.c. sources in order to compare peak and r.m.s. values;
- carry out calculations involving peak and r.m.s. values of voltage and current;
- describe what is meant by impedance;
- state the relationship between impedance, r.m.s. potential difference and r.m.s. current in a resistive a.c. circuit;
- state the relationship between r.m.s. current and frequency in a resistive a.c. circuit.

3.6 Assessment

Online assessments

Two online tests are available. Each test should take you no more than 20 minutes to complete. Both tests contain questions taken from all parts of the Topic.

Topic 4

Capacitance

Contents

4.1 Introduction

In this Topic we will be studying the capacitor, which is a device for storing electrical energy. A capacitor consists of two conducting plates. When one plate is negatively charged and the other is positively charged, then electrical energy is stored on the capacitor. We will be looking at how this process works, and how much energy can be stored on a capacitor.

Capacitors are important components in many electrical circuits. We will study how capacitive circuits respond to d.c. and a.c. signals. Towards the end of the Topic, we will be looking at some of the practical applications of capacitors.

4.2 Charge and capacitance

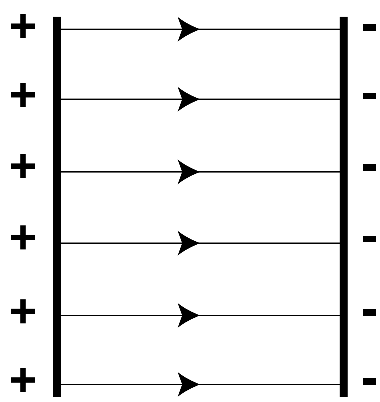
Learning Objective

To define what is meant by capacitance.
To investigate the relationship between charge, potential difference and capacitance.
To calculate the energy stored on a charged capacitor.

4.2.1 Capacitance

In the "Electric fields" Topic, we looked at the field that exists between two oppositely charged plates. The field lines are shown in Figure 4.1. Remember, the arrows on the field lines show the direction of the force that would be exerted on a positively charged particle placed in the field.

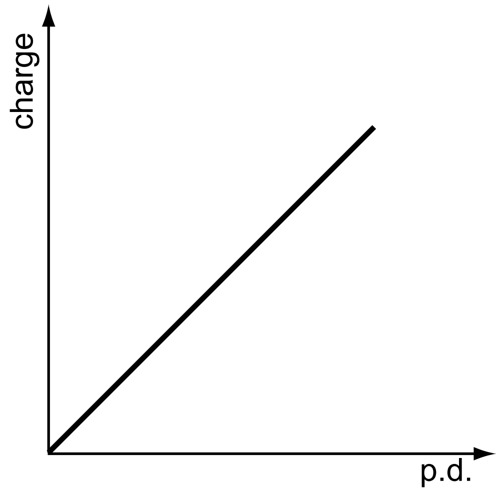
Figure 4.1: Electric field between two charged plates



. . . .

We are going to be considering the case of two parallel conducting plates, which have a fixed separation between them. Since there is an electric field between the plates, then there is a potential difference *V* across them. Investigating the relationship between *V* and the amount of charge *Q* stored on the plates will show that a plot of p.d. against charge gives a straight line graph through the origin, as in Figure 4.2.

Figure 4.2: Plot of potential difference against charge



. . . .

Since there is a linear relationship between charge (Q) and p.d. (V), we can state that

$$Q \propto V \tag{4.1}$$

. . . .

Rather than leave the relationship Equation 4.1 with a "proportional to" sign, we can put in a constant of proportionality *C* (equal to the gradient of the *Q*-*V* plot), so that Equation 4.1 becomes

$$\begin{aligned} Q &= CV \\ \text{or} \quad C &= \frac{Q}{V} \end{aligned} \tag{4.2}$$

. . . .

C is called the **capacitance**. The unit of capacitance is the farad F, where 1F = 1 C V^{-1}. The farad is named after the English physicist Michael Faraday (1791 - 1867). Equation 4.2 shows that capacitance is the ratio of charge to p.d.

A system of two parallel conductors is called a **capacitor**. In fact, any two conductors that are insulated from each other form a capacitor, but in this course we will only be studying the parallel-plate capacitor. For any parallel-plate capacitor, its capacitance *C* depends on the surface area of the plates, the distance between the plates and the insulating material (air or another insulator) that separates the plates.

It is worth pointing out that 1 F is a very large capacitance, and we will never encounter such a huge capacitance in practice. Practical capacitors have capacitances in the microfarad (1μF = 1×10^{-6} F), nanofarad (1nF = 1×10^{-9} F) or picofarad (1pF = 1×10^{-12} F) regions.

Example

A 50 μF capacitor is charged to 0.40 mC. Calculate the potential difference between the plates.

To answer this question, we will use Equation 4.2, remembering to convert the capacitance into farads, and the charge into coulombs.

$$Q = CV$$
$$\therefore V = \frac{Q}{C}$$
$$\therefore V = \frac{0.40 \times 10^{-3}}{50 \times 10^{-6}}$$
$$\therefore V = 8.0\,\text{V}$$

....

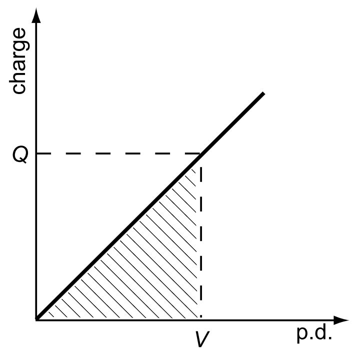
4.2.2 Energy stored in a capacitor

Let us consider the charged parallel-plate capacitor shown in Figure 4.1. Suppose we take an electron from the left-hand plate and transfer it to the right-hand plate. We have to do work in moving the electron since the electrical force acting on it opposes this motion. The more charge that is stored on the plates, the more difficult it will be to move the electron since the electric field between the plates will be larger.

The work that is done in placing charge on the plates of a capacitor is stored as potential energy in the charged capacitor. The more charge that is stored on the capacitor, the greater the stored potential energy.

Figure 4.3 shows the plot of charge against potential difference that we saw in the previous section.

Figure 4.3: Plot of charge against p.d. for a capacitor



....

It can be shown that the area under the graph (between the plotted line and the *p.d.*-axis) is equal to the work done in charging the capacitor. Since the graph is a straight line, the area between the line and the *p.d.*-axis forms a right-angled triangle. The area of a right-angled triangle = $\frac{1}{2} \times base \times height$, so for a capacitor with charge *Q* and p.d. *V*, the work done is

$$work\ done = \tfrac{1}{2} \times Q \times V$$

As we have seen, this work done is equal to the energy stored on the capacitor. Using Equation 4.2, we can state three equivalent expressions for the energy *W* stored on a capacitor:

$$\begin{aligned} W &= \tfrac{1}{2}QV \\ W &= \tfrac{1}{2}CV^2 \\ W &= \tfrac{1}{2}\frac{Q^2}{C} \end{aligned} \qquad (4.3)$$

. . . .

Energy stored on a capacitor

Paper-based exercise.

10 min

Equation 4.3 shows three expressions for the energy stored on a charged capacitor. Starting from the equation $W = \frac{1}{2}QV$, and using Equation 4.2, can you show that these three expressions are equivalent?

The three expressions in Equation 4.3 for the energy stored on a capacitor are equivalent.

Example

Let us return to the 50 μF capacitor charged to 0.40 mC. How much energy is stored on the capacitor?

We will choose the appropriate expression from Equation 4.3:

$$\begin{aligned} W &= \frac{1}{2}\frac{Q^2}{C} \\ \therefore W &= \frac{1}{2} \times \frac{\left(0.40 \times 10^{-3}\right)^2}{50 \times 10^{-6}} \\ \therefore W &= \frac{1}{2} \times \frac{1.6 \times 10^{-7}}{50 \times 10^{-6}} \\ \therefore W &= 1.6 \times 10^{-3} \text{ J} \end{aligned}$$

. . . .

Extra Help: Using the energy relationships E = QV and E = 1/2QV

Quiz 1 Capacitors

15 min

Multiple choice quiz.

First try the questions. If you get a question wrong or do not understand a question, there are 'Hints' at the the back of the book . The hints are in the same order as the questions. If you read the hint and still do not understand then ask your tutor.

Q1: 1 F is equivalent to

a) 1 C V^{-1}
b) 1 J C^{-1}
c) 1 C V
d) 1 V C^{-1}
e) 1 C J^{-1}

. . . .

Q2: The potential difference across a 500 nF capacitor is 12 V. How much charge is stored on the capacitor?

a) 3.0 pC
b) 24 pC
c) 42 nC
d) 6.0 μC
e) 72 μC

. . . .

Q3: When the charge stored on a certain capacitor is 8.0 $\times 10^{-8}$ C, the p.d. across it is 3.6 V. What is the capacitance of this capacitor?

a) 4.5 nF
b) 6.2 nF
c) 22 nF
d) 290 nF
e) 1.0 μF

. . . .

Q4: How much work is done in charging a 40 μF capacitor to 6.4 $\times 10^{-4}$ C?

a) 1.25 μJ
b) 5.1 mJ
c) 31 mJ
d) 8.0 J
e) 130 J

. . . .

Q5: When the charge on a capacitor is 1.4 μC, the potential difference across the capacitor is 0.45 V. What is the p.d. across the capacitor when the charge on it is 5.6 μC?

a) 0.11 V
b) 1.8 V
c) 1.9 V
d) 3.5 V
e) 16 V

. . . .

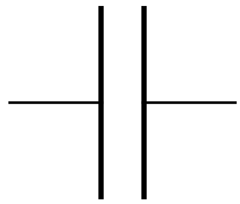
4.3 Capacitors in circuits

Learning Objective

To investigate the behaviour of capacitors in d.c. and a.c. circuits.

In this section of the Topic, we will be looking at how capacitors behave when they are connected as components in d.c. and a.c. circuits.

Figure 4.4: Capacitor circuit symbol



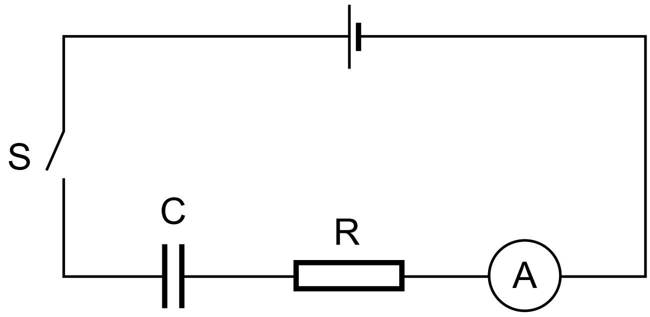
The circuit symbol for a capacitor is shown in Figure 4.4.

A capacitor is effectively a break in the circuit, and charge cannot flow across it. We will see now how this influences the current in capacitive circuits.

4.3.1 Charging a capacitor

Figure 4.5 shows a simple d.c. circuit in which a capacitor is connected in series to a battery and resistor. This is often called a series *CR* circuit.

Figure 4.5: Simple d.c. capacitive circuit



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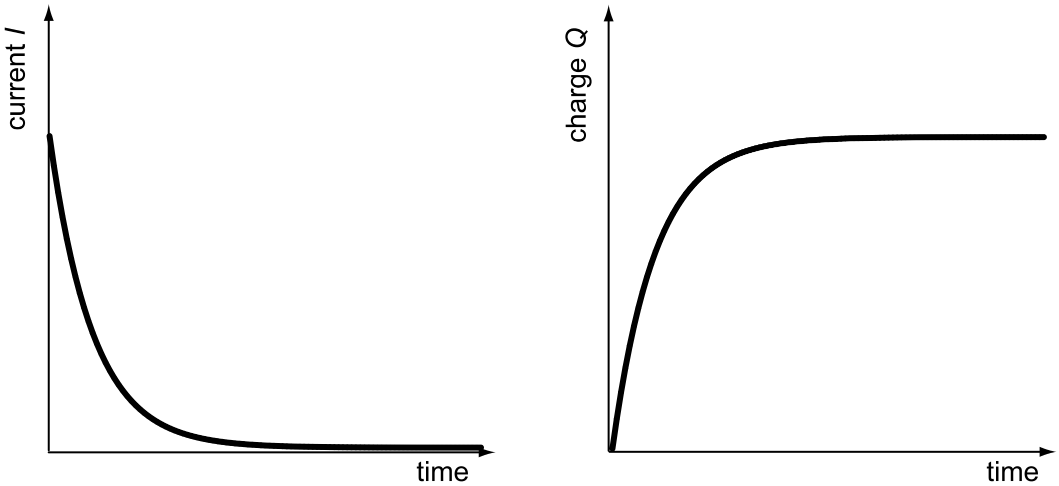
When the switch S is closed, charge can flow on to (but not across) the capacitor C. At the instant the switch is closed the capacitor is uncharged, and it requires little work to add charges to the capacitor. As we have already discussed, though, once the capacitor has some charge stored on it, it takes more work to add further charges. Figure 4.6 shows graphs of current *I* through the capacitor (measured on the ammeter) and charge *Q* on the capacitor, against time.

Figure 4.6: Plots of current and charge against time for a charging capacitor



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Since the potential difference across a capacitor is proportional to the charge on it, then a plot of p.d. against time will have the same shape as the plot of charge against time shown in Figure 4.6.

Suppose the battery in Figure 4.5 has e.m.f *E* and negligible internal resistance. The sum of the p.d.s across C and R must be equal to *E* at all times. That is to say,

$$V_C + V_R = E$$

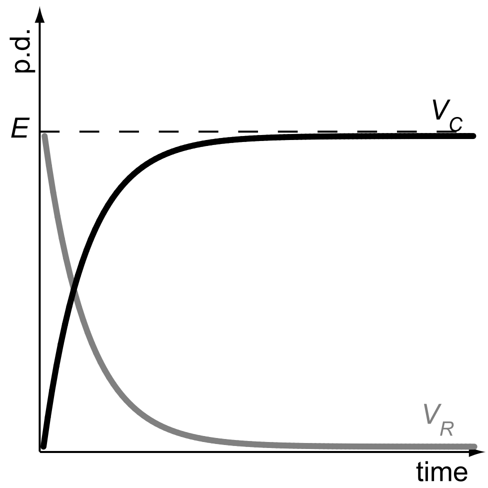
where V_C is the p.d. across the capacitor and V_R is the p.d. across the resistor. At the instant switch S is closed there is no charge stored on the capacitor, so V_C is zero, hence V_R = E. The current in the circuit at the instant the switch is closed is given by

$$I = \frac{E}{R} \tag{4.4}$$

....

As charge builds up on the capacitor, so V_C increases and V_R decreases. This is shown in Figure 4.7.

Figure 4.7: Plots of p.d. against time for a capacitive circuit



....

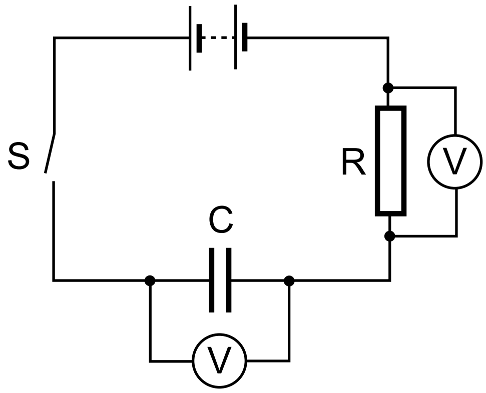
The charge and potential difference across the capacitor follow an exponential rise. (The current follows an exponential decay). The rise time (the time taken for the capacitor to become fully charged) depends on the values of the capacitance C and the resistance R. The relationship between rise time, C and R is quite complex, but it is enough for us to be able to state that the rise time increases if either C or R increases. So, for example, replacing the resistor R in the circuit in Figure 4.5 by a resistor with a greater resistance will result in the p.d. across the capacitor C rising more slowly, and the current in the circuit dropping more slowly.

How long is the rise time? For a circuit with a 1.0 μF capacitor connected in series to a 100 Ω resistor, the capacitor will charge to half its full value in around 70 μs. It will be approximately fully charged after around 0.5 ms. (You do not need to know how to calculate these times for this Course.)

Example

Consider the circuit in Figure 4.8, in which a 40 kΩ resistor and an uncharged 220 μF capacitor are connected in series to a 12 V battery of negligible internal resistance.

Figure 4.8: Capacitor and resistor in series



....

1. What is the potential difference across the capacitor at the instant the switch is closed?
2. After a certain time, the charge on the capacitor is 600 μC. Calculate the potential differences across the capacitor and the resistor at this time.

1. At the instant the switch is closed, the charge on the capacitor is zero, so the p.d. across it is also zero.
2. We can calculate the p.d. across the capacitor using Equation 4.2:

$$V = \frac{Q}{C}$$
$$\therefore V = \frac{600 \times 10^{-6}}{220 \times 10^{-6}}$$
$$\therefore V = 2.7\,\text{V}$$

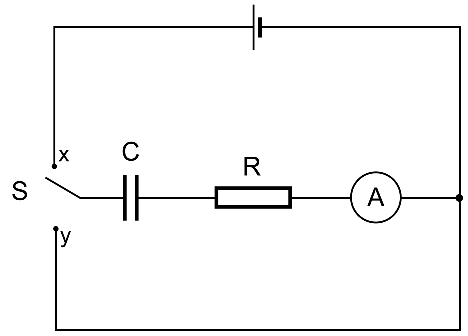
Since the p.d. across the capacitor is 2.7 V, the p.d. across the resistor is 12 - 2.7 = 9.3 V.

....

4.3.2 Discharging a capacitor

The circuit shown in Figure 4.9 can be used to investigate the charging and discharging of a capacitor.

Figure 4.9: Circuit used for charging and discharging a capacitor



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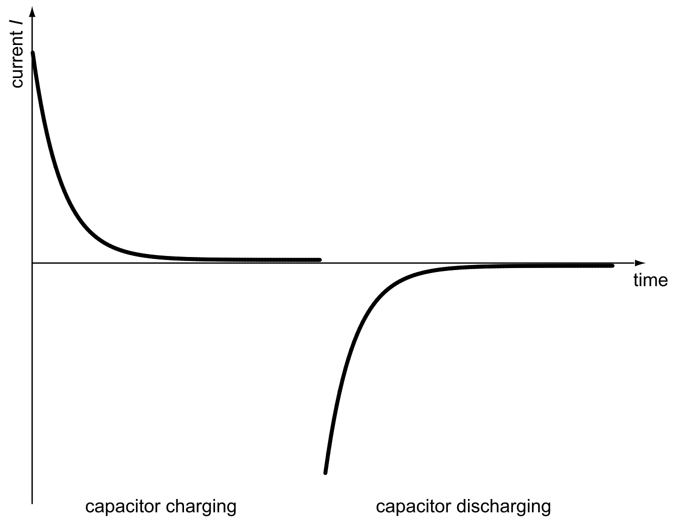
When the switch S is connected to x, the capacitor C is connected to the battery and resistor R, and will charge in the manner shown in Figure 4.6. When S is connected to y, the capacitor is disconnected from the battery, and forms a circuit with the resistor R. Charge will flow from C through R until C is uncharged. A plot of the current against time is given in Figure 4.10.

Figure 4.10: Current as the capacitor is charged, and then discharged



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Remember that the capacitor acts as a break in the circuit. Charge is *not* flowing across the gap between the plates, it is flowing from one plate through the resistor to the other plate. Note that the direction of the current reverses when we change from charging to discharging the capacitor. The energy which has been stored on the capacitor is dissipated in the resistor.

Figure 4.10 shows us that at the instant when the capacitor is allowed to discharge, the size of the current is extremely large, but dies away very quickly. This leads us to one of the applications of capacitors, which is to provide a large current for a short amount of time. One example is the use of a capacitor in a camera flash unit. The capacitor is charged by the camera's batteries. At the instant the shutter is pressed, the capacitor is allowed to discharge through the flashbulb, producing a short, bright burst of light.

Using the energy stored on a capacitor

10 min

Online simulation. This simulation shows why a capacitor is used in a camera's flash unit.

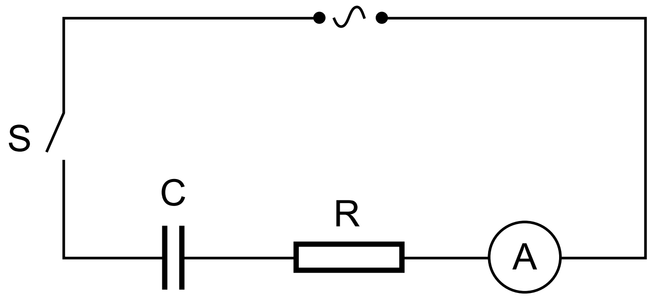
Full instructions are given on-screen.

A capacitor can be used to supply a short, high energy pulse of electrical current.

4.3.3 Capacitors in a.c. circuits

What happens when we replace the d.c. source in a *CR* circuit with an a.c. supply?

Figure 4.11: a.c. capacitive circuit



Let us consider the situation shown in Figure 4.11, in which the current is measured using an a.c. ammeter. Remember that we saw earlier when a d.c. supply is used, the current rapidly drops to zero once the switch is closed. Use the following simulation to investigate the relationship between r.m.s. current and frequency in a capacitive circuit.

The capacitor and a.c.

15 min

This is a similar online simulation to the one that was used in the "Alternating current and voltage" Topic to investigate the relationship between frequency and resistance in an a.c. circuit. As well as investigating the relationship between impedance and frequency, you should be able to describe the experimental method that is used.

Full instructions are given on-screen.

The impedance of a capacitive circuit is inversely proportional to the frequency of the supply.

We can see that the r.m.s current increases as the frequency increases - a *CR* circuit passes high frequency a.c. much better than it does low frequency a.c. or d.c. Why is this?

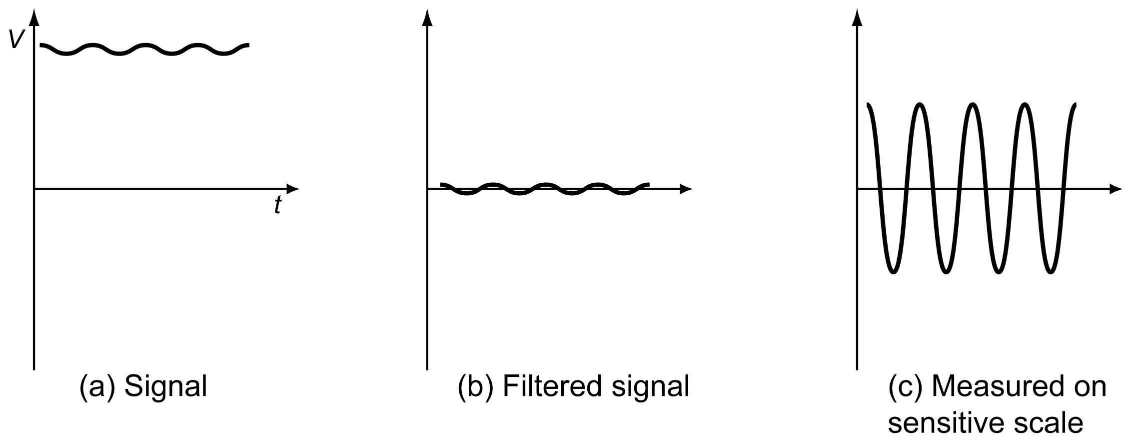
You should remember that charge does not flow across the plates of a capacitor. It accumulates on the plates, and the more charge that has accumulated, the more work is required to add extra charges. At all times, the *total* charge on the plates of the capacitor is zero. Charges are merely transferred from one plate to the other via the external circuit when the capacitor is charged. At low frequency, as the applied voltage oscillates, there is plenty of time for lots of charge to accumulate on the plates, which means the current drops more at low frequency (see Figure 4.6). At high frequency, there is only a short time for charge to accumulate on the plates before the direction of the current is reversed, and the capacitor discharges.

4.3.4 Applications of capacitors

We have already seen how capacitors can be used to store electrical energy and deliver a short, high energy burst of electricity. In this section we will look at some other applications of capacitors in electrical circuits.

In the previous section, we saw that a capacitor passes high frequency a.c. much better than low frequency a.c. or d.c. A capacitor therefore acts as a **high-pass filter** for electrical signals. That is to say, it allows a high frequency electrical signal to pass, but blocks any low frequency signals. This is particularly useful if a small a.c. voltage is superimposed on a large d.c. voltage, and we are trying to measure the a.c. part. Figure 4.12 shows how a high pass filter is used to measure such a signal (a). The d.c. component can be filtered out using a capacitor, leaving the signal shown in Figure 4.12(b). The sensitivity of the voltmeter can then be turned up to allow the a.c. signal to be measured accurately (c).

Figure 4.12: Filtered and amplified a.c. signal



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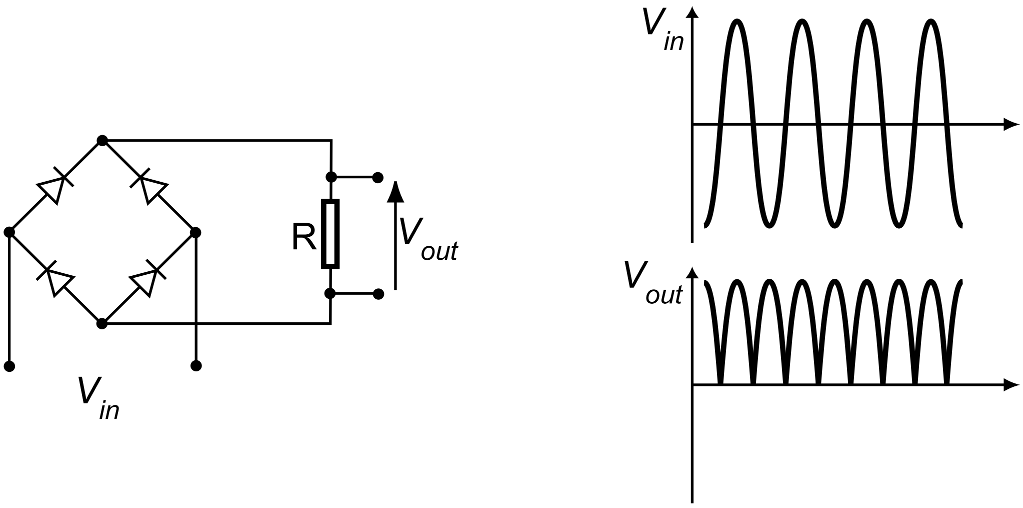
A capacitor is also used to "smooth" the output of a power supply. You may have used a benchtop power supply that takes an a.c. signal from the mains and provides a d.c. signal at its output. Inside the power supply there is a network of diodes that rectify the a.c. input - that is to say, the signal becomes positive at all times. Figure 4.13 shows a **rectifier circuit**, along with the a.c. input V_{in} and the rectified output V_{out}.

Figure 4.13: Rectifier circuit, with the input and output voltages



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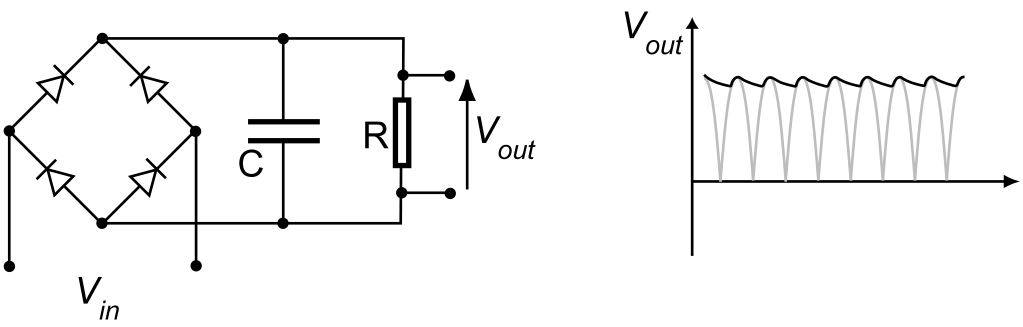
The rectified output no longer changes sign, but its magnitude still oscillates from zero to the maximum value. This output could not be used as a "steady" d.c. signal. A capacitor placed in parallel with the output resistance smooths this output. As the signal is rising, the capacitor becomes charged. As the signal falls, the capacitor discharges, adding to the signal. The result, shown in Figure 4.14, is that the output signal is smoothed, and no longer oscillates through such a large range of values.

Figure 4.14: Smoothed, rectified signal



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Quiz 2 Capacitors in circuits

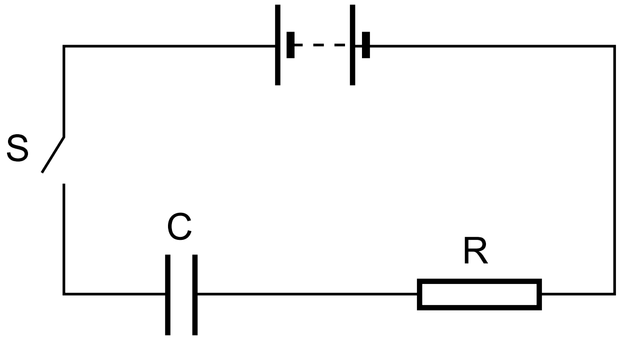
15 min

Multiple choice quiz.

First try the questions. If you get a question wrong or do not understand a question, there are 'Hints' at the back of the book . The hints are in the same order as the questions. If you read the hint and still do not understand then ask your tutor.

In the following quiz, the first three questions refer to the circuit shown in Figure 4.15. The circuit contains an uncharged 60 μF capacitor and a 36 Ω resistor connected to a 9.0 V battery of negligible internal resistance.

Figure 4.15: Circuit for multiple choice quiz



....

Q6: What is the current in the circuit at the instant the switch is closed?

a) 150 mA
b) 250 mA
c) 600 mA
d) 4.0 A
e) 6.7 A

....

Q7: What is the p.d. across the resistor at the instant when the current is 50 mA?

a) 1.4 mV
b) 450 mV
c) 1.8 V
d) 3.0 V
e) 9.0 V

....

Q8: What is the p.d. across the capacitor at the instant when the current is 50 mA?

a) 450 mV
b) 3.0 V
c) 7.2 V
d) 8.3 V
e) 9.0 V

....

Q9: A capacitor is connected in series to an a.c. power supply and an a.c. ammeter. As the frequency of the a.c. is slowly increased from 20 Hz to 2500 Hz, whilst its r.m.s. voltage remains unchanged, the current measured by the meter

a) increases.
b) decreases.
c) is constant and non-zero.
d) is zero at all times.
e) increases, then decreases.

....

Q10: A capacitor can be used in a circuit as a

a) rectifier.
b) resistor.
c) diode.
d) meter.
e) high-pass filter.

....

....

4.4 Summary

By the end of this Topic you should be able to

- state that the charge stored on two parallel conducting plates is proportional to the potential difference across the plates, and describe the principles of a method to demonstrate this;
- state that capacitance is the ratio of charge to potential difference;
- state that the unit of capacitance is the farad, and that one farad is equal to one coulomb per volt;
- perform calculations using the relationship $C = {}^{Q}/_{V}$;
- explain why work must be done to charge a capacitor;
- state that the work done in charging a capacitor is given by the area under the *Q*-*V* graph;
- state the expressions $W = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$ for the energy *W* stored on a capacitor, and carry out calculations using these expressions;
- sketch graphs of voltage and current against time for charging and discharging capacitors in series *CR* circuits;
- carry out calculations on voltage and current in series *CR* circuits;
- state the relationship between current and frequency in an a.c. *CR* circuit, and describe the principles of a method to show this relationship;
- describe and explain some applications of capacitors.

Online assessments

Two online assessments are available. Both tests contain questions taken from all parts of this Topic.

Topic 5

Analogue electronics 1

Contents

5.1 Introduction

This Topic looks at a small part of one branch of electronics called Analogue electronics. Analogue electronics deals with electrical signals that are being used to represent some physical quantity, normally one that is continually changing, such as the loudness or frequency of sounds. The other main branch of electronics is digital electronics, where signals can only have fixed values (usually only two).

The main circuit element used in analogue electronics is the **amplifier**. An amplifier is an electronic circuit with at least one input and one output, that is designed to process the waveform obtained from a physical quantity (such as a sound waveform from a microphone) and produce an output that is an enlarged (amplified) copy of this waveform.

This Topic concentrates on one particular type of amplifier called an operational amplifier (or op-amp for short). The operational amplifier was originally developed for early electronic computers that carried out computations on electrical signals used to represent numbers. Such analogue computers have now been superseded by faster digital computers. The mathematical operations of addition, subtraction, multiplication and division are all possible by the use of op-amps. Even although computers have moved on from the early days, op-amps are still employed to process analogue signals for use in digital electronic circuits.

5.2 Operational amplifiers

Learning Objective

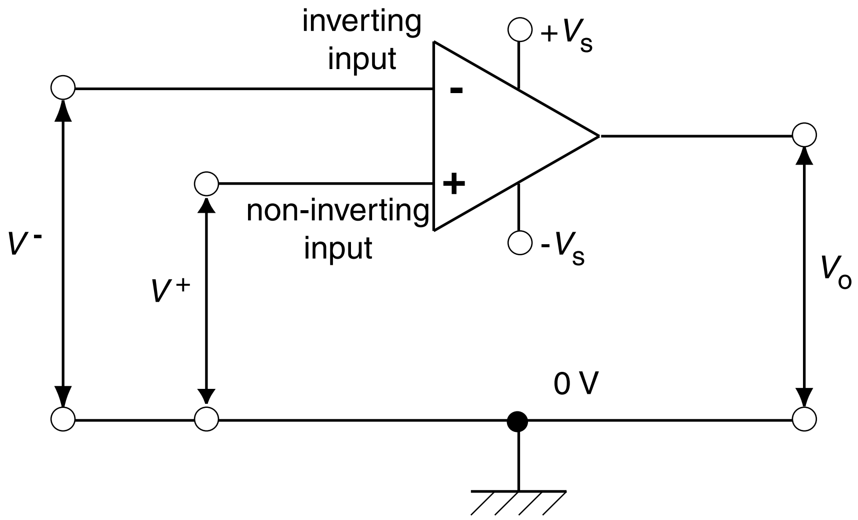
To describe what is meant by an op-amp, and draw and recognise the symbol of an op-amp.
To state what is meant by an ideal op-amp.
To state one function of an op-amp.
To state what is meant by the open-loop voltage gain of an op-amp, and carry out calculations involving the open-loop voltage gain.
To state the limitations imposed on the output of an op-amp by the supply voltage.

An op-amp is a particular form of amplifier that has two inputs and one output. The original op-amps were built using individual or discrete electronic components, although nowadays virtually all op-amps are packaged as integrated circuits (ICs - commonly called 'chips'). An integrated circuit op-amp is often used to increase the voltage of a signal. This voltage can be either d.c. or a.c. It could be a tiny voltage generated in a brain, or it could have come from a pressure-monitoring circuit used in an aircraft cabin. The amount by which an amplifier increases a voltage is known as the **voltage gain** (*A*) of the amplifier. Voltage gain is given by the expression

$$A = \frac{\text{change in output voltage}}{\text{change in input voltage}}$$

One very common integrated circuit op-amp is known as the '741'. It contains the equivalent of about 20 transistors, and other associated components. You do not need to know about the internal circuitry of an op-amp since there is a circuit symbol for it, shown in Figure 5.1.

Figure 5.1: Op-amp symbol and power supply connections



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The circuit symbol shows that there are two inputs to an op-amp. One input, the inverting input, has a negative sign and the other, the non-inverting input, has a positive sign. There is one output. Also shown in Figure 5.1 are the connections to the power supply. Most op-amps need a dual-rail balanced d.c. power supply. This type of power supply gives equal positive and negative voltage outputs. Typical values for common op-amps are $\pm$5 V, $\pm$15 V and $\pm$18 V. So a $\pm$5 V power supply has three terminals, +V_S = +5 V, 0 V and -V_S = -5 V. To save confusing the power supply terminals with the non-inverting and the inverting inputs to the op-amp, it is normal not to include them in a circuit diagram. From now on in this Topic, the supply terminals will not be shown, unless they are relevant. The 0 V output of the power supply is usually common to the input and the output voltages of the op-amp and is taken as the ground or reference voltage.

Although there is no such thing as an 'ideal' op-amp, most of the op-amps that are in production approach the ideal. But what is meant by an 'ideal' op-amp? The main properties of an ideal op-amp are as follows.

1. An infinitely-high open-loop voltage gain (A_O). An op-amp amplifies the *difference* between the voltage applied to the inverting input (V^-) and the non-inverting input (V^+). The open-loop voltage gain is the ratio of the output voltage (V_O) to the difference between the input voltages, when there is no feedback applied. (Feedback is described in more detail in the next section of these notes.)

$$V_o = A_o(V^+ - V^-) \tag{5.1}$$

....

A typical op-amp has an open-loop voltage gain of about 10^5 for d.c. and low frequency a.c. input signals (up to typically about 100 Hz), falling off as the frequency of the input signal rises.

2. An infinitely-high input **impedance** (resistance) at both of its inputs. Because of this, the input current to an ideal op-amp is zero.

In practice, the input impedance of a typical op-amp is in the range $10^6\ \Omega$ to 10^{12} Ω, giving rise to input currents of less than 0.1 μA.

3. A very low output impedance (resistance). This allows the output voltage to be efficiently transferred to the load connected to the output.

Example An op-amp has an open-loop voltage gain of 10^5. The voltage applied to the inverting input (V^-) is 15 μV, and the voltage applied to the non-inverting input (V^+) is 12 μV.
Calculate the voltage at the output of the amplifier (V_o).

$$\begin{aligned} V_o &= A_o(V^+ - V^-) \\ \therefore V_o &= 10^5 \times (12 \times 10^{-6} - 15 \times 10^{-6}) \\ \therefore V_o &= -0.3 \text{ V} \end{aligned}$$

Note that the output voltage is negative since the voltage applied to the inverting input is greater than the voltage applied to the non-inverting input of the op-amp.

. . . .

It would appear from Equation 5.1 that there is no limit to the output voltage that can be obtained from an op-amp.
Suppose, for example, that an op-amp with an open-loop voltage gain of 10^5 has voltages of +5 V and 0 V applied to the non-inverting (V^+) and inverting inputs (V^-) respectively. Putting these figures into Equation 5.1 would seem to indicate that the output voltage obtained from the op-amp would be 50 000 V. This is not the case, however, as the following on-line activity shows.

Open-loop voltage gain

20 min

In this on-line simulation, you have to find out what factor limits the output voltage that can be obtained from an op-amp. To do this, you can change the supply voltage and vary the voltages applied to both the inverting input (V^-) and to the non-inverting input (V^+) of the op-amp.

Full instructions are given in the simulation.

An op-amp cannot produce an output voltage greater than the positive supply voltage or less than the negative supply voltage.

You will see from the activity that the output of an op-amp is limited to $\pm$ the value of the supply voltage used. The greater the supply voltage, the greater the maximum output of the op-amp. As long as the difference in the input voltages causes an output voltage that is less than $\pm$ the supply voltage, then the output is an amplified copy of this difference. This is the amplifier operating in the **linear region**. Beyond this region, the amplifier is driven into **saturation**, and the output does not copy the input exactly - we say that the signal is **clipped**.

Voltage gain calculations

20 min

This is a paper-based exercise, designed to give you practice in using the expression for the voltage gain of an op-amp.

An op-amp that has an open-loop voltage gain of 10^5 is used without feedback. Calculate the output voltage V_o in each of the following cases.

	V_s V	V^- μV	V^+ μV
(a)	$\pm$ 5	0	10
(b)	$\pm$12	180	80
(c)	$\pm$15	100	300
(d)	$\pm$18	250	0

---

1. The voltage gain of an op-amp is given by the expression

$$V_o = A_o(V^+ - V^-)$$

2. If $A_o(V^+ - V^-) > \pm V_s$ then the op-amp is driven into saturation and $V_o = +V_s$ or $-V_s$

Quiz 1 Operational amplifiers

15 min

Multiple choice quiz.

First try the questions. If you get a question wrong or do not understand a question, there are 'Hints' at the the back of the book . The hints are in the same order as the questions. If you read the hint and still do not understand then ask your tutor.

Q1: An op-amp has

a) only one input and only one output.
b) only one input and two outputs.
c) two inputs and only one output.
d) two inputs and two outputs.
e) an undefined number of inputs and outputs.

....

Q2: Which of the following properties does an ideal op-amp have?
(i) An infinitely-high open-loop voltage gain.
(ii) An infinitely-high input impedance.
(iii) A very low output impedance.

a) (i) only
b) (ii) only
c) (iii) only
d) (i) and (ii) only
e) (i), (ii) and (iii)

. . . .

Q3: Which expression gives the open-loop voltage gain of an op-amp?

a) $\frac{V_o}{V^+ + V^-}$
b) $\frac{V_o}{V^+ - V^-}$
c) $\frac{V^+ + V^-}{V_o}$
d) $\frac{V^+ - V^-}{V_o}$
e) $V_o + V^+ - V^-$

. . . .

Q4: An op-amp that has an open-loop voltage gain of 10^5 is operated from a $\pm$5.0 V supply.
What is the voltage at the output of the amplifier when the voltage applied to the inverting input is 0.20 mV and the voltage applied to the non-inverting input is 0.16 mV?

a) +5.0 V
b) +4.0 V
c) 0 V
d) - 4.0 V
e) -5.0 V

. . . .

Q5: An op-amp that has an open-loop voltage gain of 10^5 is operated from a $\pm$10 V supply.
What is the voltage at the output of the amplifier when the voltage applied to the inverting input is 50 μV and the voltage applied to the non-inverting input is 200 μV?

a) +15 V
b) +10 V
c) 0 V
d) -10 V
e) -15 V

. . . .

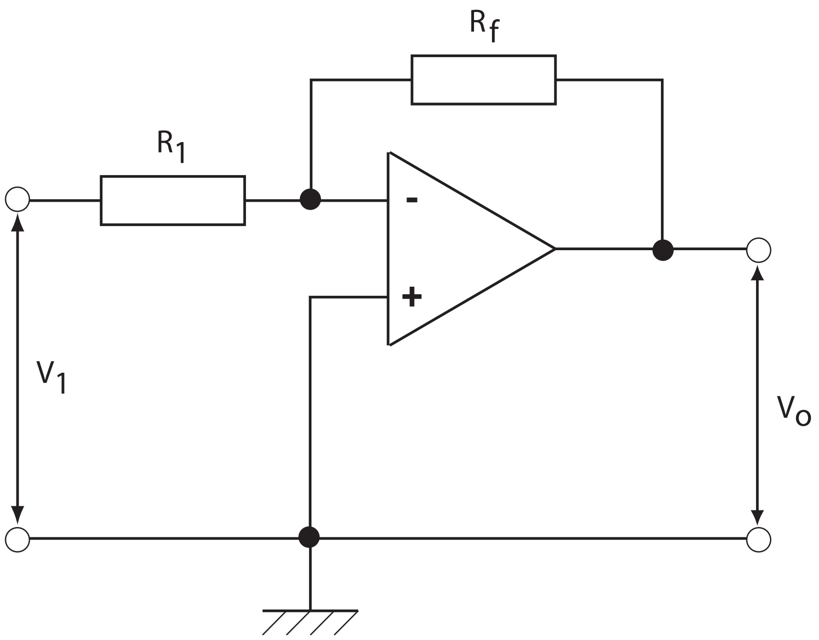
5.3 The op-amp in inverting mode

Learning Objective

To explain what is meant by negative feedback and state advantages and disadvantages of negative feedback.
To identify circuits where the op-amp is being used in the inverting mode.
To state that an op-amp connected in the inverting mode will invert the input signal.
To derive and state the inverting mode gain expression.
To carry out calculations using the inverting mode gain expression.

We have just seen that even very small voltage swings at the inputs of an op-amp can cause the amplifier to go into saturation, mainly because the voltage gain is so great. This can be prevented by applying **negative feedback** to the amplifier. Negative feedback is when a proportion of the output of the op-amp is fed back to the inverting input of the op-amp. This has the effect of reducing the overall gain of the amplifier, so that the new gain of the amplifier, called the **closed-loop gain** (*A*) is less than the open-loop gain. One way of applying negative feedback is by using the circuit shown in Figure 5.2.

Figure 5.2: Op-amp in inverting mode



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The input V_1 is applied to the inverting input of the op-amp, and the non-inverting input is connected to 0 V. This mode of use of an op-amp is called the **inverting mode**.

Applying negative feedback to an op-amp reduces the overall gain although it has the great advantage of increasing the stability of the circuit. The reduction in gain is not usually a problem since the open-loop gain is so great anyway. There are other advantages of using negative feedback:

1. the gain is constant and easily calculated;

2. the output is less easily distorted;
3. there is a wider frequency response when used with an a.c. input. This means that the gain is constant over a wider frequency range.

When negative feedback is applied to an op-amp as shown in Figure 5.2, the closed-loop gain is given by the expression

$$\frac{V_o}{V_1} = -\frac{R_f}{R_1} \tag{5.2}$$

....

There are two things to note about this expression.

- The output voltage is in the opposite sense (or inverted) compared to the input voltage. This is because the input is applied to the inverting input and so the equation has a negative sign.
- The gain of the circuit is determined only by the values of R_f and R_1, the external resistors. This makes the gain more stable and predictable, since resistors can be made with precise values whereas it is difficult to make a batch of op-amps that all have the same open-loop gain.

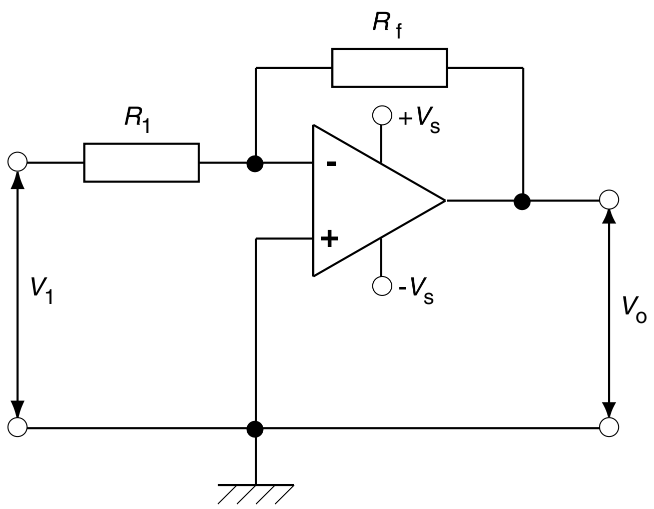
20 min

Calculations on the op-amp in inverting mode

This is a paper-based exercise, designed to give you practice in using the inverting mode gain expression of an op-amp.

An op-amp is used in the inverting mode.

Figure 5.3: The op-amp used in inverting mode



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Calculate the output voltage V_o in each of the following cases.

	V_s V	V_1 V	R_f kΩ	R_1 kΩ
(a)	±18	-3	5	1
(b)	±15	+2	20	10
(c)	±12	-1	2	10
(d)	±5	+4	10	5

---

1. The inverting mode gain expression for an op-amp is given by

$$^{V_o}/_{V_1} = -^{R_f}/_{R_1}$$

2. If $R_f > R_1$ then $V_o > V_1$ and the input signal is amplified.
3. If $R_f < R_1$ then $V_o < V_1$ and the input signal is attenuated.
4. The maximum value of V_o is limited by the supply voltages to the amplifier.

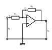
. . . .

The inverting mode gain expression

15 min

This is an optional, paper-based activity. It takes you through the derivation of the expression for the closed-loop gain of an op-amp used in the inverting mode.

Figure 5.4: Inverting mode gain expression



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We make two assumptions in deriving the inverting mode gain expression for an op-amp in inverting mode.

1. Each of the inputs of the op-amp draws zero current.
2. Both of the inputs are at the same potential. (If we make the approximation that the gain of the amplifier is infinite, then this is true as long as the amplifier is not driven into saturation.)

In Figure 5.4, the potential at the non-inverting input is zero (it is connected to the ground line), and so the potential at the inverting input can also be considered as zero. The inverting input terminal of the op-amp acts as a **virtual earth** under these conditions

$$\therefore I_1 = \frac{(V_1 - 0)}{R_1}$$

$$\text{and } I_f = \frac{(0 - V_o)}{R_f}$$

But, from assumption 1, since the op-amp draws no current at its inverting input, $I_1 = I_f$

$$\therefore \frac{V_1 - 0}{R_1} = \frac{0 - V_o}{R_f}$$
$$\text{so } \frac{V_o}{V_1} = -\frac{R_f}{R_1}$$

The closed-loop gain, *A*, is $^{V_o}/_{V_1}$, so

$$A = \frac{V_o}{V_1} = -\frac{R_f}{R_1}$$

The derivation of the expression for the closed-loop gain of an op-amp when it is used in the inverting mode.

....

Quiz 2 The op-amp in inverting mode

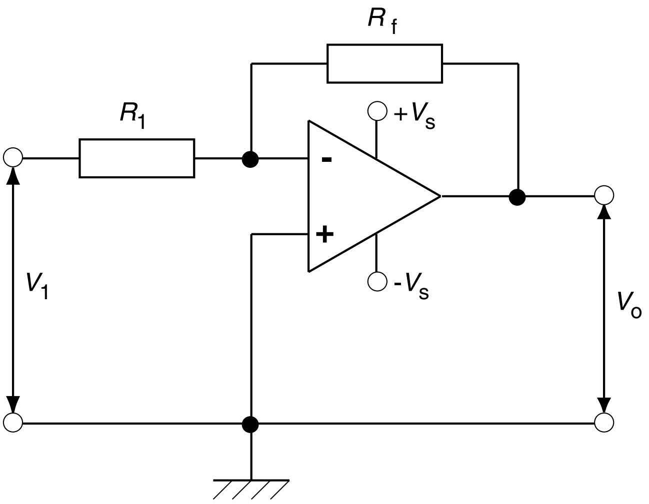
15 min

Multiple choice quiz.

First try the questions. If you get a question wrong or do not understand a question, there are 'Hints' at the the back of the book . The hints are in the same order as the questions. If you read the hint and still do not understand then ask your tutor.

All of the questions in this quiz refer to the following circuit diagram.

Figure 5.5:



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Q6: In Figure 5.5, R_f = 10 kΩ and R_1 = 1 kΩ.
What is the closed loop gain of the circuit?

a) -10
b) -0.1

c) +0.1
d) +1.0
e) +10

. . . .

Q7: In Figure 5.5, R_f = 200 kΩ, R_1 = 10 kΩ, V_1 = -200 mV, and V_s = $\pm$5.0 V. What is the output voltage, V_o?

a) -4.0 V
b) -10 mV
c) +10 mV
d) +400 mV
e) +4.0 V

. . . .

Q8: In Figure 5.5, R_f = R_1 = 47 kΩ, V_1 = +25 mV, and V_s = $\pm$5.0 V. What is the output voltage, V_o?

a) +1175 mV
b) +47 mV
c) +25 mV
d) -25 mV
e) -47 mV

. . . .

Q9: In Figure 5.5, R_f = 5 kΩ, R_1 = 20 kΩ, V_1 = -1.0 V, and V_s = $\pm$5.0 V. What is the output voltage, V_o?

a) 4.0 mV
b) 5.0 mV
c) 20 mV
d) 250 mV
e) 1000 mV

. . . .

Q10: In Figure 5.5, R_f = 10 kΩ, R_1 = 2 kΩ, V_1 = +1.5 V, and V_s = $\pm$5.0 V. What is the output voltage, V_o?

a) -1.5 V
b) -2.0 V
c) -5.0 V
d) -7.5 V
e) -10 V

. . . .

. . . .

5.4 Summary

By the end of this Topic you should be able to:

- describe what is meant by an op-amp;
- draw and recognise the circuit symbol for an op-amp;
- state that one function of an op-amp is to increase the voltage of a signal;
- state that for an ideal op-amp the input current is zero and there is no potential difference between the inverting and the non-inverting inputs;
- state what is meant by the open-loop voltage gain of an op-amp;
- carry out calculations involving the open-loop voltage gain expression $V_o = A_o(V^+ - V^-)$;
- state that an op-amp cannot produce an output voltage greater than the positive supply voltage or less than the negative supply voltage;
- explain what is meant by negative feedback and state advantages and disadvantages of negative feedback;
- identify circuits where the op-amp is being used in the inverting mode;
- state that an op-amp connected in the inverting mode will invert the input signal;
- derive and state the inverting mode gain expression ${}^{V_o}/_{V_1} = -{}^{R_f}/_{R_1}$;
- carry out calculations using the inverting mode gain expression.

5.5 Assessment

Online assessments

Two online tests are available. Each test should take you no more than 20 minutes to complete. Test one contains questions on operational amplifiers used without feedback. Test two contains questions on op-amps being used in inverting mode.

Topic 6

Analogue electronics 2

Contents

6.1 Introduction

This second Topic in Analogue electronics continues the study of the operational amplifier. The op-amp is commonly used in applications where small differences between two voltages have to be compared. When this is the case, it is used as a differential amplifier - we say it is being used in the **differential mode**. The differential mode gain expression is introduced and used, with an optional activity showing the derivation of this expression.

The second part of this Topic concentrates on the applications of op-amps, both when connected in inverting mode and differential mode. These applications range from the mathematical operations of addition, subtraction, multiplication and division, to the many applications involving the op-amp in monitoring and control situations. The Topic ends with an look at how an op-amp can be used to control external devices.

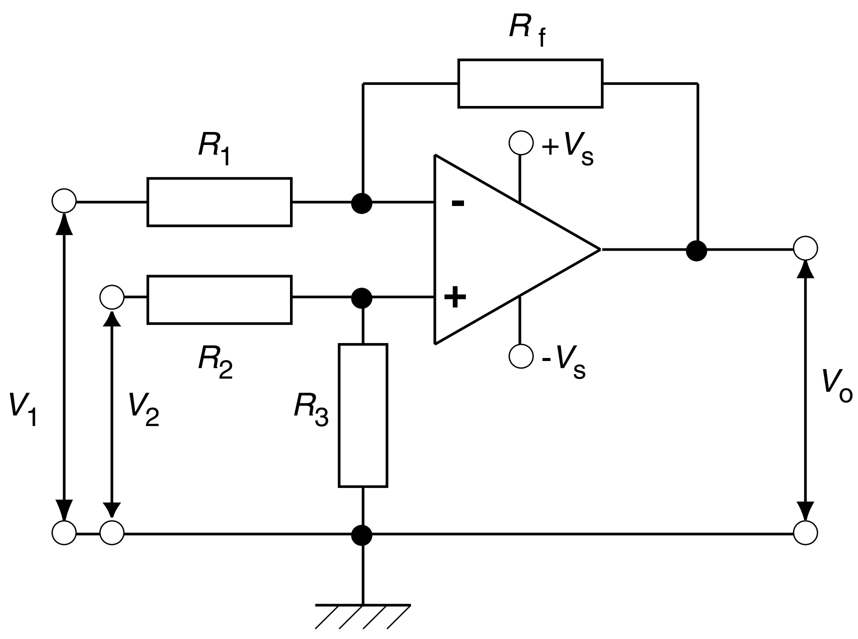
6.2 The op-amp in differential mode

Learning Objective

To identify circuits where the op-amp is being used in the differential mode.
To state that a differential amplifier amplifies the potential difference between its two inputs.
To derive and state the differential mode gain expression.
To carry out calculations using the inverting mode gain expression.

In the previous Topic, we considered the case where there was only one input to the op-amp, applied to the inverting input. Op-amps, however, have two inputs, and so they can both be used at the same time, as in Figure 6.1.

Figure 6.1: Op-amp in differential mode



. . . .

When the op-amp is used in this way, called the differential mode, the relationship linking the input voltages to the output voltage is

$$V_o = (V_2 - V_1)\,\frac{R_f}{R_1} \tag{6.1}$$

. . . .

as long as the values of the external resistors are chosen so that ${}^{R_f}/_{R_1} = {}^{R_3}/_{R_2}$.

There are several points to note about the op-amp used in differential mode.

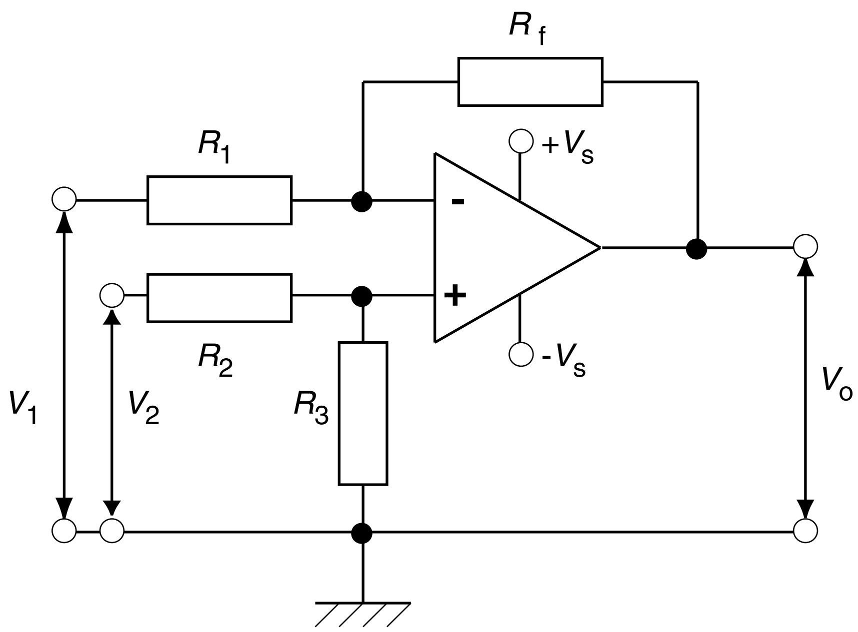
1. This relationship only holds for the special case where ${}^{R_f}/_{R_1} = {}^{R_3}/_{R_2}$, which in fact is the normal situation when the op-amp is used in differential mode.
2. As with the op-amp used in the inverting mode, the gain of this circuit is determined only by the values of the external resistors. This makes the gain more stable and predictable, since resistors can be made with precise values whereas it is difficult to make a batch of op-amps that all have the same open-loop gain. Also the open-loop gain of an op-amp is too large to be of practical use.
3. If V_2 = 0, then the differential mode gain expression becomes the same as the inverting mode gain expression, $V_o = -V_1\frac{R_f}{R_1}$.
4. If the values of R_f and R_1 are equal (which also makes R_2 and R_3 equal), then the differential mode gain equation reduces to V_o = V_2 - V_1. In this case, the circuit performs the mathematical operation of subtraction on the two input voltages. Hence the name differential mode - the output is the difference of the two inputs.
5. By a suitable choice of the values of the external resistors, this circuit can be used to *amplify* the difference between the two input voltages (when $R_f > R_1$), or to *attenuate* the difference (when $R_f < R_1$). This scaling of the difference between the input voltages is useful when the input voltages are either very large or very small, and the output voltage has to be kept within certain limits. The output voltage is of course limited by the supply voltage used for the op-amp, as was shown in the previous Topic.
6. Since the differential amplifier only amplifies the *difference* between V_1 and V_2, if an unwanted signal (sometimes called 'noise'), is present at both inputs then this signal will not be amplified. This makes the differential amplifier very useful when very small input signals (small in relation to the unwanted noise) have to be amplified. This is called 'common-mode rejection'.

Calculations on the op-amp in differential mode

20 min

This is a paper-based exercise, designed to give you practice in using the differential mode gain expression of an op-amp.

An op-amp is used in the differential mode.



Calculate the output voltage V_o in each of the following cases.
In each case $R_f/R_1 = R_3/R_2$.

	V_s V	V_1 V	V_2 V	R_f kΩ	R_1 kΩ
(a)	±18	-2	+1	5	1
(b)	±18	+2	-4	20	10
(c)	±15	-1	+14	2	10
(d)	±15	-12	-15	4	12
(e)	±12	-2	+1	10	2
(f)	±5	+4	-6	10	5

---

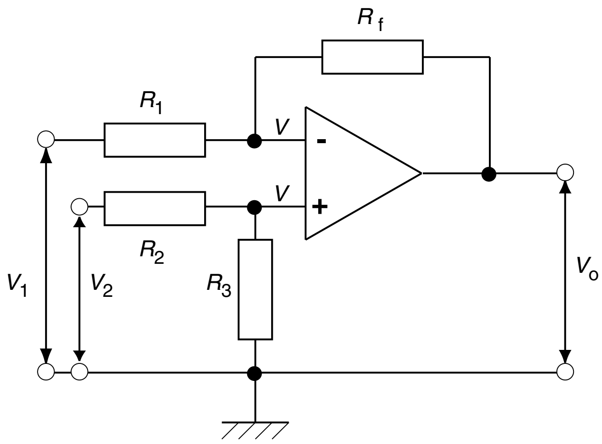
1. The differential mode gain expression for an op-amp is given by $V_o = (V_2 - V_1)\,\frac{R_f}{R_1}$.
2. If $R_f > R_1$ then $V_o > (V_2 - V_1)$ and the difference between the input signals is amplified.
3. If $R_f < R_1$ then $V_o < (V_2 - V_1)$ and the difference between the input signals is attenuated.
4. The maximum value of V_o is limited by ± the supply voltage to the amplifier.

The differential mode gain expression

This is an optional paper-based activity. It takes you through the derivation of the expression for the closed-loop gain of an op-amp used in the inverting mode.

15 min

Figure 6.2: Op-amp in the differential mode

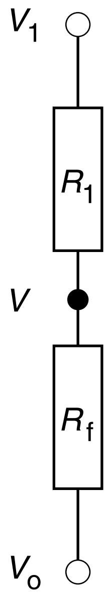


We make the same two assumptions in deriving the differential mode gain expression for an op-amp that we made when deriving the gain expression for an op-amp used in inverting mode. These are based on a simplification - treating the op-amp as an ideal op-amp.

1. Each of the inputs of the op-amp draws zero current.
2. Both of the inputs are at the same potential. (If we make the approximation that the gain of the amplifier is infinite, then this is true as long as the amplifier is not driven into saturation.)

In Figure 6.2, the potential at both inputs is equal to V.
Because of assumption 1, the inverting input draws zero current and R_1 and R_f form a potential divider, as shown in Figure 6.3, so

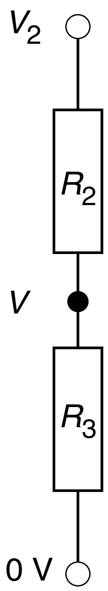
Figure 6.3:



$$
\begin{aligned}
\frac{V - V_o}{R_f} &= \frac{V_1 - V_o}{R_1 + R_f} \\
\therefore V - V_o &= \frac{R_f}{(R_1 + R_f)}(V_1 - V_o) \\
\therefore V &= \frac{R_f}{(R_1 + R_f)}(V_1 - V_o) + V_o \\
\therefore V &= \frac{R_f}{(R_1 + R_f)}(V_1 - V_o) + \frac{(R_1 + R_f)}{(R_1 + R_f)}V_o \\
\therefore V &= \frac{R_f V_1 - R_f V_o + R_1 V_o + R_f V_o}{(R_1 + R_f)} \\
\therefore V &= \frac{R_f V_1 + R_1 V_o}{(R_1 + R_f)}
\end{aligned}
$$

Also because of assumption 1, the non-inverting input draws zero current and R_2 and R_3 also form a potential divider, as shown in Figure 6.4, so

Figure 6.4:



$$\frac{V - 0}{R_3} = \frac{V_2 - 0}{(R_2 + R_3)}$$

If we make $R_f/R_1 = R_3/R_2$, then this expression can be written as

$$
\begin{aligned}
\frac{V}{R_f} &= \frac{V_2}{(R_1 + R_f)} \\
\therefore V &= \frac{R_f V_2}{(R_1 + R_f)}
\end{aligned}
$$

Equating these two expressions for *V*, we get

$$\frac{R_f V_2}{(R_1 + R_f)} = \frac{R_f V_1 + R_1 V_o}{(R_1 + R_f)}$$
$$\therefore R_f V_2 = R_f V_1 + R_1 V_o$$
$$\therefore R_1 V_o = R_f (V_2 - V_1)$$
$$\therefore V_o = (V_2 - V_1) \frac{R_f}{R_1}$$

How to derive the expression for the gain of an op-amp when it is used in the differential mode.

. . . .

Quiz 1 The op-amp in differential mode

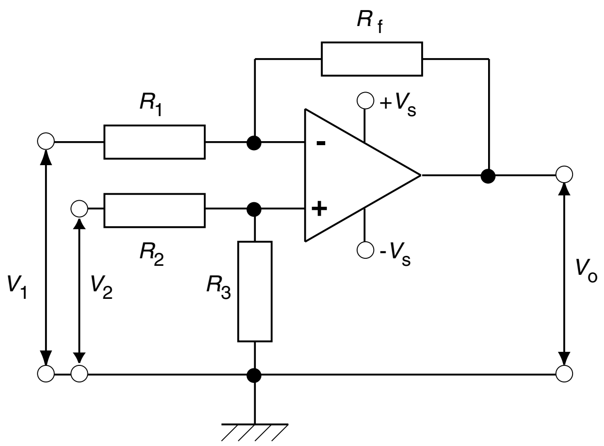
15 min

Multiple choice quiz.

First try the questions. If you get a question wrong or do not understand a question, there are 'Hints' at the the back of the book . The hints are in the same order as the questions. If you read the hint and still do not understand then ask your tutor.

All of the questions in this quiz refer to the following circuit diagram.

Figure 6.5:



. . . .

Q1: In Figure 6.5, what condition must be met for the gain of the circuit to be given by $V_o = (V_2 - V_1)\frac{R_f}{R_1}$?

a) $R_f = R_1 = R_2 = R_3$
b) ${}^{R_f}/_{R_1} = {}^{R_3}/_{R_2}$
c) $V_2 > V_1$
d) $V_o > (V_2 - V_1)$
e) $V_s > (V_2 - V_1)$

. . . .

Q2: In Figure 6.5, $R_f = R_1 = R_2 = R_3 = 10$ kΩ, $V_1 = 200$ mV and $V_2 = 300$ mV.
What is the output voltage, V_o?

a) -500 mV
b) -100 mV
c) +100 mV
d) +200 mV
e) +500 mV

. . . .

Q3: In Figure 6.5, $R_f = R_3 = 10$ kΩ, $R_1 = R_2 = 20$ kΩ, $V_1 = -5$ mV and $V_2 = +9$ mV.
What is the output voltage, V_o?

a) +2 mV
b) +7 mV
c) +8 mV
d) +14 mV
e) +28 mV

. . . .

Q4: In Figure 6.5, the difference between V_1 and V_2 can be as much as 500 mV.
If $R_1 = 20$ kΩ and $V_s = \pm 15$ V, what value of R_f would give the greatest gain without distorting the output signal?

a) 1.5 kΩ
b) 30 kΩ
c) 600 kΩ
d) 2.0 MΩ
e) 40 MΩ

. . . .

Q5: In Figure 6.5, ${}^{R_f}/_{R_1} = {}^{R_3}/_{R_2} = 10$, $V_1 = +300$ mV, $V_2 = -800$ mV and $V_s = \pm 10$ V.
What is the output voltage, V_o?

a) -3.0 V
b) -5.0 V
c) -8.0 V
d) -10 V
e) -11 V

. . . .

. . . .

6.3 Uses of op-amps

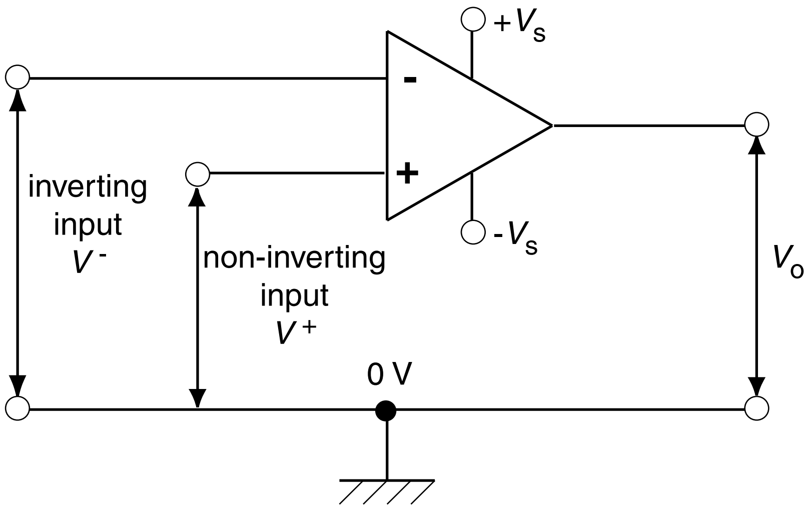
Learning Objective

To describe some of the uses of op-amps.

Earlier we noted that op-amps were developed to perform the mathematical operations of addition, subtraction, multiplication and division. We are now in a position to look at these and other uses of op-amps.

6.3.1 The comparator

Figure 6.6: The op-amp as a comparator



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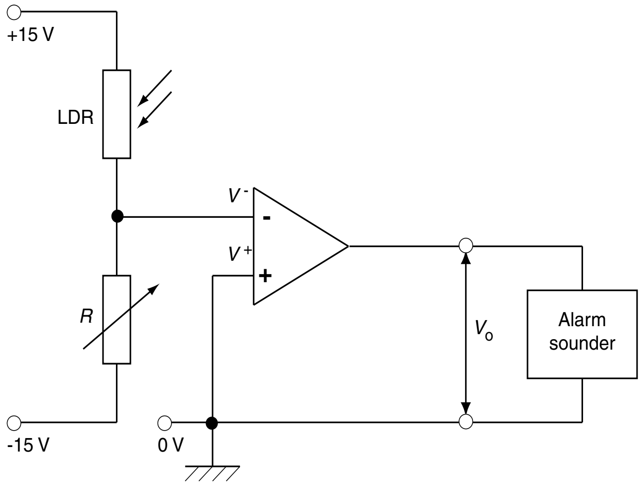
When an op-amp is used without any feedback as in Figure 6.6, the output voltage, V_o, is given by $V_o = A_o(V^+ - V^-)$. The difference between the two input voltages $(V^+ - V^-)$ is amplified and appears at the output, as a positive voltage if $V^+ > V^-$ and a negative voltage if $V^+ < V^-$.

If the difference between the two input voltages is greater than or equal to V_s/A_o, the op-amp goes into saturation and $V_o = \pm V_s$. Typically $V_s = \pm 15$ V, and $A_o = 10^5$ so the output voltage V_o would be either +15 V or -15 V if the difference between V^+ and V^- is equal to or greater than 150 μV. This causes the output of the op-amp to swing between $+V_s$ and $-V_s$ depending on whether V^+ is greater or less than V^-. This is the op-amp acting as a comparator.

The circuit shown in Figure 6.7 is an alarm circuit based around an op-amp being used as a comparator. The alarm sounder in this circuit only operates when its top terminal is at a positive potential with respect to the bottom one.

Figure 6.7: Alarm circuit using op-amp as a comparator



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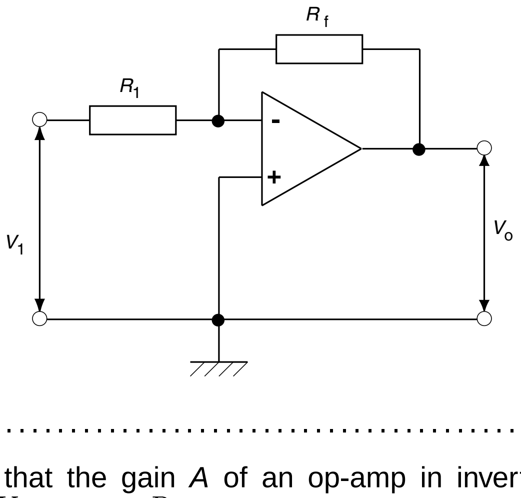
In this circuit *R* and the light dependent resistor (LDR) form a potential divider. If the resistance of *R* is set so that in light conditions the resistance of the LDR is less than that of *R* then $V^- > V^+$ and the output voltage V_o = -V_s (-15 V). In dark conditions the resistance of the LDR increases, so V^- falls and when $V^- < V^+$, V_o = +V_s (+15 V) and the alarm sounds.

6.3.2 In inverting mode

6.3.2.1 Multiplication and division

Figure 6.8: The op-amp used for multiplication and division



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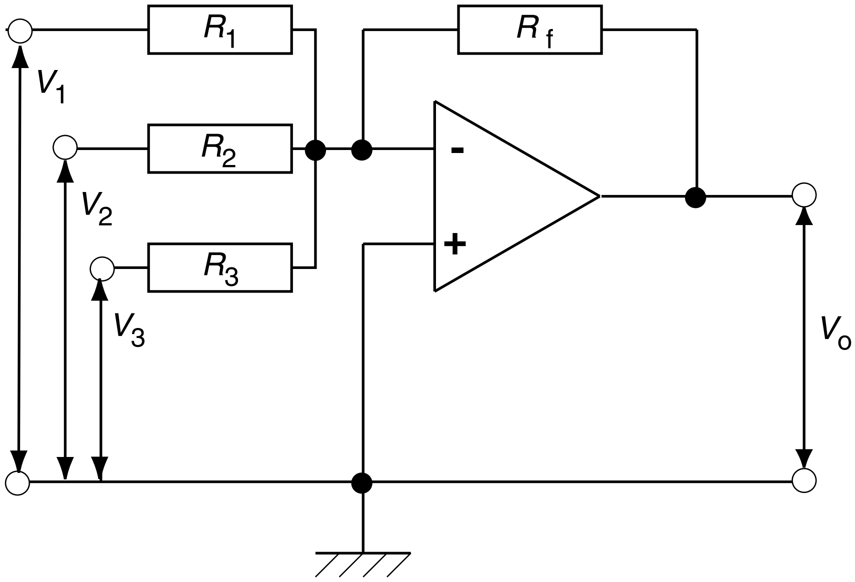
We have seen earlier that the gain *A* of an op-amp in inverting mode is given by the expression $A = {}^{V_o}/_{V_1} = -{}^{R_f}/_{R_1}$. Rearranging this expression we see that $V_o = -\left({}^{R_f}/_{R_1}\right) V_1$. By suitable choice of R_f and R_1, we can amplify or attenuate the input voltage V_1. If $R_f > R_1$ then the output voltage V_o is $\left({}^{R_f}/_{R_1}\right)$ times greater than V_1. If $R_f < R_1$ then the output voltage V_o is $\left({}^{R_f}/_{R_1}\right)$ times less than V_1. Thus the quantity to be multiplied or divided is represented by V_1 and the result is given by V_o.

6.3.2.2 Addition and d-a conversion

So far we have only considered one input connected to the op-amp when it is operating in inverting mode. However since the inverting input is a virtual earth (as a consequence of the very high open-loop voltage gain), several input resistors can be connected together to the inverting input without the voltages applied to each affecting the others. In this way, the op-amp can be used as a **summing amplifier**, with an output voltage V_o that is a function of the sum of the input voltages.

Figure 6.9: The summing amplifier



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In Figure 6.9, the op-amp is essentially acting as an inverting amplifier, with three input resistors connected to the inverting input. The usual gain expression applies, modified in this case to

$$V_o = -\left(\frac{R_f}{R_1} \times V_1 + \frac{R_f}{R_2} \times V_2 + \frac{R_f}{R_3} \times V_3\right) \quad (6.2)$$

. . . .

If $R_f = R_1 = R_2 = R_3$, then Equation 6.2 reduces to $V_o = -(V_1 + V_2 + V_3)$. In other words the output voltage is the sum of the input voltages (but with opposite polarity).

Note that by suitable choice of resistor values, the output voltage could be an amplified version of the sum of the input voltages (if $R_f >$ the input resistances), or an attenuated version (if $R_f <$ the input resistances). It is also possible to arrange for 'weighted' summation to take place if the input resistors are not of equal resistance.

The virtual earth at the inverting input of the op-amp is known as the **summing point** of the circuit. This is because the inputs applied to this point are all added together. In addition, none of the inputs influences the others, even although they are all connected together. This circuit is often used as a mixer or a pre-amplifier, since it can be used to add or 'mix' signals obtained from audio sources such as microphones and guitar pick-ups, before being amplified in a power amplifier.

15 min

The summing amplifier

This on-line simulation allows you to verify that the output of a summing amplifier is given by the expression $V_o = -(V_1 + V_2 + V_3)$. *To do this, you can change the input voltages and check that the output voltage is as expected.*

Full instructions are given in the simulation.

---

1. An op-amp connected in inverting mode can perform the mathematical operation of addition.
2. The output of a summing amplifier, when $R_f = R_1 = R_2 = R_3$ is given by $V_o = -(V_1 + V_2 + V_3)$

A similar circuit to that shown in Figure 6.9 is used to convert digital signals to analogue ones. Modern computers can only deal with digital signals (mostly in the form of voltages being ON or OFF). For example the information on an audio CD is stored as a series of 'pips' that are translated as a stream of pulses. This stream of ON or OFF pulses has to be converted back into an analogue signal before being passed to a loudspeaker. This conversion is carried out by a circuit known as a **digital to analogue converter** (DAC for short).

A DAC consists of a summing amplifier where the input voltages are digital - each input can only be HIGH or LOW. The input resistors have resistances that are in the ratios 1 : 2 : 4 : 8 ... and so on. This is known as a 'binary weighted network'.

If in Figure 6.9 $R_f = R_1 = R$; $R_2 = 2R$; $R_3 = 4R$, and V_1, V_2 and V_3 can either be HIGH or LOW only, then the output voltage V_o is given by $V_o = -\left(V_1 + \frac{1}{2}V_2 + \frac{1}{4}V_3\right)$. In this way, the streams of 3-bit binary numbers represented by V_1, V_2 and V_3 are converted into an analogue signal by weighted addition. Three inputs would only give 8 'steps' to the 'analogue' signal created ($8 = 2^3$). With more inputs, 16-, or even 32-bit binary streams can be converted into analogue signals, making the resulting signal smoother and a more accurate copy of the original.

6.3.2.3 Saturation in an inverting mode amplifier

We have already seen that an op-amp can be driven into saturation. With an op-amp connected in inverting mode, this can happen in one of two ways. Increasing the gain too much and increasing the input voltage too much can both send an inverting amplifier into saturation, as the next activity shows.

Saturation in an inverting amplifier

25 min

This on-line simulation allows you to observe the effects of driving an inverting mode amplifier into saturation.
There are two ways that this can happen. If the gain is too great for the input voltage used or if the input voltage is too great for the gain that has been set.
In this simulation, you can adjust both the gain (by altering the values of the input and feedback resistors), and the input voltage.

Full instructions are given in the simulation.

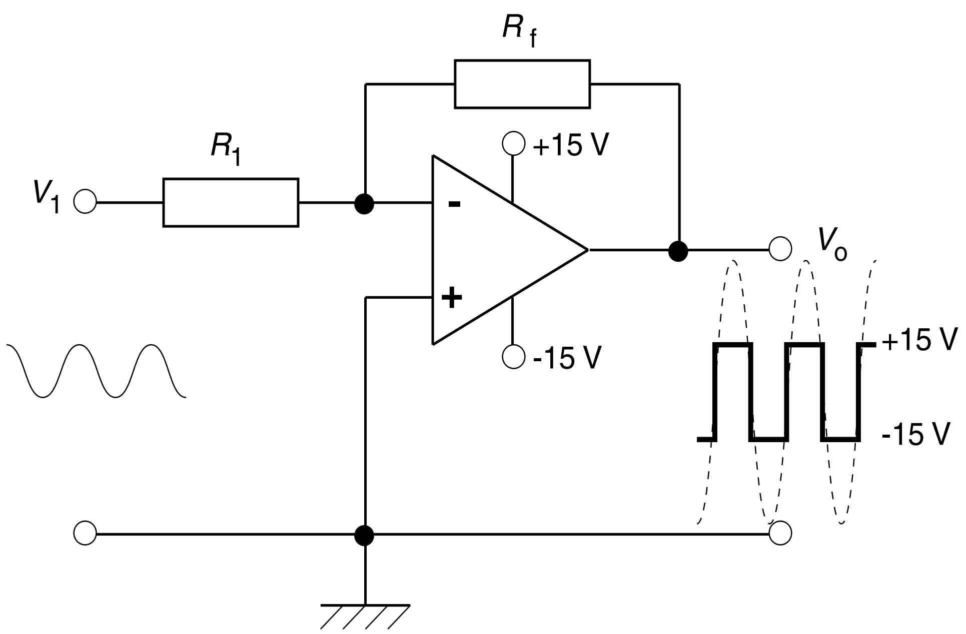
An inverting mode amplifier can be driven into saturation by

- increasing the gain of the amplifier
- increasing the magnitude of the input signal.

An amplifier that is being used to amplify an audio signal should not be driven into saturation since the output will be distorted. However there are some situations when it is advantageous to operate an amplifier under saturation conditions.

Consider an inverting mode amplifier with a gain of 100 that is operated from a ±15 V power supply. If the input signal to this amplifier is an a.c. signal with a 1 V peak amplitude, then the output voltage should be 100 V peak a.c. Since the power supply is only ±15 V, then the output voltage is limited to ±15 V also. (Actually the saturation voltage V_{sat} is slightly less than the supply voltage, but for simplicity we have taken it to equal the supply voltage.) The resulting waveform obtained from the output of the op-amp is a very close approximation to a square wave. This is shown in Figure 6.10.

Figure 6.10: Square wave generation



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6.3.3 In differential mode

The basic action of the op-amp when connected in differential mode is to perform the mathematical operation of subtraction. This was dealt with earlier in this Topic.

The differential amplier is also used in monitoring and control situations.

6.3.3.1 Monitoring

The differential amplifier can be used to amplify the very small voltage change produced when the resistance of a sensor in one limb of a Wheatstone bridge circuit changes in response to some change in physical conditions. In this way, the differential amplifier can be used to monitor changes.

The types of resistive sensors that can be used include:

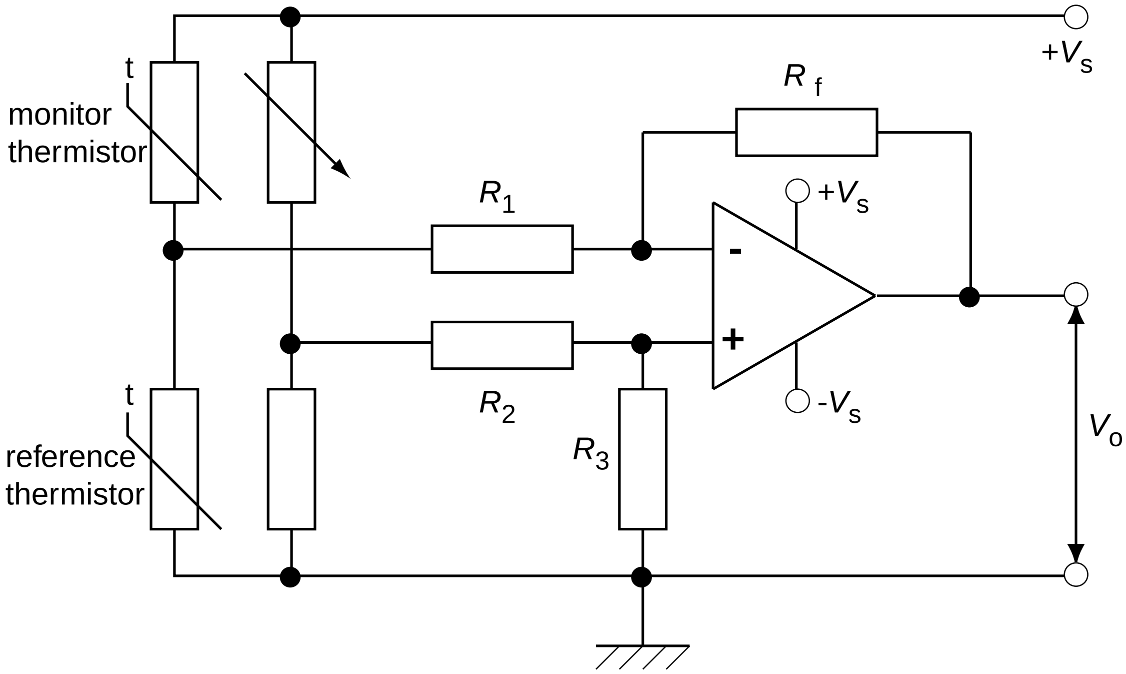
- thermistor, to respond to changes in temperature;
- light dependent resistor (LDR), to respond to changes in light intensity;
- strain gauge, to respond to strain when an object is subjected to a load.

It is usual to include a second resistive sensor of the same type in one other arm of the Wheatstone bridge, this second one not being subjected to the physical change. This second sensor acts as a reference to compensate for other physical changes that could have an effect on the output of the circuit.

Also one other resistor in the Wheatstone bridge network is sometimes variable, so that the bridge can be balanced and give zero output voltage under suitable conditions.

An op-amp circuit incorporating a Wheatstone bridge arrangement is shown in Figure 6.11. This circuit shows a thermistor, to monitor temperature changes, but any other resistive sensors could be substituted, to monitor different changes.
In this circuit, as the temperature of the monitor thermistor decreases, its resistance increases. Therefore the potential at the junction of the thermistors deceases. Since the potential at the other junction of the Wheatstone bridge is fixed, V_o increases.

Figure 6.11: Differential amplifier with Wheatstone bridge for monitoring



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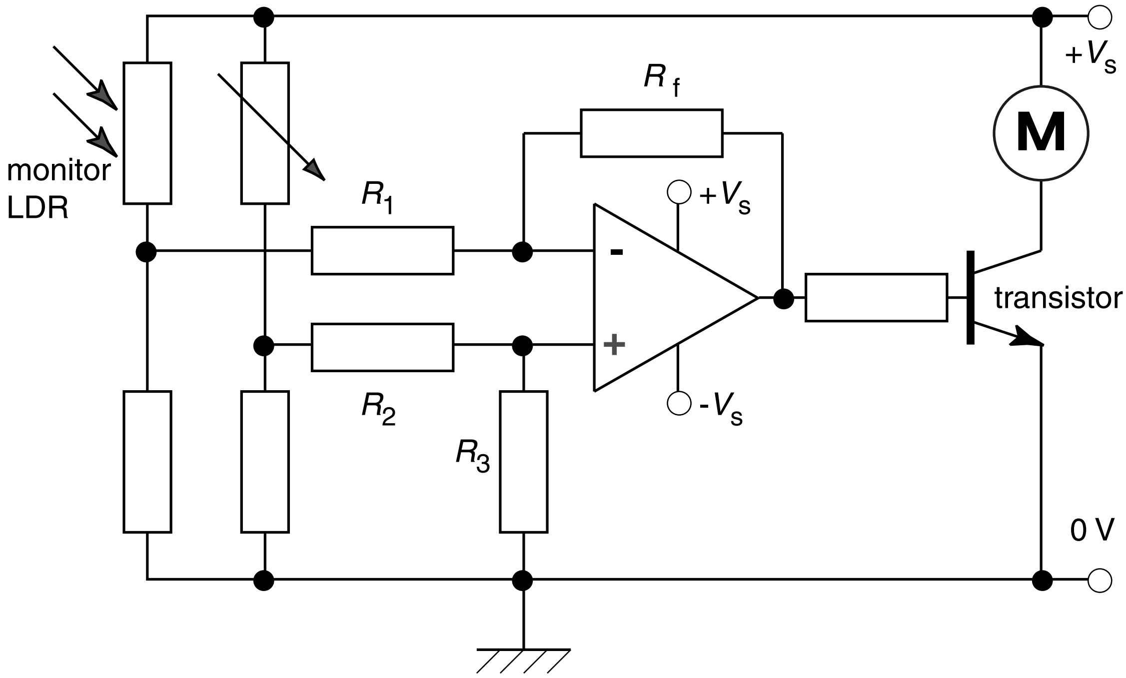
6.3.3.2 Controlling output devices

As well as being able to monitor physical changes, the differential amplifier is often used to react to the change and cause something to happen in the output part of a system. Very often, the output devices used, such as motors or heaters, draw a larger current than the output of an op-amp can handle. In order to supply sufficient current to an output device, a transistor is often connected to the output of the op-amp, and this transistor acts as a power amplifier to operate the output device.

The circuit shown in Figure 6.12 switches on the motor when the light level falling on the LDR drops below a certain level. This system could be used to draw the curtains when night falls, for example.

In this circuit, as the light level decreases the resistance of the LDR increases. Therefore the potential at the inverting input of the op-amp decreases. Since the potential at the other junction of the Wheatstone bridge is fixed, the potential at the base of the transistor increases, eventually turning on the motor.

Figure 6.12: Differential op-amp circuit used for monitoring and control



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Extra Help: Explaining the control of transistors and op-amps

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Quiz 2 Uses of op-amps

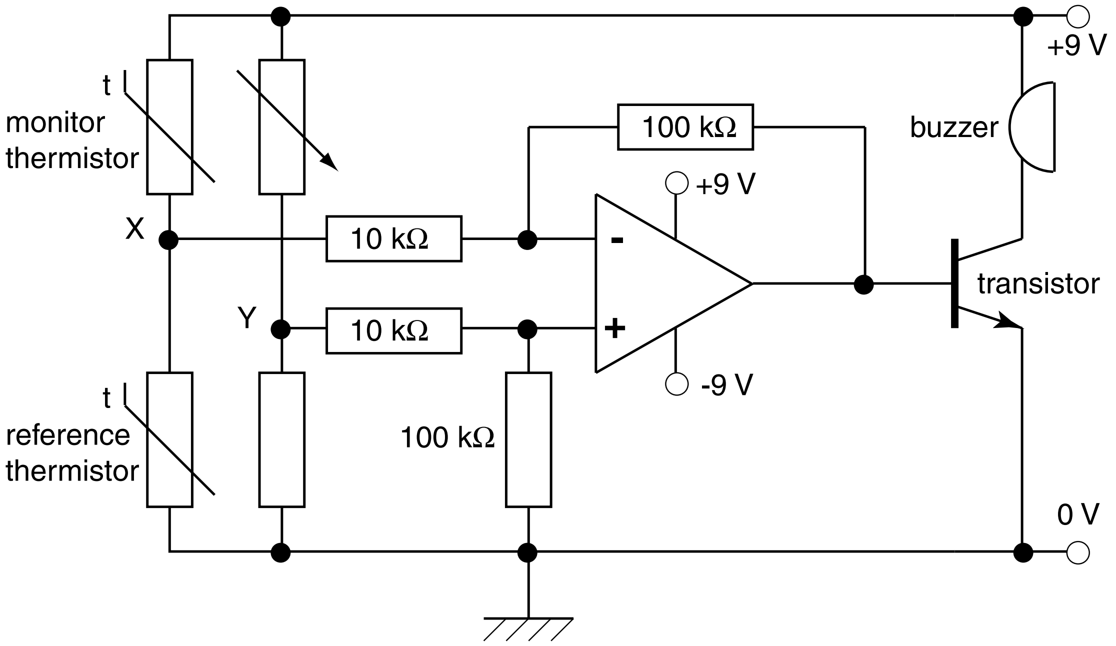
15 min

Multiple choice quiz.

First try the questions. If you get a question wrong or do not understand a question, there are 'Hints' at the the back of the book . The hints are in the same order as the questions. If you read the hint and still do not understand then ask your tutor.

The first three questions in this quiz refer to the following circuit diagram.

Figure 6.13:



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Q6: In Figure 6.13, what physical change does the circuit respond to?

a) a change in light level
b) a change in temperature
c) a change in load applied
d) a change in pressure
e) a change in sound level

. . . .

Q7: In Figure 6.13, the potential at point X is 4.1 V and the potential at point Y is 4.3 V.
What is the potential applied to the base of the transistor?

a) 2.0 V
b) 0.20 V
c) 0.02 V
d) -0.02 V
e) -2.0 V

. . . .

Q8: Why is a transistor needed in Figure 6.13?

a) because the physical change is not great enough to sound the buzzer
b) to switch the buzzer off after a pre-set time period
c) as a power amplifier to allow enough current to switch the buzzer on
d) because the op-amp needs a dual-rail power supply but the buzzer only needs a single power supply
e) because the inputs to the op-amp are connected to a Wheatstone bridge circuit

. . . .

Q9: When an op-amp is used as a summing amplifier, it is connected

a) with positive feedback.
b) with no feedback.
c) as a differential amplifier.
d) as an inverting amplifier.
e) to a Wheatstone bridge input circuit.

. . . .

Q10: An op-amp is used to obtain a square wave from a sine wave.
To do this, the op-amp is connected as

a) a differential amplifier, and operated below the saturation level.
b) a DAC.
c) a summing amplifier.
d) an inverting amplifier, and operated below the saturation level.
e) an inverting amplifier, and driven into saturation.

. . . .

. . . .

6.4 Summary

By the end of this Topic you should be able to:

- identify circuits where the op-amp is being used in the differential mode;
- state that a differential amplifier amplifies the potential difference between its two inputs;
- derive and state the differential mode gain expression $V_o = (V_2 - V_1)\, {}^{R_f}\!/_{R_1}$;
- carry out calculations using the differential mode gain expression;
- describe the use of an op-amp as a comparator;
- explain how the mathematical operations of multiplication, division, addition and subtraction can be carried out by an op-amp;
- describe what a D to A converter is and how it operates;
- describe the effects of saturation in an inverting mode amplifier when the gain of the amplifier is increased;
- describe the effects of saturation in an inverting mode amplifier when the magnitude of the input signal is increased;

- explain how saturation in an inverting mode amplifier can be used to generate a square wave from a sine wave;
- describe how to use a differential amplifier with resistive sensors connected in a Wheatstone bridge arrangement;
- describe how an op-amp can be used to control external devices via a transistor.

6.5 Assessment

Online assessments

Two online tests are available. Each test should take you no more than 20 minutes to complete. Test one contains questions on op-amps being used in differential mode. Test two contains questions on the uses of op-amps.

Glossary

a.c.

Alternating current. The current from an a.c. supply constantly changes direction.

alternator

An a.c. generator.

amplifier

An electronic circuit with at least one input and one output, that is designed to process the waveform obtained from a physical quantity (such as a sound) and produce an output that is an enlarged (amplified) copy of this waveform.

capacitance

The ratio of electric charge to potential difference between any two conductors separated by an insulating material. The capacitance of a system of conductors describes the ability of the system to store electric charge.

capacitor

Two (or more) conductors separated by an insulator that can be used to store charge.

clipped

An output signal is clipped when the peak values that it should have are greater than $\pm$ the supply voltage.

closed-loop gain

The effective gain of an amplifier when feedback is applied. Given the symbol A.

common-mode rejection

The property of an op-amp, when it is used in the differential mode, of ignoring or 'rejecting' a signal or voltage that is present on both inputs.

conductor

A material through which electric charge can flow.

d.c.

Direct current. The current from a d.c. supply always moves in the same direction around an electric circuit.

differential mode

A mode of operation of an op-amp where the output of the op-amp V_o depends on the difference between the voltage V_1 applied to the inverting input, and the voltage V_2 applied to the non-inverting input.

digital to analogue converter

A circuit that converts digital signals into an analogue waveform.

electric field

The region around a charged object where the charge exerts a force on other charges.

electromotive force

The electromotive force of a source is the electrical potential energy that is given to each unit of charge that passes through the source.

frequency

The number of complete events that happen in a given time period. The unit used to measure frequency is the hertz (Hz) where one hertz is one wave or one cycle per second.

fundamental unit of charge *e*

The magnitude of charge carried by one electron or one proton. Equal to 1.60×10^{-19} coulombs.

high-pass filter

An electrical filter that allows high frequency signals to pass, but blocks low frequency signals.

impedance

The opposition of a component to an alternating current in an a.c. circuit. The ratio of r.m.s. potential difference across a component to r.m.s. current through it, measured in Ω.

internal resistance

The opposition to current in a source of electrical energy.

inverting mode

A mode of operation of an op-amp where the single input voltage V_1 is applied to the inverting input, and a proportion of the output is fed back to the inverting input. The non-inverting input is connected to the 0 V line.

linear region

The part of an amplifier's operating range where the output waveform is the same shape as the input waveform.

load resistor

The resistor, or combination of resistors, that forms the external part of an electrical circuit.

lost volts

The potential difference that is used to drive a current through the internal resistance of a source. Lost volts is given by the expression Ir where r is the internal resistance of the source.

negative feedback

Negative feedback is when a proportion of the output of an op-amp is fed back to the inverting input of the op-amp.

noise

An unwanted electrical signal picked up at the inputs of an op-amp. Most frequently mains pick-up.

period

The time to make one complete wave. Period is measured in seconds.

potential difference

The potential difference between two points is a measure of the work done in moving one coulomb of charge between the two points.

rectifier circuit

A circuit that uses diodes to convert an a.c. signal to a d.c. signal.

resistance

The opposition that a conductor offers to a current through it. Defined as the ratio of potential difference across the conductor to the current through it.

saturation

An amplifier is driven into saturation when the input signal is so large that the output is at its greatest value, ie $\pm V_s$.

short-circuit current

The maximum current that a source can supply. The current drawn from the supply when there is zero resistance in the external circuit (when the terminals of the source are joined together or 'short-circuited').

summing amplifier

A circuit that has an op-amp connected in the inverting mode with more than one input resistor connected to the inverting input. By suitable choice of resistors, the output is the sum of the input voltages.

summing point

The virtual earth at the inverting input of an op-amp connected as a summing amplifier. So called because it is the point at which all the inputs are added together without influencing each other.

terminal potential difference (t.p.d.)

The terminal potential difference is the potential difference that appears across the terminals of a source when the source is supplying a current to a circuit. It is the potential difference that appears across the external resistance, or load resistor, in the circuit.

virtual earth

A point in a circuit that is at earth potential, although it is not actually connected to earth.

voltage gain

The amount by which an amplifier increases a voltage. Voltage gain is given by the expression

$$A = \frac{\text{change in output voltage}}{\text{change in input voltage}}$$

Wheatstone bridge circuit

A resistor network, consisting of a series/parallel combination that can be used to measure resistance when balanced. In the out-of-balance condition, a small p.d. that is proportional to the change in resistance is produced.

Hints for activities

Topic 1: Electric fields

Quiz 1 Charges, forces and fields

Hint 1:

The magnitude of the charge on a proton is equal to the fundamental charge.

Hint 2:

Read the following paragraph and try again

An object can be charged by adding negatively-charged particles such as electrons, in which case it becomes negatively charged. An object may also be charged by removing electrons, making it positively charged. A negatively-charged object attracts a positively-charged object and objects that have similar charges repel each other.

Hint 3:

The definition of an**electric field** is: a region in space where a charge experiences a force.

Hint 4:

On screen simulation.

Hint 5:

Electric field strength is the force per unit charge

Quiz 2 Work done and potential difference

Hint 1:

The definition of **potential difference** is: the potential difference between two points is the work done in moving one coulomb of charge between the two points.

Hint 2:

In all Physics relationships, units are equivalent on both sides of an equation - apply this to $E = QV$.

Hint 3:

This is a straight application of $E = QV$.

Hint 4:

First calculate the total charge of 40 excess electrons - this is equal to the charge on the oil drop. The use $E = QV$.

Hint 5:

Electrical energy is converted to kinetic energy - see Mechanics and Properties of Matter, section titled Energy, if you can't remember the formula for kinetic energy.

Topic 2: Resistors in circuits

Quiz 1 e.m.f. and internal resistance

Hint 1:

$$\begin{aligned} E =& IR + Ir \\ E =& V + Ir \\ E - V =& Ir \\ r =& \frac{E - V}{I} \end{aligned}$$

Hint 2:

$$\begin{aligned} E =& IR + Ir \\ E =& V + Ir \\ E - V =& Ir \\ r =& \frac{E - V}{I} \end{aligned}$$

In the equation above, the term V is the potential difference that appears at the terminals of the source. For this reason it is called the terminal potential difference (t.p.d.). The term Ir represents the potential difference that is ’lost’ across the internal resistance of the source, and never appears in the external circuit. This term is often called the ’lost volts’. It is worth noting that both E and r are properties of the source and are constant (at least in the short term, if the source is not abused). On the other hand both the terminal potential difference and the lost volts depend on the current taken from the source, and so are not constant.

Hint 3:

This is a straight application of

$$r = \frac{E - V}{I}$$

.

Hint 4:

When a battery is short-circuited, the internal resistance of the battery is the only resistance in the circuit.

Hint 5:

First find the total resistance in the circuit and use this to find the current.

Quiz 2 Resistors in series and parallel

Hint 1:

See the section titled Resistors in series.

Hint 2:

See the section titled Resistors in parallel.

Hint 3:

First work out the total resistance of the circuit.

Hint 4:

The equivalent resistance of any number of resistors connected in parallel is always less than the smallest resistance.

Hint 5:

The equivalent resistance of two identical resistors connected in parallel is equal to half the resistance of either.

Quiz 3 The Wheatstone bridge

Hint 1:

See the section titled The balanced Wheatstone bridge circuit.

Hint 2:

This is a straight application of the relationship for a balanced Wheatstone bridge refer to the section with this title.

Hint 3:

"The ratio R_1:R_2 is equal to 3:2."

Hint 4:

See the section titled The out-of-balance Wheatstone bridge

Hint 5:

The out-of-balance p.d depends on the **change** in resistance in one branch.

Topic 3: Alternating current and voltage

Quiz 1 Frequency of a.c.

Hint 1:

See the introduction at the beginning of this topic.

Hint 2:

$$T = \frac{1}{f}$$

Hint 3:

This is a straight application of the relationship

$$T = \frac{1}{f}$$

.

Hint 4:

This is a straight application of the relationship

$$T = \frac{1}{f}$$

.

Hint 5:

First count the number of squares for one complete cycle - e.g between adjacent troughs. Multiply this by the time-base setting to get the period.

Quiz 2 Voltage and frequency

Hint 1:

$$V_{r.m.s.} = \frac{V_m}{\sqrt{2}}$$
$$V_m = \sqrt{2} \times V_{r.m.s.}$$

Hint 2:

This is a straight application of

$$V_{r.m.s.} = \frac{V_m}{\sqrt{2}}$$
$$V_m = \sqrt{2} \times V_{r.m.s.}$$

.

Comparing the energy transformed by a.c. and d.c. supplies.

Hint 3:

The peak voltage must not be greater than the voltage at which the insulation in the capacitor breaks down.

Hint 4:

Power varies with the **square** of the r.m.s. current.

Hint 5:

Changing frequency has no effect on the current in circuit that contains only resistors.

Topic 4: Capacitance

Quiz 1 Capacitors

Hint 1:

"In all Physics relationships, units are equivalent on both sides - apply this to the following relationship.

$$Q = CV$$

or $$C = \frac{Q}{V}$$

Hint 2:

This is a straight application of the following relationship from the section titled Capacitance.

$$Q = CV$$
$$\text{or} \quad C = \frac{Q}{V}$$

Hint 3:

This is a straight application of the following relationship from the section titled Capacitance.

$$Q = CV$$
$$\text{or} \quad C = \frac{Q}{V}$$

Hint 4:

This is a straight application of the following relationship from the section titled Energy stored in a capacitor.

$$W = \tfrac{1}{2}QV$$
$$W = \tfrac{1}{2}CV^2$$
$$W = \tfrac{1}{2}\frac{Q^2}{C}$$

Hint 5:

Use the data given in the first sentence to find the capacitance - then use this with the data given in the second sentence.

Quiz 2 Capacitors in circuits

Hint 1:

This is a straight application of the following relationship from the section titled Charging a capacitor.

$$I = \frac{E}{R}$$

Hint 2:

This is a straight application of Ohm's Law .

$$R = \frac{V}{I}$$

Hint 3:

The sum of the p.d. across the capacitor plus the p.d across the resistor is equal to the p.d across the battery.

Hint 4:

Run through the following interactive simulation and try again.

Hint 5:

See the Section titled on Applications of capacitors.

Topic 5: Analogue electronics 1

Quiz 1 Operational amplifiers

Hint 1:

See the section titled Operational amplifiers.

Hint 2:

See the section titled Operational amplifiers for the properties of an ideal op-amp.

Hint 3:

See the following equation, or review the section titled Operational amplifiers.

$$V_o = A_o(V^+ - V^-)$$

Hint 4:

This is a straight application of the following relationship.

$$V_o = A_o(V^+ - V^-)$$

Hint 5:

Remember the op-amp can become saturated - see the end of the section titled Operational amplifiers.

Quiz 2 The op-amp in inverting mode

Hint 1:

This is a straight application of the following relationship.

$$\frac{V_o}{V_1} = -\frac{R_f}{R_1}$$

Hint 2:

This is a straight application of the following relationship.

$$\frac{V_o}{V_1} = -\frac{R_f}{R_1}$$

Hint 3:

This is a straight application of the following relationship.

The op-amp in inverting mode, with R_1 equal to R_f.

$$\frac{V_o}{V_1} = -\frac{R_f}{R_1}$$

Hint 4:

Did you notice that R_1 is greater than R_f.

Hint 5:

Did you remember about saturation of the op-amp?

Topic 6: Analogue electronics 2

Quiz 1 The op-amp in differential mode

Hint 1:

See the section titled The op-amp in differential mode - in particular the following 'point to note' :

This relationship only holds for the special case where ${R_f}/{R_1} = {R_3}/{R_2}$, which in fact is the normal situation when the op-amp is used in differential mode.

Hint 2:

This is a straight application of

$$V_o = (V_2 - V_1)\,\frac{R_f}{R_1}$$

.

Hint 3:

This is a straight application of

$$V_o = (V_2 - V_1)\,\frac{R_f}{R_1}$$

.

Hint 4:

The output signal is distorted if the op-amp becomes saturated - this means the output voltage cannot be greater than V_s.

Hint 5:

Did you remember about saturation of the op-amp?

Quiz 2 Uses of op-amps

Hint 1:

The clue is in the name **therm**istor!

Hint 2:

This is a straight application of

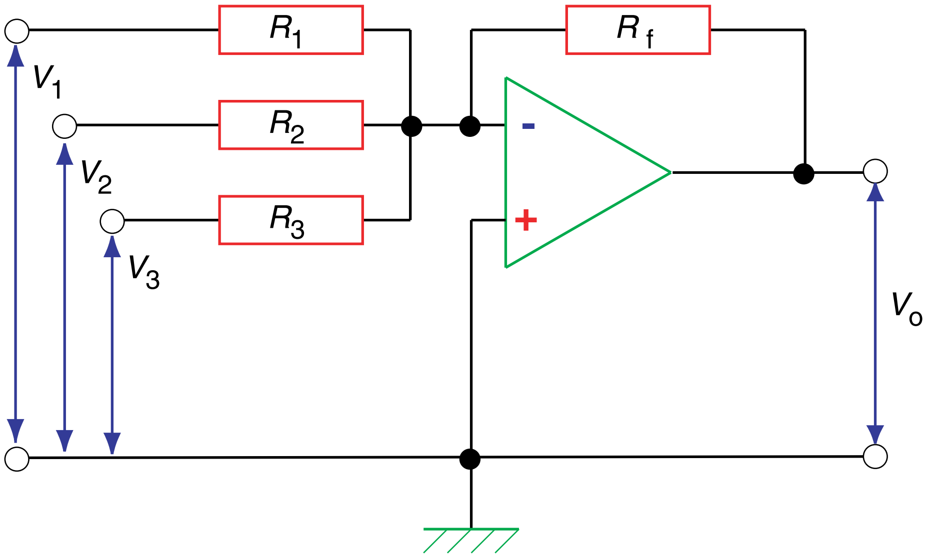
$$V_o = (V_2 - V_1)\frac{R_f}{R_1}$$

Hint 3:

See 'Controlling output devices' in the section titled Uses of op-amps in differential mode.

Hint 4:

Look at the following diagram - which input or inputs are being used in the summing amplifier?



The summing amplifier

Hint 5:

Look at the following animation - vary the amplitude of the input wave and observe what happens to the output.

Square wave generation

Answers to questions and activities

1 Electric fields

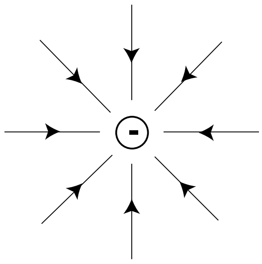
Quiz 1 Charges, forces and fields (page 4)

Q1: d) 6.25 x 10^{18}

Q2: e) Both spheres are positively charged.

Q3: a) experiences a force.

Q4: c)



Q5: b) N C^{-1}

Quiz 2 Work done and potential difference (page 10)

Q6: a) a measure of the work done in moving one coulomb of charge between the two points.

Q7: e) joule per coulomb

Q8: c) 40 J

Q9: b) 30 V

Q10: d) 4.2 x 10^{6} m s^{-1}

2 Resistors in circuits

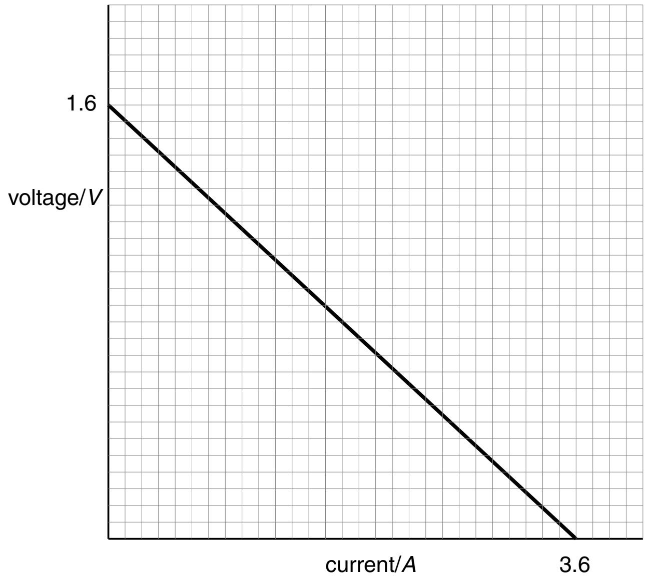
Ohmic conductors (page 15)

Using the relationship $R = {}^{V}/_{I}$, the following values of resistance are obtained for each set of potential difference and current values.

Potential difference (V)	1.0	2.3	3.7	4.8	7.3	8.9
Current (A)	0.18	0.41	0.66	0.86	1.3	1.6
Resistance (Ω)	5.6	5.6	5.6	5.6	5.6	5.6

Since the resistance is constant, then the conductor obeys Ohm's law and is ohmic.

Measuring the e.m.f. and internal resistance of a source (page 18)



1. The e.m.f. *E* is the terminal potential difference when the cell is not driving a current, so this is the intercept of the graph on the voltage axis.

$$E = 1.6 \text{ V}$$

2. The internal resistance *r* is given by $r = \frac{E-V}{I}$, and this is the negative of the gradient of the graph.

$$r = -\frac{1.6 - 0.5}{0 - 2.5}$$
$$r = -\frac{1.1}{-2.5}$$
$$r = 0.44\ \Omega$$

3. The short-circuit current is the current when all of the e.m.f. appears across the internal resistance. From the graph, this is the intercept on the current axis when the terminal potential difference is zero.

$$I_{sc} = \frac{E}{r}$$
$$I_{sc} = \frac{1.6}{0.44}$$
$$I_{sc} = 3.6 \text{ A}$$

Quiz 1 e.m.f. and internal resistance (page 19)

Q1: c) $\frac{E-V}{I}$

Q2: d) Ir

Q3: a) 0.10 Ω

Q4: e) 0.608 Ω

Q5: c) 7.5 V

Quiz 2 Resistors in series and parallel (page 24)

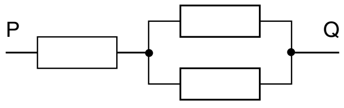
Q6: a) $R = R_1 + R_2 + R_3$

Q7: e) $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

Q8: c) 10 mA

Q9: b) 9.48 Ω

Q10: d)



Quiz 3 The Wheatstone bridge (page 29)

Q11: d) $\frac{R_1}{R_2} = \frac{R_3}{R_4}$

Q12: a) 20 Ω

Q13: b) R_3 120 Ω; R_4 80 Ω

Q14: c) proportional to ΔR

Q15: c) 25 mV

3 Alternating current and voltage

Quiz 1 Frequency of a.c. (page 37)

Q1: e) (i) and (iii) only

Q2: b) $T = \frac{1}{f}$

Q3: d) 20 ms

Q4: d) 40 Hz

Q5: a) 125 Hz

Quiz 2 Voltage and frequency (page 44)

Q6: b) $V_m = \sqrt{2} \times V_{r.m.s.}$

Q7: d) 14 V

Q8: b) 11 V

Q9: a) 100 W

Q10: e) 130 mA

4 Capacitance

Energy stored on a capacitor (page 51)

We will begin with the equation $W = \frac{1}{2}QV$. We can use Equation 4.2, $Q = CV$, and hence substitute for *Q*:

$$\begin{aligned} W &= \tfrac{1}{2}QV \\ \therefore W &= \tfrac{1}{2} \times (CV) \times V \\ \therefore W &= \tfrac{1}{2}CV^2 \end{aligned}$$

We can also rearrange Equation 4.2:

$$\begin{aligned} Q &= CV \\ \therefore V &= \frac{Q}{C} \end{aligned}$$

We can now substitute for *V* in the energy equation:

$$\begin{aligned} W &= \tfrac{1}{2}QV \\ \therefore W &= \tfrac{1}{2}Q \times \frac{Q}{C} \\ \therefore W &= \tfrac{1}{2}\frac{Q^2}{C} \end{aligned}$$

So we have shown that the three equations for the energy stored on a capacitor are equivalent.

Quiz 1 Capacitors (page 52)

Q1: a) 1 C V^{-1}

Q2: d) 6.0 μC

Q3: c) 22 nF

Q4: b) 5.1 mJ

Q5: b) 1.8 V

Quiz 2 Capacitors in circuits (page 60)

Q6: b) 250 mA

Q7: c) 1.8 V

Q8: c) 7.2 V

Q9: a) increases.

Q10: e) high-pass filter.

5 Analogue electronics 1

Voltage gain calculations (page 67)

(a)

$$\begin{aligned} V_o =& A_o(V^+ - V^-) \\ \therefore V_o =& 10^5(10 \times 10^{-6} - 0) \\ \therefore V_o =& +1 \text{ V} \end{aligned}$$

(b)

$$\begin{aligned} V_o =& A_o(V^+ - V^-) \\ \therefore V_o =& 10^5(80 \times 10^{-6} - 180 \times 10^{-6}) \\ \therefore V_o =& -10 \text{ V} \end{aligned}$$

(c)

$$\begin{aligned} V_o =& A_o(V^+ - V^-) \\ \therefore V_o =& 10^5(300 \times 10^{-6} - 100 \times 10^{-6}) \\ \therefore V_o =& +20 \text{ V} \end{aligned}$$

Since $V_o > +V_s$ the output voltage is limited to +15 V (the positive supply voltage).

(d)

$$\begin{aligned} V_o =& A_o(V^+ - V^-) \\ \therefore V_o =& 10^5(0 - 250 \times 10^{-6}) \\ \therefore V_o =& -25 \text{ V} \end{aligned}$$

Since $V_o > -V_s$ the output voltage is limited to -18 V (the negative supply voltage).

Quiz 1 Operational amplifiers (page 67)

Q1: c) two inputs and only one output.

Q2: e) (i), (ii) and (iii)

Q3: b) $\frac{V_o}{V^+ - V^-}$

Q4: d) - 4.0 V

Q5: b) +10 V

Calculations on the op-amp in inverting mode (page 70)

(a)

$$\begin{aligned} V_o =& -V_1\left(\frac{R_f}{R_1}\right) \\ \therefore V_o =& -(-3)\left(\frac{5000}{1000}\right) \\ \therefore V_o =& +15 \text{ V} \end{aligned}$$

$V_o > V_1$ so the signal is amplified.

(b)

$$V_o = -\,V_1 \left(\frac{R_f}{R_1}\right)$$

$$\therefore V_o = -\,2 \left(\frac{20\ 000}{10\ 000}\right)$$

$\therefore V_o = -\,4$ V

$V_o > V_1$ so the signal is amplified.

(c)

$$V_o = -\,V_1 \left(\frac{R_f}{R_1}\right)$$

$$\therefore V_o = -\,(-1) \left(\frac{2000}{10\ 000}\right)$$

$\therefore V_o = +\,0.2$ V

$V_o < V_1$ so the signal is attenuated.

(d)

$$V_o = -\,V_1 \left(\frac{R_f}{R_1}\right)$$

$$\therefore V_o = -\,4 \left(\frac{10\ 000}{5000}\right)$$

$\therefore V_o = -\,8$ V

Since $V_o > -V_s$ the output voltage is limited to -5 V (the negative supply voltage).

Quiz 2 The op-amp in inverting mode (page 72)

Q6: a) -10

Q7: e) +4.0 V

Q8: d) -25 mV

Q9: d) 250 mV

Q10: c) -5.0 V

6 Analogue electronics 2

Calculations on the op-amp in differential mode (page 77)

(a)

$$\begin{aligned} V_o &= (V_2 - V_1)\,\frac{R_f}{R_1} \\ \therefore V_o &= (1 - (-2))\,\frac{5000}{1000} \\ \therefore V_o &= 3 \times 5 \\ \therefore V_o &= +\,15 \text{ V} \end{aligned}$$

$V_o > (V_2 - V_1)$ so the signal is amplified.

(b)

$$\begin{aligned} V_o &= (V_2 - V_1)\,\frac{R_f}{R_1} \\ \therefore V_o &= (-4 - 2)\,\frac{20000}{10000} \\ \therefore V_o &= -\,6 \times 2 \\ \therefore V_o &= -\,12 \text{ V} \end{aligned}$$

$V_o > (V_2 - V_1)$ so the signal is amplified.

(c)

$$\begin{aligned} V_o &= (V_2 - V_1)\,\frac{R_f}{R_1} \\ \therefore V_o &= (14 - (-1))\,\frac{2000}{10000} \\ \therefore V_o &= 15 \times \frac{1}{5} \\ \therefore V_o &= +\,3 \text{ V} \end{aligned}$$

$V_o < (V_2 - V_1)$ so the signal is attenuated.

(d)

$$\begin{aligned} V_o &= (V_2 - V_1)\,\frac{R_f}{R_1} \\ \therefore V_o &= (-15 - (-12))\,\frac{4000}{12000} \\ \therefore V_o &= -\,3 \times \frac{1}{3} \\ \therefore V_o &= -\,1 \text{ V} \end{aligned}$$

$V_o < (V_2 - V_1)$ so the signal is attenuated.

(e)

$$\begin{aligned} V_o &= (V_2 - V_1)\,\frac{R_f}{R_1} \\ \therefore V_o &= (1 - (-2))\,\frac{10000}{2000} \\ \therefore V_o &= 3 \times 5 \\ \therefore V_o &= +\,15 \text{ V} \end{aligned}$$

Since $V_o > V_s$ the output voltage is limited to +12 V (the positive supply voltage).

(f)

$$\begin{aligned} V_o &= (V_2 - V_1)\,\frac{R_f}{R_1} \\ \therefore V_o &= (-6 - 4)\,\frac{10000}{5000} \\ \therefore V_o &= -\,10 \times 2 \\ \therefore V_o &= -\,20 \text{ V} \end{aligned}$$

Since $V_o > -V_s$ the output voltage is limited to -5 V (the negative supply voltage).

Quiz 1 The op-amp in differential mode (page 81)

Q1: b) ${}^{R_f}/_{R_1} = {}^{R_3}/_{R_2}$

Q2: c) +100 mV

Q3: b) +7 mV

Q4: c) 600 kΩ

Q5: d) -10 V

Quiz 2 Uses of op-amps (page 90)

Q6: b) a change in temperature

Q7: a) 2.0 V

Q8: c) as a power amplifier to allow enough current to switch the buzzer on

Q9: d) as an inverting amplifier.

Q10: e) an inverting amplifier, and driven into saturation.