SCHOLAR Study Guide

SQA Advanced Higher Physics
Unit 3: Wave Phenomena

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SCHOLAR Study Guide Unit 3: SQA Advanced Higher Physics

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Contents

Topic 1

Introduction to Waves

Contents

Prerequisite knowledge

- *Radian measurement of angles (Mechanics topic 3)*
- *Simple harmonic motion (Mechanics topic 8)*

Learning Objectives

By the end of this topic you should be able to:

- *use all the terms commonly employed to describe waves;*
- *derive an equation describing travelling sine waves, and solve problems using this equation;*
- *show an understanding of the difference between travelling and stationary waves;*
- *calculate the harmonics of a number of stationary wave systems.*

1.1 Introduction

A wave is a travelling disturbance that carries energy from one place to another, but with no net displacement of the medium. You should already be familiar with transverse waves, such as light waves, where the oscillations are perpendicular to the direction in which the waves are travelling; and longitudinal waves, such as sound waves, where the oscillations of the medium are parallel to the direction in which the waves are travelling.

We begin this topic with a review of the basic definitions and terminology used to describe waves. We will discuss what terms such as 'frequency' and 'amplitude' mean in the context of light and sound waves. Sections 1.3 and 1.4 deal with travelling and stationary waves, and we will derive and use mathematical expressions to describe both of these sorts of waves.

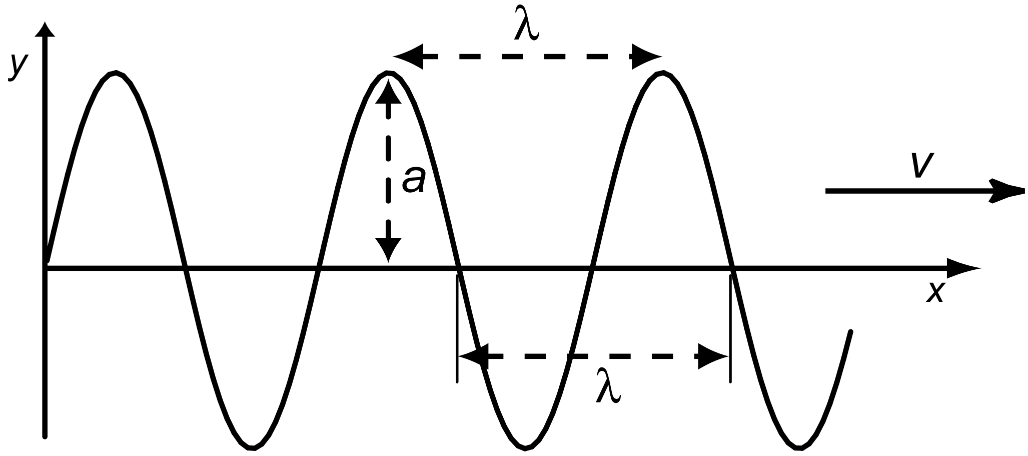
1.2 Definitions

Learning Objective

To introduce all the terms and definitions used to describe wave phenomena

The easiest way to explain wave phenomena is to visualise a train of waves travelling along a rope. The plot of displacement (*y*) against distance (*x*) is then exactly the same as the rope looks while the waves travel along it. We will be concentrating on sine and cosine waves as the most common sorts of waves, and the simplest to describe mathematically (see Figure 1.1).

Figure 1.1: Travelling sine wave



. . . .

As a train of waves passes along the rope, each small portion of the rope is oscillating in the *y*-direction. There is no movement of each portion of the rope in the *x*-direction, and when we talk about the **speed** v of the wave, we are talking about the speed at which the disturbance travels in the *x*-direction.

The **wavelength** λ is the distance between two identical points in the wave cycle, such as the distance between two adjacent crests. The **frequency** f is the number of complete waves passing a point on the *x*-axis in a given time period. When this time period is one second, f is measured in hertz (Hz), equivalent to s^{-1}. 1 Hz is therefore equivalent to one complete wave per second. The relationship between these three quantities is

$$v = f\lambda \tag{1.1}$$

....

The **periodic time** T (or simply the **period** of the wave) is the time taken to complete one oscillation, in the same way that the periodic time we use to describe circular motion is the time taken to complete one revolution. The period is related to the frequency by the simple equation.

$$T = \frac{1}{f} \tag{1.2}$$

....

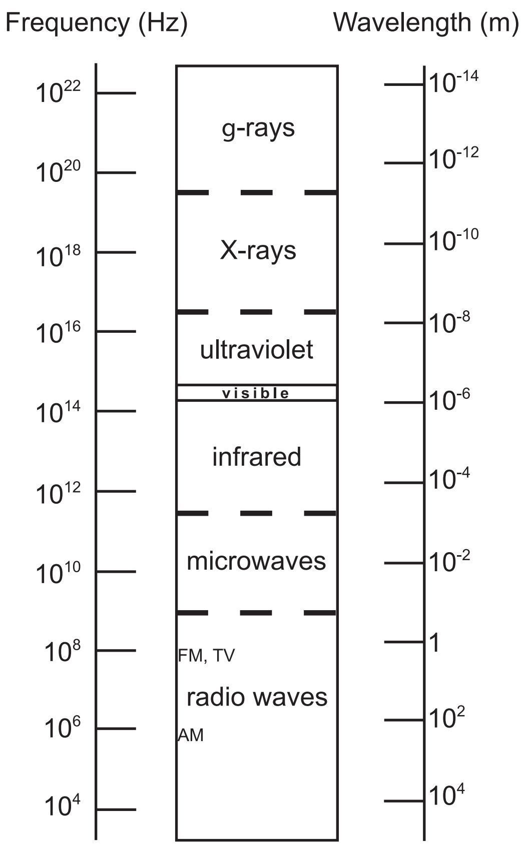
The **amplitude** a of the wave is the maximum displacement in the *y*-direction. As the waves pass along the rope, the motion of each portion of the rope follows the simple harmonic motion relationship

$$y = a\sin(2\pi ft)$$

We will use this expression to work out a mathematical relationship to describe wave motion later in this topic.

We normally use the wavelength to describe a light wave, or any member of the electromagnetic spectrum. The visible spectrum extends from around 700 nm (red) to around 400 nm (blue) (1 nm = 10^{-9} m). Longer wavelengths go through the infrared and microwaves to radio waves. Shorter wavelengths lead to ultraviolet, X-rays and gamma radiation (see Figure 1.2). The frequency of visible light is of the order of 10^{14} Hz (or 10^{5} GHz, where 1 GHz = 10^{9} Hz).

Figure 1.2: The electromagnetic spectrum



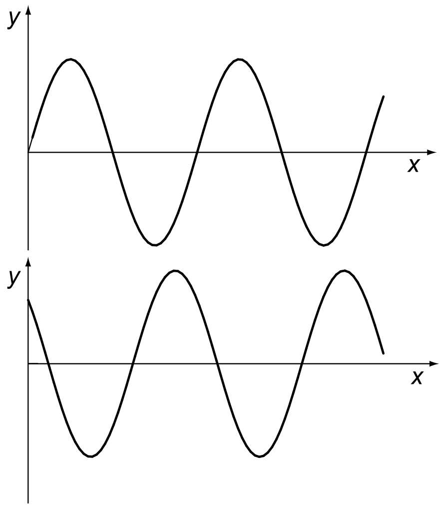
....

Sound waves are usually described by their frequency (or **pitch**). The human ear can detect sounds in the approximate range 20 Hz - 20 000 Hz. Sound waves with frequencies greater than 20 000 Hz are called ultrasonic waves, whilst those with frequencies lower than 20 Hz are infrasonic. The musical note middle C, according to standard concert pitch, has frequency 261 Hz.

The **irradiance** I of a wave tells us the amount of power falling on unit area, and is measured in W m^{-2}. The irradiance is proportional to a^2. In practical terms, this means the brightness of a light wave, or the loudness of a sound wave, depends on the amplitude of the waves, and increases with a^2.

Finally in this Section, let us consider the two waves in Figure 1.3.

Figure 1.3: Two out-of-phase sine waves



. . . .

In terms of amplitude, frequency and wavelength, these waves are identical, yet they are ‘out-of-step' with each other. We say they are **out of phase** with each other. A wave is an oscillation of a medium and the **phase** of the wave tells us how far through an oscillation a point in the medium is. Coherent waves **coherent waves** have the same speed and frequency (and similar amplitudes) and so there is a **constant phase relationship between two coherent waves.**

Quiz 1 Properties of waves

15 min

First try the questions. If you get a question wrong or do not understand a question, there are Hints. The hints are in the same order as the questions. If you read the hint and still do not understand then ask your tutor. All references in the hints are to online materials.

Useful data:

Speed of light c	3.00×10^{8} m s^{-1}

Q1: Which one of the following quantities should be increased to increase the volume (loudness) of a sound wave?

a) frequency
b) wavelength
c) speed
d) phase
e) amplitude

....

Q2: The visible spectrum has the approximate wavelength range 400 - 700 nm. What is the approximate frequency range of the visible spectrum?

a) 2.1×10^{14} - 1.2×10^{15} Hz
b) 4.3×10^{14} - 7.5×10^{14} Hz
c) 1.3×10^{15} - 2.3×10^{15} Hz
d) 4.3×10^{20} - 7.5×10^{20} Hz
e) 1.3×10^{20} - 2.3×10^{20} Hz

....

Q3: Which of these sets of electromagnetic waves are listed in order of **increasing** wavelength?

a) X-rays, infrared, microwaves.
b) radio waves, gamma rays, visible.
c) infrared, visible, ultraviolet.
d) X-rays, infrared, ultraviolet.
e) microwaves, visible, infrared.

....

Q4: What is the frequency of a beam of red light from a helium-neon laser, which has wavelength 633 nm?

a) 190 Hz
b) 2.11×10^{5} GHz
c) 4.74×10^{5} GHz
d) 2.11×10^{8} GHz
e) 4.74×10^{8} GHz

....

Q5: A laser produces light waves of average amplitude a m. The irradiance of the beam is 20 W m^{-2}. What is the irradiance if the average amplitude is increased to $3a$ m?

a) 6.7 W m^{-2}
b) 23 W m^{-2}
c) 60 W m^{-2}
d) 180 W m^{-2}
e) 8000 W m^{-2}

....

1.3 Travelling waves

Learning Objective

To derive an equation to represent a travelling sinusoidal wave

In this section, we will attempt to find a mathematical expression for a **travelling wave**. The example we shall consider is that of a train of waves being sent along a rope in the *x*-direction, but the expression we will end up with applies to all transverse travelling waves.

We will start by considering what happens to a small portion of the rope as the waves travel through it. From the definition of a wave we know that although the waves are travelling in the *x*-direction, there is no net displacement of each portion of the rope in that direction. Instead each portion is performing simple harmonic motion (SHM) in the *y*-direction, about the y = 0 position, and the SHM of each portion is slightly out of step (or phase) with its neighbours.

The displacement of one portion of the rope is given by the SHM equation

$$y = a\sin(2\pi f t)$$

where y is the displacement of a particle at time t, a is the amplitude and f is the frequency of the waves.

The wave disturbance is travelling in the *x*-direction with speed v. Hence at a distance x from the origin, the disturbance will happen after a time delay of x/v. So the disturbance at a point x after time t is exactly the same as the disturbance at the point x = 0 at time ($t - x/v$).

We can therefore find out exactly what the disturbance is at point x at time t by replacing t by ($t - x/v$) in the SHM equation

$$y = a\sin 2\pi f\left(t - \frac{x}{v}\right)$$

We can re-arrange this equation, substituting for v = $f\lambda$

$$\begin{aligned} y =& a\sin 2\pi f\left(t - \frac{x}{v}\right) \\ \therefore y =& a\sin 2\pi\left(ft - \frac{fx}{v}\right) \\ \therefore y =& a\sin 2\pi\left(ft - \frac{fx}{f\lambda}\right) \\ \therefore y =& a\sin 2\pi\left(ft - \frac{x}{\lambda}\right) \end{aligned} \tag{1.3}$$

. . . .

Note that we are taking the sine of the angle $2\pi\left(ft - {}^{x}/_{\lambda}\right)$. This angle is expressed in

radians. You should also note that this expression assumes that at t = 0, the displacement at x = 0 is also 0.

For a wave travelling in the -x direction, Equation 1.3 becomes

$$y = a \sin 2\pi \left(ft + \frac{x}{\lambda}\right) \tag{1.4}$$

. . . .

We can now calculate the displacement of the rope, or any other medium, at position x and time t if we know the wavelength and frequency of the waves.

Travelling waves and the wave equation

20 min

At this stage there is an online activity.

This activity explores how the appearance and speed of a travelling wave depend on the different parameters in the wave equation.

You should understand how the appearance and speed of a travelling wave depend on the different parameters in the wave equation.

. . . .

Examples

1. A periodic wave travelling in the *x*-direction is described by the equation

$$y = 0.2 \sin(4\pi t - 0.1x)$$

What are (a) the amplitude, (b) the frequency, (c) the wavelength, and (d) the speed of the wave? (All quantities are in S.I. units.)

To obtain these quantities, we first need to re-arrange the expression for the wave into a form more similar to the general expression given for a travelling wave. The general expression is

$$y = a \sin 2\pi \left(ft - \frac{x}{\lambda}\right)$$

We are given

$$y = 0.2 \sin(4\pi t - 0.1x)$$

Re-arranging

$$y = 0.2 \sin 2\pi \left(2t - \frac{0.1x}{2\pi}\right)$$

So by comparison, we can see that:

a) the amplitude a = 0.2 m
b) the frequency f = 2 Hz
c) the wavelength $\lambda = {}^{2\pi}/_{0.1} = 63$ m
d) By calculation, the speed of the wave v = $f\lambda$ = 2 $\times$ 63 = 130 m s^{-1}

. . . .

2. Consider again the travelling wave in the previous example, described by the equation

$$y = 0.2\sin\left(4\pi t - 0.1x\right)$$

Calculate the displacement of the medium in the *y*-direction caused by the wave at the point x = 25 m when the time t = 0.30 s.

To calculate the *y*-displacement, put the data into the travelling wave equation. Remember to take the sine of the angle measured in *radians*.

$$\begin{aligned} y &= 0.2\sin\left(4\pi t - 0.1x\right) \\ \therefore y &= 0.2\sin\left((4 \times \pi \times 0.30) - (0.1 \times 25)\right) \\ \therefore y &= 0.2\sin 1.2699 \\ \therefore y &= 0.2 \times 0.9551 \\ \therefore y &= 0.19\,\text{m} \end{aligned}$$

. . . .

Quiz 2 Travelling waves

15 min

First try the questions. If you get a question wrong or do not understand a question, there are Hints. The hints are in the same order as the questions. If you read the hint and still do not understand then ask your tutor. All references in the hints are to online materials.

Q6: The equation representing a wave travelling along a rope is

$$y = 0.5\sin 2\pi\left(0.4t - \frac{x}{12}\right)$$

At time t = 2.50 s, what is the displacement of the rope at the point x = 7.00 m?

a) 0.00 m
b) 0.23 m
c) 0.25 m
d) 0.40 m
e) 0.50 m

....

Q7: Waves are being emitted in the *x*-direction at frequency 20 Hz, with a wavelength of 1.0 m. If the displacement at x = 0 is 0 when t = 0, which one of the following equations could describe the wave motion?

a) $y = \sin 2\pi\left(t - 20x\right)$
b) $y = 2\sin 2\pi\left(20t - x\right)$
c) $y = \sin 2\pi\left(\frac{t}{20} - x\right)$
d) $y = 3\cos 2\pi\left(t - 20x\right)$
e) $y = 20\sin 2\pi\left(t - 20x\right)$

....

Q8: A travelling wave is represented by the equation

$$y = 4\cos 2\pi\left(t - 2x\right)$$

What is the displacement at x = 0 when t = 0?

a) 0 m
b) 1 m
c) 2 m
d) 3 m
e) 4 m

....

Q9: A travelling wave is represented by the equation

$$y = \sin 2\pi\left(12t - 0.4x\right)$$

What is the frequency of this wave?

a) 0.4 Hz
b) 0.833 Hz
c) 2.0 Hz
d) 2.5 Hz
e) 12 Hz

....

Q10: A travelling wave is represented by the equation

$$y = 2\sin 2\pi\left(5t - 4x\right)$$

What is the speed of this wave?

a) 0.80 m s^{-1}
b) 1.25 m s^{-1}
c) 4.0 m s^{-1}
d) 5.0 m s^{-1}
e) 20 m s^{-1}

....

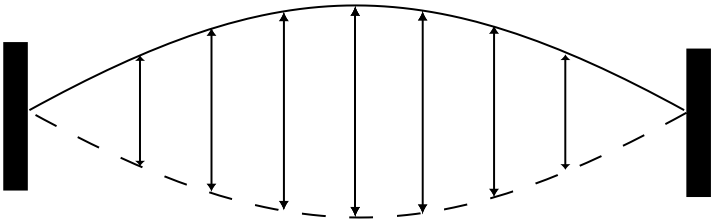
1.4 Stationary waves

Learning Objective

To describe different systems of stationary waves

We have discussed travelling waves. There is another situation we need to consider, that of stationary or standing waves. Consider a guitar string, or a piece of elastic stretched between two fixed supports. If we pluck the string or elastic, it oscillates at a certain frequency, whilst both ends remain fixed in position (see Figure 1.4).

Figure 1.4: Oscillations of a stretched string held fixed at both ends



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This type of wave is called a **stationary wave**. The name arises from the fact that the waves do not travel along the medium, as we saw in the previous section. Instead, the points of maximum and zero oscillation are fixed. Many different wave patterns are allowed, as will be discussed shortly.

At the fixed ends of the medium (the guitar string, the elastic band or whatever), no oscillations occur. These points are called **nodes**. The points of maximum amplitude oscillations are called **antinodes**. If you have trouble remembering which is which, the nodes are the points where there is ‘node-disturbance'.

10 min

Stationary Waves

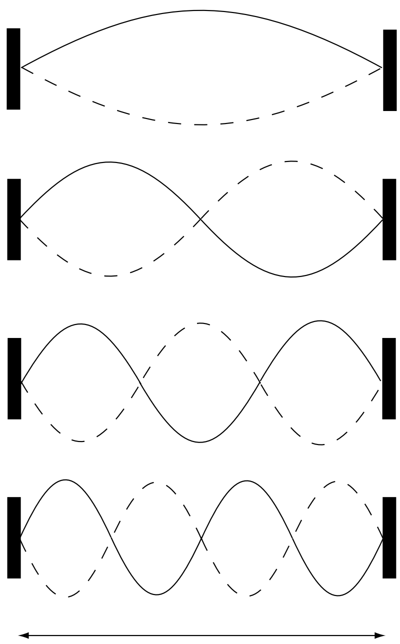
Learning Objective

To study stationary waves

Oscillations of several modes of a guitar string

Because of the condition of having a node at each end, we can build up a picture of the allowed modes of oscillation, as shown.

Figure 1.5: First four harmonics of a transverse standing wave



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The wavelength of the oscillations is twice the distance between adjacent nodes, so the longest possible wavelength in Figure 1.5 is $\lambda_1 = 2L$. This is called the fundamental mode, and oscillates at the fundamental frequency $f_1 = {}^{v}/_{\lambda_1} = {}^{v}/_{2L}$.

Looking at Figure 1.5, we can see that the allowed wavelengths are given by

$$\lambda_1 = 2L,\ \lambda_2 = {}^{2L}/_{2},\ \lambda_3 = {}^{2L}/_{3}.....$$
$$\lambda_n = {}^{2L}/_{n} = {}^{\lambda_1}/_{n} \quad (1.5)$$

....

The allowed frequencies are therefore given by

$$f_1 = {}^{v}/_{2L},\ f_2 = {}^{2v}/_{2L},\ f_3 = {}^{3v}/_{2L}.....$$
$$f_n = {}^{nv}/_{2L} = nf_1 \quad (1.6)$$

....

These different frequencies are called the **harmonics** of the system, and f_1 is the first harmonic (also called the fundamental mode, as stated earlier). f_2 is called the second harmonic (or the first overtone), f_3 the third harmonic (or second overtone) and so on.

The equation for travelling waves does **not** also describe the motion of stationary waves. It can be proved mathematically that the equation for stationary waves is the superposition of two travelling waves of equal amplitude travelling in opposite directions.

When working out the equation for travelling waves, we stated that every small portion of the medium was performing SHM slightly out of phase with its neighbours, but with the same amplitude. An important difference between stationary and travelling waves is that for stationary waves, each portion of the medium between nodes oscillates **in phase** with its neighbours, but with slightly **different** amplitude.

As with the travelling waves, stationary waves can also be set up for longitudinal as well as transverse waves. Equation 1.5 and Equation 1.6 equally apply for longitudinal and transverse stationary waves.

Example

An organ pipe has length L = 2.00 m and is open at both ends. The fundamental stationary sound wave in the pipe has an antinode at each end, and a node in the centre. Calculate the wavelength and frequency of the fundamental note produced. (*Take the speed of sound in air v = 340 m s*$^{-1}$.)

The distance between two antinodes (like the distance between two nodes) is equal to λ/2.

$$\therefore \frac{\lambda}{2} = 2.00$$
$$\therefore \lambda = 4.00\,\text{m}$$

To calculate the frequency of the fundamental, use n = 1 in the equation

$$f_n = \frac{nv}{2L}$$
$$\therefore f = \frac{340}{2 \times 2.00}$$
$$\therefore f = \frac{340}{4.00}$$
$$\therefore f = 85.0\,\text{Hz}$$

We could, of course, have calculated f using $f = {}^{v}/_{\lambda}$ which gives the same answer.

. . . .

20 min

Longitudinal stationary waves

When someone blows across the top of a bottle, a sound wave is heard. This is because a stationary wave has been set up in the air in the bottle. The oscillating air molecules form stationary longitudinal waves. In this exercise you will work out the wavelength of different longitudinal stationary waves.

a) Consider a pipe of length L, with one end open and the other closed. The stationary waves formed by this system have an antinode at the open end and a node at the closed end.

   i Sketch the fundamental wave in the pipe and calculate its wavelength.

   ii Sketch the next two harmonics.

b) If the pipe is open at both ends, the air molecules are free to oscillate at either end, so there will be an antinode at each end.

   i For a pipe of length L which is open at both ends, sketch the fundamental wave and calculate its wavelength.

   ii Sketch the next two harmonics.

You should be able to build up a picture of the harmonics of stationary waves in open and closed pipes.

....

20 min

Quiz 3 Stationary waves

First try the questions. If you get a question wrong or do not understand a question, there are Hints. The hints are in the same order as the questions. If you read the hint and still do not understand then ask your tutor. All references in the hints are to online materials.

speed of sound in air	340 m s^{-1}

Q11: The third harmonic of a plucked string has frequency 429 Hz. What is the frequency of the fundamental?

a) 1.30 Hz
b) 47.7 Hz
c) 143 Hz
d) 429 Hz
e) 1290 Hz

....

Q12: Which *one* of the following statements is true?

a) Large amplitude oscillations occur at the nodes of a stationary wave.
b) Every point between adjacent nodes of a stationary wave oscillates in phase.
c) The amplitude of the stationary wave oscillations of a plucked string is equal at all points along the string.

d) The distance between adjacent nodes is equal to the wavelength of the stationary wave.
e) Stationary waves only occur for transverse, not longitudinal waves.

. . . .

Q13: A string is stretched between two clamps placed 1.50 m apart. What is the wavelength of the fundamental note produced when the string is plucked?

a) 0.67 m
b) 0.75 m
c) 1.50 m
d) 2.25 m
e) 3.00 m

. . . .

Q14: The fourth harmonic of a standing wave is produced on a stretched string 2.50 m long. How many antinodes are there?

a) 2.5
b) 3
c) 4
d) 5
e) 10

. . . .

Q15: A stationary sound wave is set up in an open pipe 1.25 m long. What is the frequency of the third harmonic note in the pipe?

a) 182 Hz
b) 264 Hz
c) 340 Hz
d) 408 Hz
e) 816 Hz

. . . .

1.5 Summary

This topic has covered some of the basics of wave motion which you may have already known, such as the relationship between the speed, frequency and wavelength of a wave.

We then looked in turn at travelling and stationary waves. A general equation for travelling waves was derived, which allows us to calculate the disturbance of a medium at a given displacement and time. We saw that there are several differences between stationary and travelling waves.

By the end of this topic, you should be able to:

- state that in wave motion energy is transferred with no net mass transport;
- state that the irradiance of a wave is proportional to the square of its amplitude;
- state that a sine or cosine wave is the simplest mathematical form of a wave;
- explain that the relationship

  $$y = a\sin 2\pi\left(ft - \frac{x}{\lambda}\right)$$

  represents a travelling wave, and perform calculations using this equation;
- explain the meaning of phase difference;
- explain what is meant by a stationary wave;
- define the terms ‘node' and ‘antinode'.

1.6 End of topic test

End of topic test

30 min

At this stage there is an end of topic test available online. If however you do not have access to the internet you may try the questions which follow.

The following data should be used when required:

speed of light in a vacuum *c*	3.00×10^{8} m s^{-1}
speed of sound	340 m s^{-1}
acceleration due to gravity *g*	9.8 m s^{-2}

Q16: A laser produces a monochromatic (single wavelength) beam of light with wavelength 557 nm.

Calculate the frequency of the light in Hz.

....

Q17: A beam of red light (λ= 655 nm) is focussed onto a detector which measures a light irradiance of 2.43×10^{6} W m^{-2}.

Calculate the measured irradiance in W m^{-2} when the amplitude of the waves is doubled.

. . . .

Q18: Suppose a knot is tied in a horizontal piece of string. A train of transverse vertical sine waves are sent along the string, with amplitude 15.5 cm and frequency 1.4 Hz.

Calculate the total distance (in cm) through which the knot moves in 5.0 s. (NOT the displacement, but the total distance through which it has moved.)

. . . .

Q19: A travelling wave is represented by the equation

$y = 2 \sin 2\pi \left(1.2t - \frac{x}{3.5}\right)$

All the quantities in this equation are in SI units.

What is the value of the speed of the wave, in m s^{-1}?

. . . .

Q20: A transverse wave travelling along a rope is represented by the equation

$y = 0.55 \sin 2\pi \left(0.42t - \frac{x}{2.5}\right)$

All the quantities in this equation are in SI units.

Calculate the displacement in m at the point x = 0.50 m when t = 1.0 s.

. . . .

Q21: A guitar string is 0.68 m long.

Calculate the wavelength in m of the 4th harmonic.

. . . .

Q22: A string is stretched between two clamps held 1.75 m apart. The string is made to oscillate at its third harmonic frequency.

Calculate the distance in m between two adjacent nodes.

. . . .

Q23: A plank of wood is placed over a pit 14 m wide. A girl stands on the middle of the plank and starts jumping up and down, jumping upwards from the plank every 2 s. The plank oscillates with a large amplitude in its fundamental mode, the maximum amplitude occurring at the centre of the plank.

Calculate the speed in m s^{-1} of the transverse waves on the plank.

. . . .

Topic 2

The principle of superposition and the Doppler effect

Contents

Prerequisite knowledge

- *General waves definitions (Waves topic 1).*
- *Relative velocity.*
- *Line spectrum of an element.*

Learning Objectives

By the end of this topic, you should be able to:

- *apply the principle of superposition;*
- *show an understanding of the principle of Fourier's theorem;*
- *derive and use expressions for the Doppler effect in sound waves;*
- *show an understanding of how the Doppler effect is used in different applications involving light and sound waves.*

2.1 Introduction

This topic covers two very important effects associated with waves - superposition and the Doppler effect. We start by looking at the principle of superposition, which tells us what happens when two or more waves overlap at a point in space.

The second part of this topic is spent looking at the Doppler effect. Here we are dealing with just one wave, and we will see how it appears to be modified by the motion of either the wave source or the observer detecting the wave. This phenomenon has many practical applications, and we will study a few of them at the end of the topic.

2.2 Superposition and Fourier series

Learning Objective

To describe the superposition of coherent waves, and explain the meaning of constructive and destructive interference

There are two sections in this subtopic covering:

- the principle of superposition
- the fourier series.

2.2.1 Principle of superposition

The **principle of superposition** tells us what happens if two or more waveforms overlap. This might happen when you are listening to stereo speakers, or when two light beams are focused onto a screen. The result at a particular point is simply the sum of all the disturbances at that point.

Figure 2.1: (a) Constructive interference, (b) destructive interference of two sine waves

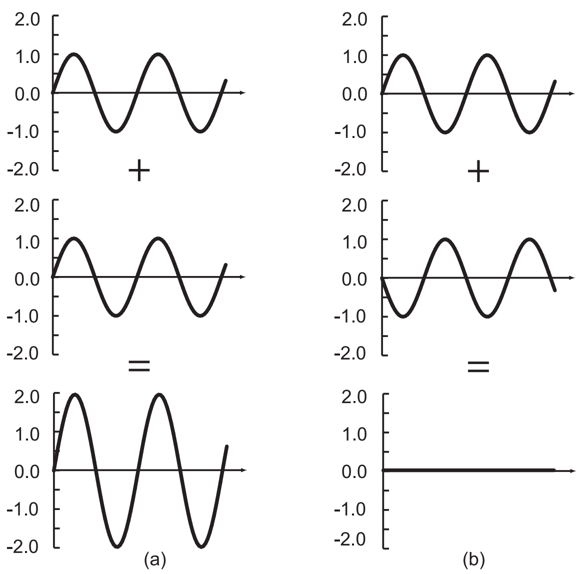

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Figure 2.1 shows plots of displacement against **time** at a certain point for two coherent sine waves with the same amplitude. Using the principle of superposition, the lowest graphs show the resultant wave at that point. In both cases, the resultant wave has an amplitude equal to the sum of the amplitudes of the two interfering waves. If the two waves are exactly in phase, as shown in Figure 2.1(a), **constructive interference** occurs, the amplitude of the resultant wave is greater than the amplitude of either individual wave. If they are exactly out-of-phase ('in anti-phase'), the sum of the two disturbances is zero at all times, hence there is no net disturbance. This is called **destructive interference**, as shown in Figure 2.1(b).

The **phase difference** between two waves can be expressed in fractions of a wavelength or as a fraction of a circle, with one whole wavelength being equivalent to a phase difference of 360° or 2π radians. Two waves that are in anti-phase would therefore have a phase difference of $\lambda/2$ or 180° or π radians.

Use the following exercise to investigate the superposition of two coherent waves with phase differences other than 0, π and 2π radians.

Superposition of two waves

20 min

At this stage there is an online activity.

This activity illustrates constructive and destructive interference of identical waves. It also explores what happens when the phase difference lies between 0° and 180°.

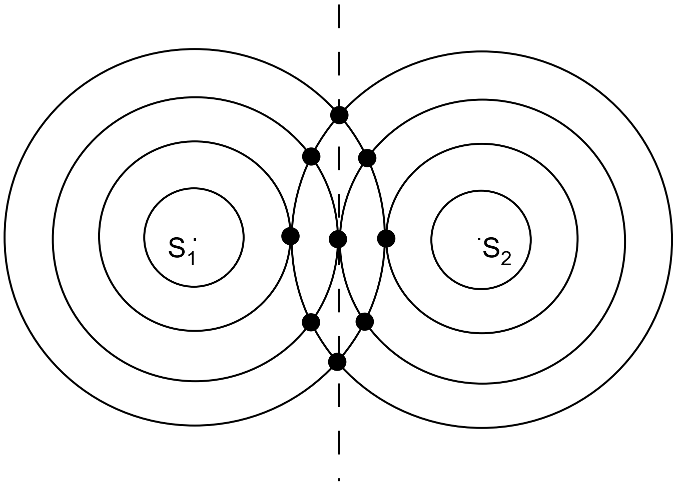
Two sine waves can combine to form a wave with a larger amplitude or they can cancel each other out.

....

As a practical example of interference, let us look at what happens when coherent waves of identical amplitude are emitted in phase by two loudspeakers. It should be clear that at a point equidistant from each speaker, two waves travelling with the same speed will arrive at exactly the same time. Constructive interference will occur, and the amplitude of the resultant wave will be the sum of the amplitudes of the two individual waves.

Figure 2.2 shows the **wavefronts** from two sources S_1 and S_2, producing waves with identical wavelengths. The wavefronts join points of identical phase, such as the crests of a wave, and the distance between adjacent wavefronts from the same source is equal to λ. Where wavefronts from the two sources overlap (shown by the solid black dots), constructive interference occurs.

Figure 2.2: Interference of waves from two sources



. . . .

In fact constructive interference will occur at any point where the difference in path length between the waves from each of the two sources is equal to a whole number of wavelengths. At any such point, the arrival of the crest of a wave from the left-hand speaker will coincide with the arrival of a crest from the right-hand speaker, leading to constructive interference.

Put mathematically, the condition for constructive interference of two waves is

$$|l_1 - l_2| = n\lambda \tag{2.1}$$

. . . .

where l_1 and l_2 are the distances from source to detector of the two waves, and n is a whole number. (The $||$ around $l_1 - l_2$ means the 'absolute' value, ignoring the minus sign if $l_2 > l_1$.)

If the path difference between the two waves is an odd number of half-wavelengths (λ/2, 3λ/2, etc.) then destructive interference occurs. The crest of a wave from one speaker will now arrive at the same time as the trough of the wave from the other speaker. If the amplitudes of the two waves are the same, the result of adding them together is zero as in Figure 2.1(b). In this case

$$|l_1 - l_2| = \left(n + \frac{1}{2}\right)\lambda \tag{2.2}$$

. . . .

In terms of phase, we can state that constructive interference occurs when the two waves are in phase, and destructive interference occurs when the two waves are in anti-phase.

Check your understanding of constructive and destructive interference using the superposition shown in Figure 2.2. Suppose S_1 and S_2 are emitting coherent sound waves in phase. What would you expect to hear if you walked from S_1 to S_2? (Answer given below the next worked example.)

Example

Two radio transmitters A and B are broadcasting the same signal in phase, at wavelength 750 m. A receiver is at location C, 6.75 km from A and 3.00 km from B. Does the receiver pick up a strong or weak signal?

The distance from A to C is 6750 m, and the wavelength is 750 m, so in wavelengths, the distance A to C is 6750/750 = 9.00 wavelengths.

Similarly, B to C is 3000 m or 3000/750 = 4.00 wavelengths.

So the path difference AC - BC = 5.00 wavelengths, and since this is a whole number of wavelengths, constructive interference will occur and the receiver will pick up a strong signal.

....

Walking from speaker S_1 to S_2 in Figure 2.2, you would hear the sound rising and falling in loudness, as you moved through regions of constructive and destructive interference.

2.2.2 Fourier series

One very important application of the principle of superposition is in the Fourier analysis of a waveform. Fourier's theorem, first proposed in 1807, states that any periodic wave can be represented by a sum of sine and cosine waves, with frequencies which are multiples (harmonics) of the wave in question. If you had been wondering why so much emphasis has been placed on studying sine and cosine waves, it is because any periodic waveform we might encounter is made up of a superposition of sine and cosine waves.

Put mathematically, any wave can be described by a Fourier series

$$y(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n2\pi ft) + \sum_{n=1}^{\infty} b_n \sin(n2\pi ft) \qquad (2.3)$$

....

Fourier analysis is an important technique in many areas of physics and engineering. For example, the response of an electrical circuit to a non-sinusoidal electrical signal can be determined by breaking the signal down into its Fourier components.

Fourier series

At this stage there is an online activity.

20 min

This activity demonstrates how any periodic wave function can be made up from a combination of sine and cosine waves.

You should be aware that any periodic wave function can be made up from a combination of sine and cosine waves.

....

20 min

Quiz 1 Superposition

First try the questions. If you get a question wrong or do not understand a question, there are Hints. The hints are in the same order as the questions. If you read the hint and still do not understand then ask your tutor. All references in the hints are to online materials.

Q1: Two sine waves are exactly out of phase at a certain point in space, so they undergo destructive interference. If one wave has amplitude 5.0 cm and the other has amplitude 2.0 cm, what is the amplitude of the resultant disturbance?

a) 0 cm
b) 2.0 cm
c) 2.5 cm
d) 3.0 cm
e) 7.0 cm

....

Q2: A listener is standing midway between two loudspeakers, each broadcasting the same signal in phase. Does the listener hear

a) a loud signal, owing to constructive interference?
b) a quiet signal, owing to destructive interference?
c) a quiet signal, owing to constructive interference?
d) a loud signal, owing to destructive interference?
e) no signal at all?

....

Q3: A radio beacon is transmitting a signal (λ = 200 m) to an aeroplane. When the aeroplane is 4.50 km from the beacon what is the separation between the beacon and the aeroplane in numbers of wavelengths?

a) 0.0225 wavelengths
b) 0.044 wavelengths
c) 22.5 wavelengths
d) 44.0 wavelengths
e) 900 wavelengths

....

Q4: Fourier's theorem tells us that

a) only coherent waves can be added together.
b) any periodic wave is a superposition of harmonic sine and cosine waves.
c) any periodic wave is a superposition of stationary and travelling waves.

d) all sine and cosine waves have the same phase.
e) any periodic wave is made up of a set of sine waves, all with the same amplitude.

. . . .

Q5: Two separate sources emit sinusoidal travelling waves of equal wavelength λ, which are in phase at their respective sources. A detector is located at a distance l_1 from one source and l_2 from the other. The amplitude is at a maximum at the detector if the distance $l_1 - l_2$ is

a) a multiple of π.
b) an odd multiple of π/2.
c) an odd multiple of λ/4.
d) an odd multiple of λ/2.
e) a multiple of λ.

. . . .

2.3 The Doppler effect

Learning Objective

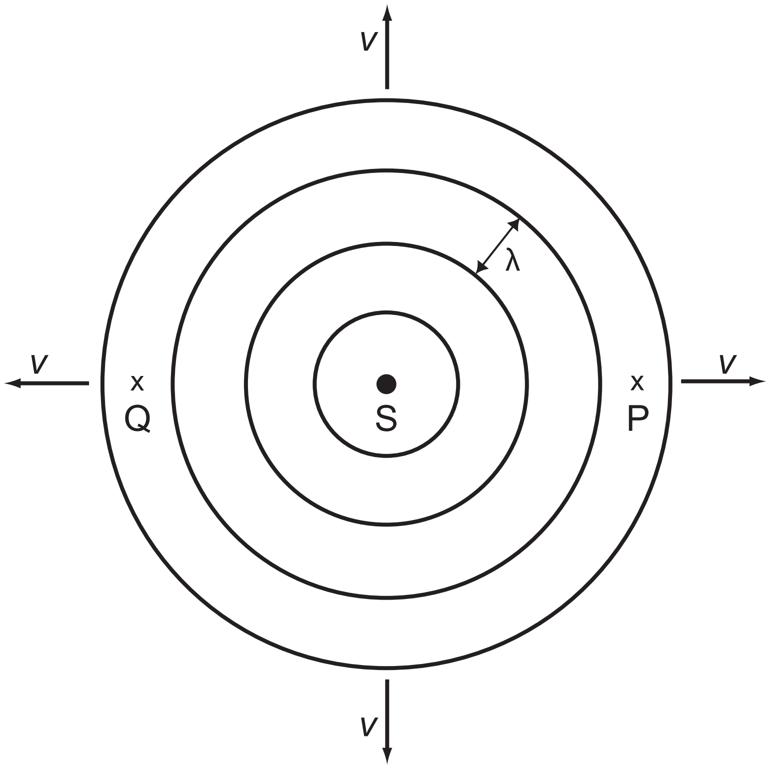
To understand and perform calculations on the Doppler effect for a moving source or a moving observer

Have you ever noticed the change of note when an ambulance with a siren comes towards and goes past you, or the way that a car engine sounds different once the car has gone past you? This change in the sound waves that you hear is called the Doppler effect. The frequency of the sound waves being emitted by the siren or the engine is different to the frequency of the waves that you hear because of the relative motion of the source of the waves (the siren, the engine or whatever) and the observer or listener. In this Section we will be investigating the Doppler effect to find out how this change in frequency occurs.

2.3.1 The Doppler effect with moving source

Consider a source S producing sound waves with wavelength λ and frequency f. Figure 2.3 shows the waves being emitted in all directions. P and Q are two stationary observers listening to the waves. Both observers will hear sound at the same frequency f.

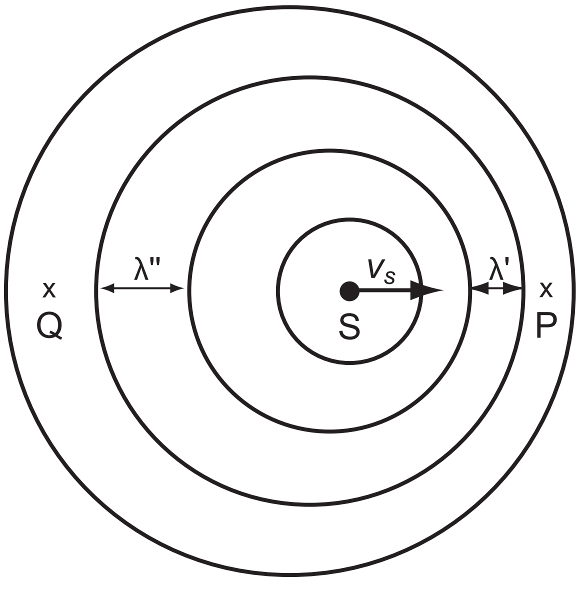
Figure 2.3: Waves produced by a stationary source



. . . .

The waves travel at speed v, where $v = f\lambda$, and the source S is stationary. Now let us consider what happens if S is moving whilst emitting waves.

Figure 2.4: Waves produced by a moving source



. . . .

In Figure 2.4, S is moving to the right with speed v_s. The waves produced in the direction SP are compressed into a smaller space than before, as the source is moving in the same direction as the waves. λ' is therefore smaller than the emitted wavelength λ. In the direction SQ, the wavelength appears to be greater, as the source is moving in the opposite direction to which the waves are travelling. This means that λ'' is greater than λ.

When we are dealing with sound waves, the frequency of the wave tells us its pitch. Does this apparent change of wavelength mean that the frequency of the waves has shifted?

Let us look first at the situation where the source S is moving towards the observer P. In a period of time t, the number of waves emitted by S is ft. The first wave emitted has travelled a distance vt in this time, whilst in the same time the source has moved a distance v_st in the same direction. This means that ft waves are compressed into a distance $vt - v_st = (v - v_s)t$.

The distance between the waves is equal to the apparent wavelength λ' observed by an observer at P, hence

$$\begin{aligned} \lambda' &= \frac{(v - v_s)\,t}{ft} \\ \therefore \lambda' &= \frac{(v - v_s)}{f} \end{aligned}$$

Since the waves are travelling at speed v, then the apparent frequency f' is given by

$$\lambda' = \frac{v}{f'}$$

Substituting for λ',

$$\begin{aligned} \frac{v}{f'} &= \frac{(v - v_s)}{f} \\ \therefore \frac{f'}{v} &= \frac{f}{(v - v_s)} \\ \therefore f' &= f\frac{v}{(v - v_s)} \end{aligned} \qquad (2.4)$$

....

So the frequency of the waves heard at P depends on the speed v_s of the source. This frequency shift, called the **Doppler effect**, means that the apparent frequency is higher than the actual frequency of the emitted waves when the source is moving towards the observer. Make sure you understand why Equation 2.4 implies that $f' > f$.

We can now look at the situation when the source S is moving away from the observer at Q. As before, the source emits ft waves in time t, the first of which travels a distance

vt in that period of time. The source S moves a distance $v_s t$ in the opposite direction, so these waves are spread out over a distance $vt + v_s t = (v + v_s)t$. In this case, the distance between the waves is the apparent wavelength λ''.

$$\begin{aligned} \lambda'' &= \frac{(v\ +\ v_s)t}{ft} \\ \therefore\ \lambda'' &= \frac{(v\ +\ v_s)}{f} \end{aligned}$$

Again we can substitute for $\lambda'' = {}^{v}/_{f''}$ in this equation

$$\begin{aligned} \frac{v}{f''} &= \frac{(v + v_s)}{f} \\ \therefore \frac{f''}{v} &= \frac{f}{(v + v_s)} \\ \therefore f'' &= f\frac{v}{(v + v_s)} \end{aligned} \tag{2.5}$$

In this case, the motion of the source means that the apparent frequency f'' is less than the emitted frequency.

Doppler effect

15 min

At this stage there is an online activity.

This activity demonstrates the Doppler effect.

You should understand why the motion of the source means the detected waves have a different frequency than the emitted waves. You should also understand why the frequency is raised in one case and lowered in the other.

2.3.2 The Doppler effect with moving observer

We will now consider what happens when the source S is stationary, but the observer is moving towards or away from S with speed v_o. Referring back to Figure 2.3, the wavelength of the waves is not changed, and $\lambda = {}^{v}/_{f}$.

When the observer is moving towards S, waves arrive at the observer at a higher rate than if the observer were stationary, so there will be a shift in frequency again. We will call this apparent frequency f'.

Since the observer is moving towards S, the relative velocity v' of the waves (relative to the observer) is v + v_o. But since the wavelength isn't changed, we can state that $\lambda = {}^{v'}/_{f'}$. Equating these two expressions for λ gives us

$$\begin{aligned}\frac{v'}{f'} &= \frac{v}{f}\\ \therefore \frac{(v+v_o)}{f'} &= \frac{v}{f}\\ \therefore \frac{f'}{(v+v_o)} &= \frac{f}{v}\\ f' &= f\frac{(v+v_o)}{v}\end{aligned} \tag{2.6}$$

. . . .

Make sure you understand why Equation 2.6 means that $f' > f$.

The final case we will look at is that of the observer moving away from the stationary source with speed v_o. The procedure here is the same as for deriving Equation 2.6, except that the velocity v'' of the waves relative to the observer is now is v - v_o. The apparent frequency f'' is derived as follows

$$\begin{aligned}\frac{v''}{f''} &= \frac{v}{f}\\ \therefore \frac{(v-v_o)}{f''} &= \frac{v}{f}\\ \therefore \frac{f''}{(v-v_o)} &= \frac{f}{v}\\ f'' &= f\frac{(v-v_o)}{v}\end{aligned} \tag{2.7}$$

. . . .

Again, you should understand that Equation 2.7 implies that $f'' < f$, and you should be able to explain why.

Example

The alarm of a parked car is emitting sound waves of frequency 540 Hz. If you are driving towards this car at a steady speed of 15.0 m s^{-1}, what would be the frequency of the waves you hear? (Assume the speed of sound in air is 340 m s^{-1}.)

We have a situation where the source is stationary and the observer is moving towards it. The appropriate Doppler relationship is Equation 2.6

$$\begin{aligned}f' &= f\frac{(v+v_o)}{v}\\ \therefore f' &= 540 \times \frac{(340+15)}{340}\\ \therefore f' &= 540 \times \frac{355}{340}\\ \therefore f' &= 564\,\text{Hz}\end{aligned}$$

The frequency you would hear is 564 Hz. Remember to check your answer - if the observer is moving towards a stationary source, would you expect the apparent frequency to be higher or lower that the emitted frequency?

. . . .

Extra Help: Solving problems on the Doppler effect

At this stage there is an online activity which provides extra practice with Doppler effect problems.

. . . .

Quiz 2 Doppler effect

20 min

First try the questions. If you get a question wrong or do not understand a question, there are Hints. The hints are in the same order as the questions. If you read the hint and still do not understand then ask your tutor. All references in the hints are to online materials.

Useful data:

speed of sound in air	340 m s^{-1}

Q6: The rise in pitch heard when a source of sound waves approaches a stationary listener is due to a shift in

a) speed.
b) frequency.
c) amplitude.
d) phase.
e) coherence.

. . . .

Q7: Granny toots her car horn to say goodbye as she drives off down the road at 10 m s^{-1}. What is the frequency of the note heard by her grandchildren as she drives away, if the emitted sound waves have frequency 510 Hz?

a) 324 Hz
b) 337 Hz
c) 495 Hz
d) 510 Hz
e) 526 Hz

. . . .

Q8: A car driver travels at 24.0 m s^{-1} towards a stationary source emitting sound waves of frequency 495 Hz. What is the frequency of the waves heard by the car driver?

a) 459 Hz
b) 462 Hz
c) 495 Hz
d) 530 Hz

e) 544 Hz

....

Q9: A man is blowing a whistle, producing a note of frequency 480 Hz. The note is heard by a woman standing 50 m away. The apparent frequency heard by the woman is

a) higher when the man is walking towards her.
b) unchanged when the man is walking away from her.
c) higher when she is walking away from the man.
d) lower when she stands 20 m from the man.
e) higher when she stands 20 m from the man.

....

Q10: A car is being driven towards a stationary observer, with the horn sounding. The apparent frequency heard by the observer is 480 Hz. If the actual frequency of the car horn is 465 Hz, how fast is the car travelling?

a) 3.20 m s^{-1}
b) 10.6 m s^{-1}
c) 15.0 m s^{-1}
d) 22.7 m s^{-1}
e) 320 m s^{-1}

....

2.3.3 Applications of the Doppler Effect

The Doppler effect can be used to:

- measure the speed at which distant stars are moving relative to the Earth
- measure the speed of blood flow using ultrasound waves.

2.3.3.1 Light and other electromagnetic radiation

In astronomy, the Doppler effect is used to measure the speed at which distant stars are moving relative to the Earth. The line spectrum of the star as observed on Earth is compared to that of the appropriate elements taken in the laboratory. The lines in the star's spectrum are found to be different to the laboratory spectrum, with all the wavelengths shifted in one direction. The shift in wavelength tells us how fast the star is moving, and whether it is moving towards or away from Earth.

For a star moving away from the Earth at speed v_s, the Doppler-shifted wavelength was shown in the derivation of Equation 2.5 to be

$$\lambda'' = \frac{(c + v_s)}{f}$$

for light waves travelling with speed c. We can rearrange this equation to allow us to compare the emitted λ with the Doppler-shifted λ' by using the relation $\lambda = {}^{c}/_{f}$

$$\begin{aligned}\lambda'' &= \frac{(c+v_s)}{f}\\ \therefore \lambda'' &= \frac{c}{f}+\frac{v_s}{f}\\ \therefore \lambda'' &= \frac{c}{f}\left(1+\frac{v_s}{c}\right)\\ \therefore \lambda'' &= \lambda\left(1+\frac{v_s}{c}\right)\end{aligned} \tag{2.8}$$

Equation 2.8 implies that λ'' is greater than λ, so a line in the star spectrum appears at a longer wavelength than the corresponding line in the laboratory spectrum. This means that a line in the visible part of the spectrum is Doppler-shifted towards the red end of the spectrum. The line is said to be **red-shifted**, and the fact that a spectrum is red-shifted tells us that the star is moving away from the Earth.

Similarly, a star moving towards the Earth would have its spectrum **blue-shifted**. You should note that Equation 2.8 and the similar equation that can be obtained for the blue-shift are only valid when v_s is much less than c. If this is not the case then relativistic calculations have to be used.

Another application of the Doppler effect for electromagnetic radiation is in Doppler radar. In this case microwave radiation with frequency of the order of 10^9 Hz is used. A signal is emitted from a radar 'gun' towards a car travelling towards the gun. Since the car is moving towards the stationary source, the waves are Doppler shifted as they arrive at the car, and the reflected signal is at the new frequency. The reflected signal is detected by a receiver attached to the gun, and since it comes from a 'source' moving towards a stationary detector, it suffers a second Doppler shift. The shift in frequency of the detected signal tells the gun operator how fast the car is going.

20 min

Doppler effect in light

This problem is often set to demonstrate the speeds necessary to observe the Doppler effect using visible light waves. It involves a physicist caught by the police for driving through a red light. When the police pull him over, his excuse is that it wasn't his fault: 'Because of the speed I was driving at, the red light was Doppler-shifted as I drove towards it and it appeared to be green.'

Assuming the red light has wavelength 6.5×10^{-7} m and the green light has wavelength 5.4×10^{-7} m, perform a non-relativistic calculation to estimate the speed at which the car must have been travelling to observe this Doppler shift.

This exercise shows why the Doppler effect for light waves is not observed in everyday life, unlike the Doppler-shifting of sound waves.

2.3.3.2 Sound

Ultrasound waves (frequencies in the MHz range, beyond the range of human hearing) are used to measure the speed of blood flow by making use of the Doppler effect. In a similar way to the radar speed gun, the flow meter emits an ultrasonic signal which is reflected by red blood cells. The reflected signal is measured by the flow meter and its frequency is compared to that of the original signal. In narrowed arteries, the blood is forced to flow faster, so the Doppler shift will be greater in such regions than in normal arteries.

2.4 Summary

The principle of superposition describes what happens when two or more waves overlap at a point in space. When two coherent waves overlap, interference takes place. Constructive interference, when the two waves are exactly in phase, results in large amplitude oscillations. If the two waves are exactly out of phase, the waves interfere destructively, resulting in small amplitude oscillations, or even no oscillations at all.

In the Doppler effect, the relative motion of source and observer leads to the frequency of the detected wave being shifted, compared to the frequency of the emitted wave. This effect has many practical uses, some of which have been outlined in the text.

By the end of this topic you should be able to:

- state what is meant by the constructive and destructive interference of two or more waves;
- state that any periodic wave can be described by the superposition of harmonic sine or cosine waves (Fourier's theorem);
- state that the Doppler effect is the change in frequency observed when a source of waves is moving relative to an observer;
- derive and apply expressions for the Doppler effect for moving source or observer, and use these expressions to calculate Doppler shifts;
- describe some applications of the Doppler effect.

2.5 End of topic test

End of topic test

30 min

At this stage there is an end of topic test available online. If however you do not have access to the internet you may try the questions which follow.

The following data should be used when required:

speed of light in a vacuum *c*	3.00×10^{8} m s1
speed of sound	340 m s^{-1}
acceleration due to gravity *g*	9.8 m s^{-2}

Q11: Two coherent sine waves overlap at a point A. The amplitude of one wave is 9.8 cm, and the amplitude of the other wave is 6.2 cm.

Calculate the minimum possible amplitude of the resultant disturbance at A.

. . . .

Q12: Two in-phase speakers A and B are emitting a signal of wavelength 1.06 m. A tape recorder is placed on the straight line between the speakers, 2.53 m from speaker A.

Calculate the shortest distance from the tape recorder that speaker B should be placed, to ensure constructive interference where the signal is recorded.

. . . .

Q13: Two loudspeakers are emitting a single frequency sound wave in phase. A listener is seated 5.7 m from one speaker and 2.1 m from the other.

Calculate the minimum frequency of the sound waves that would allow constructive interference where the listener is seated.

. . . .

Q14: A car is being driven towards a stationary pedestrian at 16.5 m s^{-1}. The car driver sounds the horn, which produces a note at frequency 488 Hz.

Calculate the frequency of the note heard by the pedestrian.

. . . .

Q15: The siren of a fire engine is Doppler-shifted from 505 Hz to 489 Hz as it drives away at constant speed from a stationary observer.

Calculate the speed of the fire engine.

. . . .

Q16: A car driver travelling towards a stationary source of sound waves hears a note at 312 Hz.

If the source is emitting sound waves at 299 Hz, calculate how fast the car is travelling.

. . . .

Q17: A car travelling at constant speed passes by a stationary pedestrian as the car driver sounds the horn.

As the car approaches the pedestrian she hears the horn sounding at 521 Hz. Once the car has driven past her, she hears the horn sounding at 478 Hz.

1. Calculate the speed of the car.
2. Calculate the frequency of the sound waves emitted by the car horn.

....

Topic 3

Interference by division of amplitude

Contents

Prerequisite knowledge

- *Refractive index.*
- *Frequency and wavelength (Waves topic 1).*

Learning Objectives

By the end of this topic you should be able to:

- *show an understanding of coherence between light waves;*
- *explain the difference between path length and optical path length, and calculate the latter;*
- *show an understanding of interference by division of amplitude;*
- *describe thin film interference - experimental conditions, analysis and applications;*
- *describe wedge fringes - how fringes are formed by an air wedge, and analysis of the fringes.*

3.1 Introduction

Interference of light waves is responsible for the rainbow colours seen on an oil film on a puddle of water, or in light reflected by a soap bubble. In this Topic we will be looking at the conditions under which these interference effects can take place.

The topic begins with some of the background work necessary to fully understand interference. Once this is completed, we proceed to two methods of producing interference, thin film interference and wedge fringes. In both cases the experimental conditions will be described, followed by an analysis which will enable us to determine the conditions for constructive and destructive interference of different wavelengths of light.

3.2 Coherence and optical path difference

Learning Objective

To understand coherence between light waves, and to calculate the optical path difference between two light rays

The section begins with the concept of coherence and ends with the method of calculating the optical path difference between two light rays.

3.2.1 Coherence

We briefly discussed coherent waves in the 'Introduction to Waves' topic. Two waves are said to be coherent if they have a constant phase relationship. For two waves travelling in air to have a constant phase relationship, they must have the same frequency and wavelength. At any given point, the phase difference between the two waves will be fixed.

It is easier to produce coherent sound waves or microwaves than it is to produce coherent visible electromagnetic waves. Both sound and microwaves can be generated electronically, with loudspeakers or antennae used to emit the waves. The electronic circuits used to generate these waves can 'frequency lock' and 'phase lock' two signals to ensure they remain coherent. In contrast to this, light waves are produced by transitions in individual atoms, and are usually emitted with random phase.

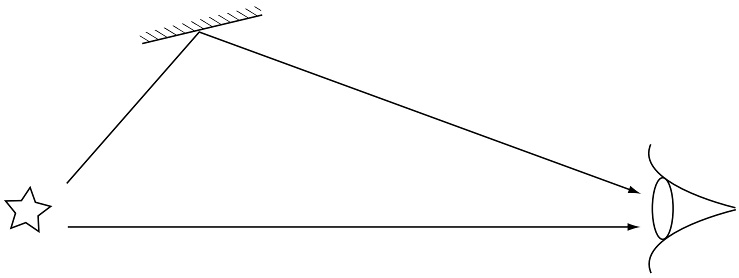
For us to see interference effects, we require two or more sources of coherent light waves. The best source of coherent radiation is a laser, which emits light at a single wavelength, usually in a collimated (non-diverging) beam. Another way to produce coherent light is to split a wave, for example by reflection from a glass slide. Some of the light will be transmitted, the rest will be reflected, and the two parts must be coherent with each other.

Filament light bulbs and strip lights do not emit coherent radiation. Such sources are called extended sources, or incoherent sources. They emit light of many different wavelengths, and light is emitted from every part of the tube or filament.

3.2.2 Optical path difference

In the Section on Superposition in the second Waves topic, we solved problems in which waves emitted in phase arrived from two different sources. By measuring the paths travelled by both waves, we could determine whether they arrived in phase (interfering constructively) or out of phase (interfering destructively). We are now going to look at a slightly different situation, in which waves emitted in phase by the same source arrive at a detector via different routes.

Figure 3.1: Two light rays travelling along different paths

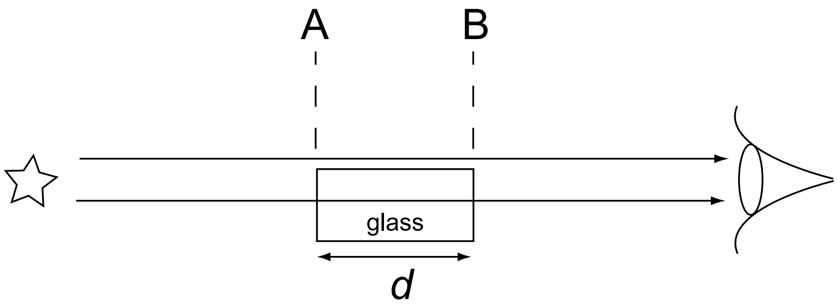


. . . .

In Figure 3.1 we can see two waves from the same source arriving at the same detector. The situation is similar to that which we saw in the second Topic. If the **path difference** is a whole number of wavelengths (λ, 2λ, 3λ...) then the two waves will arrive in phase. If the path difference is an odd number of half wavelengths (λ/2, 3λ/2, 5λ/2...) then the two waves arrive out of phase at the detector and destructive interference takes place.

A further complication can arise if one of the rays passes through a different medium

Figure 3.2: Two light rays travelling different optical path lengths



. . . .

In Figure 3.2, the lower ray passes through a glass block between A and B. An **optical path difference** Δd exists between the two rays, even though they have both travelled the same distance, because the wavelength of the light changes as it travels through the glass. The refractive index of glass, n_{glass}, is greater than the refractive index of air, so the waves travel at a slower speed in the glass. The frequency of the waves does not change, so the wavelength in glass λ_{glass} must be smaller than the wavelength in air, λ. If the refractive index of air is 1.00, then λ_{glass} is given by

$$\begin{aligned} n_{glass} &= \frac{c}{v_{glass}} \\ \therefore n_{glass} &= \frac{f \times \lambda}{f \times \lambda_{glass}} \\ \therefore n_{glass} &= \frac{\lambda}{\lambda_{glass}} \\ \therefore \lambda_{glass} &= \frac{\lambda}{n_{glass}} \end{aligned} \tag{3.1}$$

....

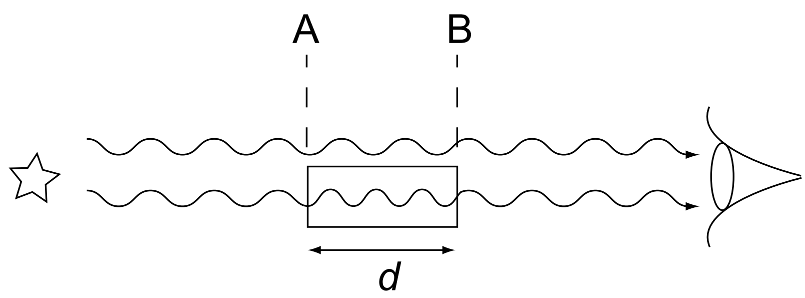
In Figure 3.2, the upper ray is travelling in air all the way from source to detector. The number of wavelengths contained between A and B will be $^{d}/_{\lambda}$.

For the lower ray, in the same distance, the number of wavelengths will be

$$\frac{d}{\lambda_{glass}} = \frac{d}{^{\lambda}/_{n_{glass}}} = \frac{d \times n_{glass}}{\lambda}$$

So there will be more wavelengths between A and B in the glass than in the same distance in air. This is shown in Figure 3.3. For a light ray travelling a distance d in a material of refractive index n, the **optical path length** is $n \times d$. To determine the phase difference between two waves travelling between the same source and detector, the **optical path difference** must be a whole number of wavelengths for constructive interference, and an odd number of half-wavelengths for destructive interference. In Figure 3.3, the optical path difference is the difference between the optical path lengths of both waves travelling from A to B. For the upper ray, the optical path length of AB = $n_{air} \times d = d$, since $n_{air} \approx 1.00$. For the lower ray, the optical path length of AB is $n_{glass} \times d$. Thus the optical path difference is $(n_{glass}-n_{air}) \times d$.

Figure 3.3: Optical path difference between air and glass



....

In general, for two rays of light travelling the same distance d in media with refractive indices n_1 and n_2, the interference conditions are

constructive interference (waves emerge in phase)	$(n_1 - n_2)\, d = m\lambda$
destructive interference (waves emerge exactly out of phase)	$(n_1 - n_2)\, d = \left(m + \frac{1}{2}\right) \lambda$

m is an integer. Note that we will talk about an "optical path difference" even when we are studying radiation from outside the visible part of the electromagnetic spectrum.

Example

Two beams of microwaves with a wavelength of 6.00×10^{-3} m are emitted from a source. One ray of the waves travels through air to a detector 0.050 m away. Another ray travels the same distance through a quartz plate to the detector. Do the waves interfere constructively or destructively at the detector, if the refractive index of quartz is 1.54?

The optical path difference Δd in this case is

$$\begin{aligned} \Delta d &= (n_{quartz} - n_{air}) \times d \\ \therefore \Delta d &= (1.54 - 1.00) \times 0.050 \\ \therefore \Delta d &= 0.54 \times 0.050 \\ \therefore \Delta d &= 0.027\text{m} \end{aligned}$$

To find out how many wavelengths this optical path length is, divide by the wavelength in air

$$\frac{0.027}{\lambda} = \frac{0.027}{6.00 \times 10^{-3}} = 4.5 \text{ wavelengths}$$

So the waves arrive at the detector exactly out of phase, and hence interfere destructively.

. . . .

Quiz 1 Coherence and optical paths

20 min

First try the questions. If you get a question wrong or do not understand a question, there are Hints. The hints are in the same order as the questions. If you read the hint and still do not understand then ask your tutor. All references in the hints are to online materials.

Q1: Two light waves are coherent if

a) they have the same speed.
b) their amplitudes are identical.
c) the difference in their frequencies is constant.
d) their phase difference is constant.
e) the difference in their wavelengths is constant.

. . . .

Q2: A radio transmitter emits waves of wavelength 500 m, A receiver dish is located 4.5 km from the transmitter. What is the path length, in wavelengths, from the transmitter to the receiver?

a) 0.11 wavelengths
b) 0.5 wavelengths
c) 2 wavelengths
d) 5 wavelengths
e) 9 wavelengths

....

Q3: Light waves of wavelength 450 nm travel 0.120 m through a glass block

(n = 1.50).

What is the optical path length travelled?

a) 8.10×10^{-9} m
b) 4.00×10^{-4} m
c) 0.180 m
d) 0.800 m
e) 4.00×10^{5} m

....

Q4: Two light rays travel in air from a source to a detector. Both travel the same distance from source to detector, but one ray travels for 2.50 cm of its journey through a medium of refractive index 1.35.

What is the optical path difference between the two rays?

a) 8.75×10^{-3} m
b) 3.38×10^{-3} m
c) 0.0250 m
d) 14.0 m
e) Depends on the wavelength of the light rays.

....

Q5: In a similar set-up to question 4, two microwaves (λ = 1.50 cm) from the same source arrive with a phase difference of exactly 4.00 wavelengths at the detector. The waves have travelled the same distance, but one has gone through a sheet of clear plastic (n = 1.88) while the other has travelled through air all the way.

What is the thickness of the plastic sheet?

a) 0.0319 m
b) 0.0682 m
c) 0.220 m
d) 4.55 m
e) 14.7 m

....

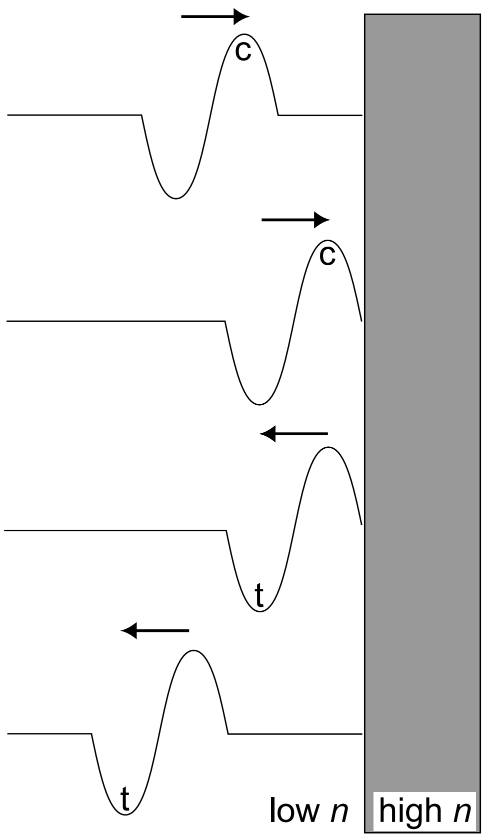
3.3 Reflection and interference by division of amplitude

Learning Objective

To state the conditions in which a wave changes its phase upon reflection
To explain what is meant by interference by division of amplitude

The phase of a wave may be changed when it is reflected. For light waves, we must consider the refractive indices of the two media involved. For example, consider a light wave travelling in air (n = 1.00) being reflected by a glass surface (n = 1.50). In this case the wave is travelling in a low refractive index medium, and is being reflected by a higher refractive index medium. Whenever this happens, there is a phase change of 180° (π radians). A wave crest becomes a wave trough on reflection. This is shown in Figure 3.4, where we can see a crest (labelled c) before reflection comes back as a trough (t) after reflection.

Figure 3.4: Phase change upon reflection at a higher refractive index material

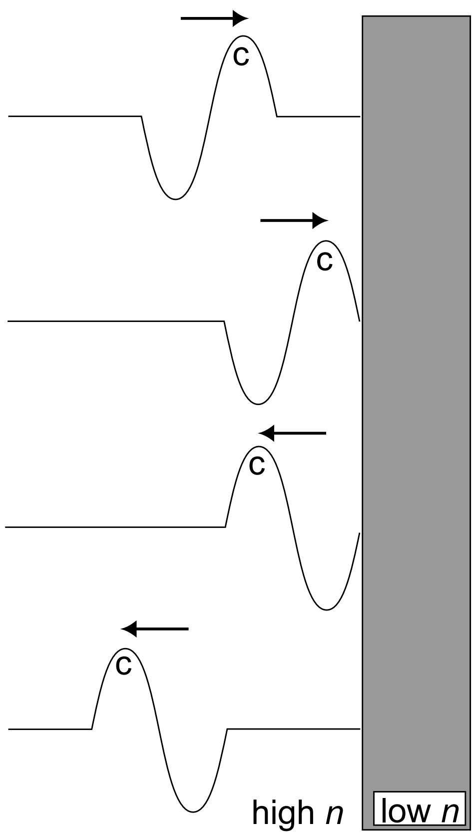


. . . .

You would see exactly the same effect happening if you sent a wave along a rope that was fixed at one end. As in Figure 3.4, a wave that has a crest leading a trough is reflected back as a wave with a trough leading a crest.

There is no phase change when a light wave travelling in a higher index material is reflected at a boundary with a material that has a lower index. If our light wave is travelling in glass, the phase of the wave is not changed when it is reflected at a boundary with air. A wave crest is still a wave crest upon reflection. This reflection is shown in Figure 3.5.

Figure 3.5: No phase change upon reflection at a lower refractive index material



....

A wave sent along a rope that is unsecured at the end would behave in exactly the same way. A wave which travels as a crest leading a trough remains as a wave with the crest leading the trough upon reflection.

The importance of the phase change in certain reflections is that the optical path of the reflected wave is changed. If the phase of a wave is changed by 180° it is as if the wave has travelled an extra $\lambda/2$ distance compared to a wave whose phase has not been changed by the reflection.

In the remainder of this topic we will be discussing **interference by division of amplitude**. This is rather a long title for a simple idea. You should already know what we mean in Physics by interference. Division of amplitude means that each wave is being split, with some of it travelling along one path, and the remainder following a different path. When these two parts are recombined, it is the difference in optical paths taken by the two waves that determines their phase difference, and hence how they interfere when they recombine.

3.4 Thin film interference

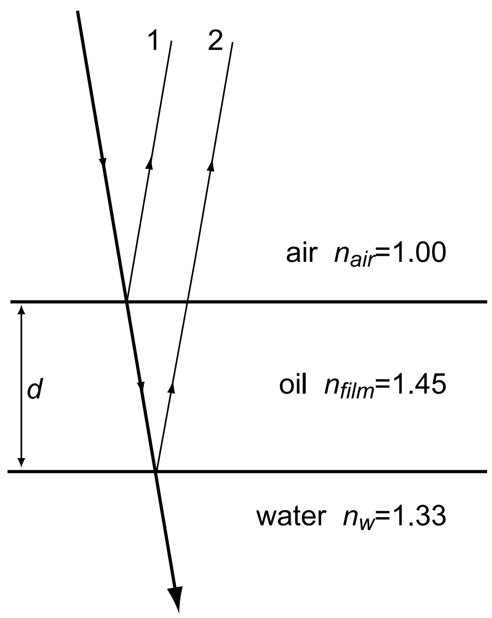
Learning Objective

To describe thin film interference

One example of interference by division of amplitude with which you are probably familiar is called thin film interference. When oil or petrol is spilt onto a puddle of water, we see a multicoloured film on the surface of the puddle. This is due to the thin film of oil formed on the surface of the puddle. Sunlight is being reflected from the film and the puddle. The film appears multicoloured because of constructive and destructive interference of the sunlight falling on the puddle. In this Section we will examine this effect more closely.

Figure 3.6 shows light waves falling on a thin film of oil on the surface of water.

Figure 3.6: Light reflected from a thin film of oil on water



....

Light is reflected back upwards from both the air-oil boundary (ray 1) and the oil-water boundary underneath it (ray 2). Someone looking at the reflected light will see the superposition of these two rays. Note that this situation is what we have called interference by division of amplitude. For any light wave falling on the oil film, some of the wave is reflected from the surface of the oil film. Some light is reflected by the water surface, and some is transmitted into the water. The two reflected rays travel different paths before being recombined. Under what conditions will the two rays interfere constructively or destructively?

To keep the analysis simple, we will assume the angle of incidence in Figure 3.6 is 0°, so that ray 2 travels a total distance of $2d$ in the film. The optical path difference between rays 1 and 2 is therefore $2n_{film}d$. But there is another source of path difference. Ray 1 has undergone a λ/2 phase change, since it has been reflected at a higher refractive index medium.

The total optical path difference is therefore equal to

$$2n_{film}d + \frac{\lambda}{2}$$

For constructive interference, this optical path difference must equal a whole number of wavelengths.

$$2n_{film}d + \frac{\lambda}{2} = m\lambda$$

where m = 1, 2, 3...

$$\begin{aligned} 2n_{film}d &= m\lambda - \frac{\lambda}{2} \\ \therefore 2n_{film}d &= \left(m - \tfrac{1}{2}\right)\lambda \\ \therefore d &= \frac{\left(m - \frac{1}{2}\right)\lambda}{2n_{film}} \end{aligned} \tag{3.2}$$

Equation 3.2 gives us an expression for the values of film thickness d for which reflected light of wavelength λ will undergo constructive interference, and hence be reflected strongly.

For destructive interference, the optical path difference must equal an odd number of half-wavelengths

$$2n_{film}d + \frac{\lambda}{2} = \left(m + \tfrac{1}{2}\right)\lambda$$

where m = 1, 2, 3...

$$\begin{aligned} \therefore 2n_{film}d &= \left(m + \tfrac{1}{2}\right)\lambda - \frac{\lambda}{2} \\ \therefore 2n_{film}d &= m\lambda \\ \therefore d &= \frac{m\lambda}{2n_{film}} \end{aligned} \tag{3.3}$$

Equation 3.3 tells us for which values of d reflected light of wavelength λ will undergo destructive interference, and hence be reflected weakly.

Example

Using Figure 3.6, what is the *minimum* thickness of oil film which would result in destructive interference of green light (λ = 525 nm) falling on the film with angle of incidence 0°?

For destructive interference we will use Equation 3.3, with λ = 525 nm = 5.25×10^{-7} m. For the minimum film thickness, we set m equal to 1.

$$d = \frac{m\lambda}{2n_{film}}$$
$$\therefore d = \frac{1 \times 5.25 \times 10^{-7}}{2 \times 1.45}$$
$$\therefore d = \frac{1 \times 5.25 \times 10^{-7}}{2.90}$$
$$\therefore d = 1.81 \times 10^{-7}\text{m}$$

....

If we have sunlight falling on an oil film, then we have a range of wavelengths present. Also, an oil film is unlikely to have the same thickness all along its surface. So we will have certain wavelengths interfering constructively where the film has one thickness, whilst the same wavelengths may be interfering destructively at a part of the surface where the film thickness is different. The overall effect is that the film appears to be multi-coloured.

Thin film interference

30 min

At this stage there is an online activity.

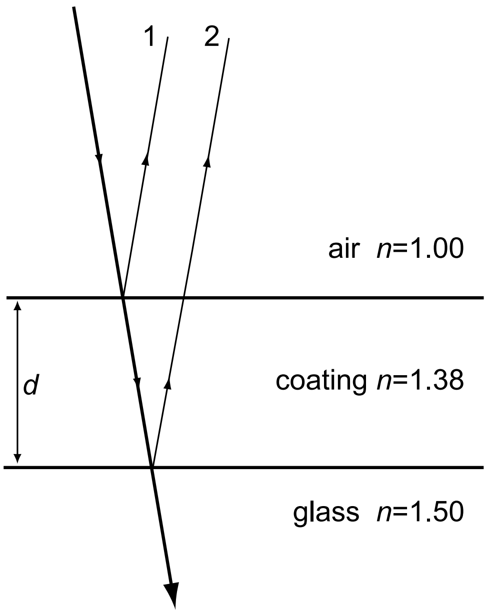
This activity shows interference as the thickness of a thin film is varied, along with a calculation.

For a monochromatic incident beam, the thickness of the film determines whether constructive or destructive interference takes place. Destructive interference occurs for film thicknesses of $\lambda/2n_{film}$, λ/n_{film}, $3\lambda/2n_{film}$...

....

An important application of thin film interference is anti-reflection coatings used on camera lenses. A thin layer of a transparent material such as magnesium fluoride (n = 1.38) is deposited on a glass lens (n = 1.50), as shown in Figure 3.7.

Figure 3.7: Anti-reflection coating deposited on a glass lens



....

Once again, if we consider the light falling with an angle of incidence of 0° on the lens, then the optical path difference between rays 1 and 2 is $2 \times n_{coating} \times d$. Both rays are reflected by media with a greater value of refractive index, so they **both** undergo the same λ/2 phase change on reflection and there is no extra optical path difference as there was with the oil film on water. So for destructive interference

$$\begin{aligned} 2n_{coating}d &= \left(m + \tfrac{1}{2}\right)\lambda \\ \therefore d &= \frac{\left(m + \tfrac{1}{2}\right)\lambda}{2n_{coating}} \end{aligned} \tag{3.4}$$

....

In this equation, m = 0, 1, 2...

The minimum coating thickness that will result in destructive interference is given by putting m = 0 in Equation 3.4:

$$\begin{aligned} d &= \frac{\left(0 + \tfrac{1}{2}\right)\lambda}{2n_{coating}} \\ \therefore d &= \frac{\lambda}{4n_{coating}} \end{aligned} \tag{3.5}$$

....

So a coated **(bloomed) lens** can be made non-reflecting for a specific wavelength of light, and Equation 3.5 gives the minimum coating thickness for that wavelength.

Example

For the coated lens shown in Figure 3.7, what is the minimum thickness of magnesium fluoride that can be used to make the lens non-reflecting at λ = 520 nm?

Use Equation 3.5

$$d = \frac{\lambda}{4n_{coating}}$$
$$\therefore d = \frac{5.20 \times 10^{-7}}{4 \times 1.38}$$
$$\therefore d = 9.42 \times 10^{-8}\text{m}$$

. . . .

Since most lenses are made to operate in sunlight, the thickness of the coating is designed to produce destructive interference in the centre of the visible spectrum. The previous example produced an anti-reflection coating in the green part of the spectrum, which is typical of commercial coatings. The extremes of the visible spectrum - red and violet - do not undergo destructive interference upon reflection, so a coated lens often looks reddish-purple under everyday lighting.

Quiz 2 Thin film interference

20 min

First try the questions. If you get a question wrong or do not understand a question, there are Hints. The hints are in the same order as the questions. If you read the hint and still do not understand then ask your tutor. All references in the hints are to online materials.

refractive index of air	1.00
refractive index of water	1.33

Q6: Two coherent light rays emitted from the same source will interfere constructively if

a) they undergo a phase change on reflection.
b) they travel in materials with different refractive indices.
c) their optical path difference is an integer number of wavelengths.
d) their optical path difference is an odd number of half-wavelengths.
e) they have different wavelengths.

....

Q7: What is the minimum thickness of oil film (n = 1.48) on water that will produce destructive interference of a beam of light of wavelength 620 nm?

a) 1.05×10^{-7} m
b) 2.09×10^{-7} m
c) 2.33×10^{-7} m
d) 3.14×10^{-7} m
e) 2.07×10^{-6} m

....

Q8: A soap film, with air on either side, is illuminated by electromagnetic radiation normal to its surface. The film is 2.00×10^{-7} m thick, and has refractive index 1.40. Which wavelengths will be intensified in the reflected beam?

a) 200 nm and 100 nm
b) 560 nm and 280 nm
c) 1120 nm and 373 nm
d) 1120 nm and 560 nm
e) 2240 nm and 747 nm

....

Q9: Why do lenses coated with an anti-reflection layer often appear purple in colour when viewed in white light?

a) You cannot make an anti-reflection coating to cut out red or violet.
b) The coating is too thick to work properly.
c) The green light is coherent but the red and violet light is not.
d) The coating is only anti-reflecting for the green part of the visible spectrum.
e) Light only undergoes a phase change upon reflection at that wavelength.

....

Q10: What is the minimum thickness of magnesium fluoride (n = 1.38) that can form an anti-reflection coating on a glass lens for light with wavelength 500 nm?

a) 9.06×10^{-8} m
b) 1.36×10^{-7} m
c) 1.81×10^{-7} m
d) 2.72×10^{-7} m
e) 5.00×10^{-7} m

....

There is one final point to add before we leave thin film interference. At the beginning of this topic, we discussed coherence, and stated that we required coherent radiation to observe interference effects. But we have seen here that we can produce interference with an incoherent source such as sunlight. How is this possible? The answer is that the process of interference by division of amplitude means that we are taking one wave, splitting it up and then recombining it with itself. We do not have one wave interfering with another, and so we do not need a source of coherent waves. Division of amplitude means we can use an extended source to produce interference.

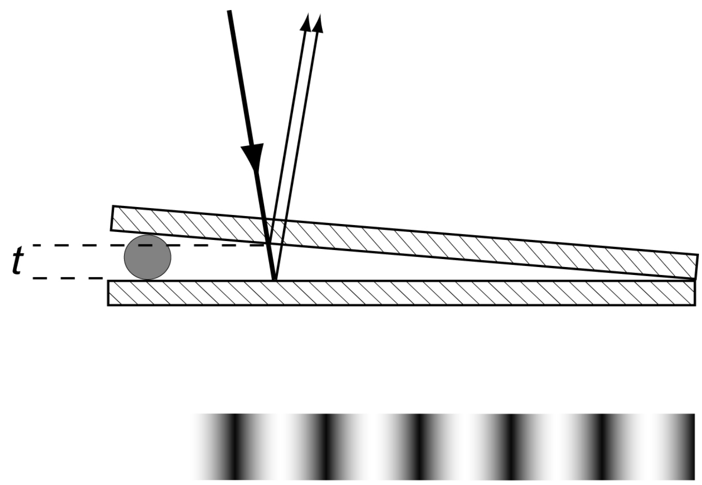
3.5 Wedge fringes

Learning Objective

To describe how an air wedge can produce interference fringes

Interference fringes can also be produced by a thin wedge of air. If we place a thin piece of foil between two microscope slides at one end, we form a thin air wedge. This is shown in Figure 3.8, where the size of the wedge angle has been exaggerated for clarity.

Figure 3.8: The side and top views of a wedge fringes experiment



When the wedge is illuminated with a monochromatic source, bright and dark bands are seen in the reflected beam. Interference is taking place between rays reflected from the lower surface of the top slide and the upper surface of the bottom slide. Because of the piece of foil, the thickness of the air wedge is increasing from right to left, so the optical path difference between the reflected rays is increasing. A bright fringe is seen when the optical path difference leads to constructive interference, and a dark fringe occurs where destructive interference is taking place.

We can calculate the wedge thickness required to produce a bright or dark fringe. We will be considering light falling normally onto the glass slides. Remember that the angle between the slides is extremely small, so it can be assumed to be approximately zero in this analysis. The path difference between the two rays in Figure 3.8 is the extra distance travelled **in air** by the ray reflected from the lower slide, which is equal to $2t$.

Do either of the rays undergo a phase change upon reflection?

The answer is yes - the ray reflected by the lower slide is travelling in a low refractive index medium (air) and being reflected at a boundary with a higher n medium (glass) so this ray does undergo a $\lambda/2$ phase change. The other ray does not, as it is travelling in the higher n medium. So the total optical path difference between the two rays is

$$2t + \frac{\lambda}{2}$$

For constructive interference and a bright fringe,

$$\begin{aligned} 2t + \frac{\lambda}{2} &= m\lambda \\ \therefore 2t &= (m - \tfrac{1}{2})\lambda \\ \therefore t &= \frac{(m - \frac{1}{2})\lambda}{2} \end{aligned} \tag{3.6}$$

....

In this case, m = 1, 2, 3...

For destructive interference, leading to a dark fringe

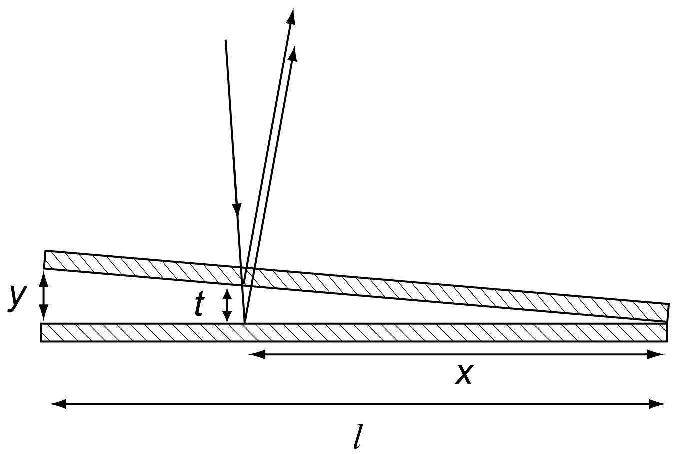
$$\begin{aligned} 2t + \frac{\lambda}{2} &= \left(m + \tfrac{1}{2}\right)\lambda \\ \therefore 2t &= m\lambda \\ \therefore t &= \frac{m\lambda}{2} \end{aligned} \tag{3.7}$$

....

Here, m is a whole number again, but we can also put m = 0, which corresponds to t = 0. Check back to Figure 3.8, and you can see that the m = 0 case gives us a dark fringe where the two slides are touching and t = 0.

Finally, the fringe separation can be determined if we know the size and separation of the glass slides.

Figure 3.9: Wedge separation analysis



....

The length of the glass slides shown in Figure 3.9 is l m, and the slides are separated by y m at one end. The m^{th} dark fringe is formed a distance x m from the end where the slides are in contact. By looking at similar triangles in Figure 3.9, we can see that

$$\frac{t}{x} = \frac{y}{l}$$
$$\therefore x = \frac{tl}{y}$$

Substituting for t using Equation 3.7

$$x = \frac{m\lambda l}{2y}$$

The distance Δx between the m^{th} and $(m+1)^{\text{th}}$ dark fringes (the fringe separation) is therefore

$$\Delta x = \frac{(m+1)\,\lambda l}{2y} - \frac{m\lambda l}{2y}$$
$$\therefore \Delta x = \frac{\lambda l}{2y} \tag{3.8}$$

. . . .

Wedge fringes

20 min

A wedge interference experiment is being carried out. The wedge is formed using two microscope slides, each of length 8.0 cm, touching at one end. At the other end, the slides are separated by a 0.020 mm thick piece of foil. What is the fringe spacing when the experiment is carried out using:

1. a helium-neon laser, at wavelength 633 nm?
2. an argon laser, at wavelength 512 nm?

The fringe separation depends on the wavelength of the incident light. The fringe spacing decreases as the wavelength decreases.

. . . .

Quiz 3 Wedge fringes

20 min

First try the questions. If you get a question wrong or do not understand a question, there are Hints. The hints are in the same order as the questions. If you read the hint and still do not understand then ask your tutor. All references in the hints are to online materials.

Q11: In a wedge interference experiment carried out in air using monochromatic light, the 8^{th} bright fringe occurs when the wedge thickness is 1.80×10^{-6} m.

What is the wavelength of the light?

a) 225 nm
b) 240 nm
c) 450 nm
d) 480 nm
e) 960 nm

. . . .

Q12: An air-wedge interference experiment is being carried out using a mixture of blue light (λ = 420 nm) and red light (λ = 640 nm).

What is the colour of the bright fringe that is seen closest to where the glass slides touch?

a) Blue
b) Red
c) Purple - both blue and red appear at the same place
d) It is dark, as the two colours cancel each other out
e) Impossible to say without knowing the thickness of the wedge

. . . .

Q13: A wedge interference experiment is carried out using two glass slides 10.0 cm long, separated at one end by 1.00×10^{-5} m.

What is the fringe separation when the slides are illuminated by light of wavelength 630 nm?

a) 1.00×10^{-3} m
b) 1.25×10^{-3} m
c) 1.58×10^{-3} m
d) 3.15×10^{-3} m
e) 6.30×10^{-3} m

. . . .

Q14: What is the minimum air-wedge thickness that would produce a bright fringe for red light of wavelength 648 nm?

a) 81.0 nm
b) 162 nm
c) 324 nm
d) 648 nm
e) 972 nm

. . . .

Q15: During a wedge fringes experiment with monochromatic light, air between the slides is replaced by water.

What would happen to the fringes?

a) The bright fringes would appear brighter.
b) The fringes would move closer together.
c) The fringes would move further apart.

d) There would be no difference in their appearance.
e) The colour of the fringes would change.

. . . .

3.6 Summary

Interference of light waves can be observed when two (or more) coherent beams are superposed. This usually requires a source of coherent light waves. Division of amplitude - splitting a wave into two parts which are later re-combined - can be used to produce interference effects without requiring a coherent source.

Two experimental arrangements for viewing interference by division of amplitude have been presented in this topic. In thin film interference, one ray travels an extra distance through a different medium. The thickness and refractive index of the film must be known in order to predict whether constructive or destructive interference takes place. In a thin wedge interference experiment, only the thickness of the wedge is required.

By the end of this topic you should be able to:

- state the condition for two light beams to be coherent;
- explain why the conditions for coherence are more difficult to achieve for light than for sound and microwaves;
- define the term "optical path difference" and relate it to phase difference;
- state what is meant by the principle of division of amplitude, and describe how the division of amplitude allows interference to be observed using an extended source;
- state the conditions under which a light wave will undergo a phase change upon reflection;
- derive expressions for maxima and minima to be formed in a "thin film" reflection, and perform calculations using these expressions;
- explain the formation of coloured fringes when a thin film is illuminated by white light;
- explain how a lens can be made non-reflecting for a particular wavelength of light;
- derive the expression for the minimum thickness of a non-reflecting coating, and carry out calculations using this expression;
- explain why a coated ("bloomed") lens appears coloured when viewed in daylight;
- derive the expression for the distance between fringes formed by "thin wedge" reflection, and carry out calculations using this expression;

3.7 End of topic test

End of topic test

30 min

At this stage there is an end of topic test available online. If however you do not have access to the internet you may try the questions which follow.

The following data should be used when required:

speed of light in a vacuum *c*	3.00×10^8 m s1
speed of sound	340 m s^{-1}
acceleration due to gravity *g*	9.8 m s^{-2}

Q16: Monochromatic light of wavelength 462 nm travels through a glass lens of thickness 23.5 mm and refractive index 1.52.

Calculate the optical path length through the lens, giving your answer in mm.

....

Q17: Two light rays from a coherent source travel through the same distance to reach a detector. One ray travels 29.2 mm through glass of refractive index 1.55 . The other ray travels in air throughout.

Calculate the optical path difference between the rays, in mm.

....

Q18: A soap bubble consists of a thin film of soapy water (n = 1.29) surrounded on both sides by air.

Calculate the minimum film thickness (in m) if light of wavelength 620 nm is strongly reflected by the film.

....

Q19: White light is incident on an oil film (thickness 2.71×10^{-7} m, refractive index 1.35) floating on water (refractive index 1.33). The white light has a wavelength range of 350 - 750 nm.

Calculate the two wavelengths in this range which undergo destructive interference upon reflection.

....

Q20: A 1.02×10^{-7} m coating of a transparent polymer (n = 1.34) is deposited on a glass lens to make an anti-reflection coating.

f the lens has refractive index 1.52, calculate the wavelength of light in the range 350 - 750 nm for which the coating is anti-reflecting. (Give your answer in nm).

....

Q21: In a standard air wedge interference experiment, calculate the thickness (in m) of the wedge which gives the 10th bright fringe for light of wavelength 462 nm.

....

Q22: Wedge fringes are formed in the air gap between two glass slides of length 12.5 cm, separated at one end by a 10.0 μm piece of paper. The wedge is illuminated by monochromatic light of wavelength 504 nm.

Calculate the distance between adjacent dark fringes, giving your answer in m.

. . . .

Q23: Two glass slides are laid together, separated at one end by a 15.0 μm sliver of foil. The slides each have length 10.0 cm. When illuminated by monochromatic light, a series of light and dark reflection fringes appear, with fringe separation 1.85 mm.

Calculate the wavelength (in nm) of the light.

. . . .

Topic 4

Interference by division of wavefront

Contents

Prerequisite knowledge

- *A prerequisite for this topic is an understanding of coherent light waves.*

Learning Objectives

By the end of this topic you should be able to:

- *show an understanding of the difference between interference by division of wavefront and division of amplitude;*
- *describe the Young's slits experiment;*
- *derive the expression for fringe spacing in a Young's slits experiment, and use this expression to determine the wavelength of a monochromatic source.*

4.1 Introduction

The work in this topic will concentrate on the experiment known as Young's slits, which is an example of interference by division of wavefront. We will see the difference between this process and interference by division of amplitude.

The experimental arrangement for producing interference will be described, followed by an analysis of the interference pattern. This analysis will reveal that the spacing between interference fringes is in direct proportion to the wavelength of the light.

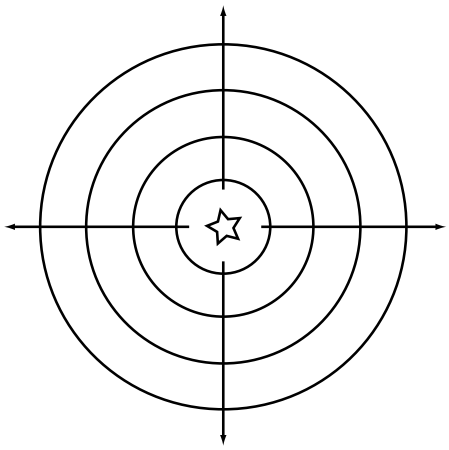
4.2 Interference by division of wavefront

Learning Objective

To define interference by division of wavefront

Light emitted from a point source radiates uniformly in all directions. This is often illustrated by showing the wavefronts perpendicular to the direction of travel, as in Figure 4.1.

Figure 4.1: Wavefronts emitted by a point source



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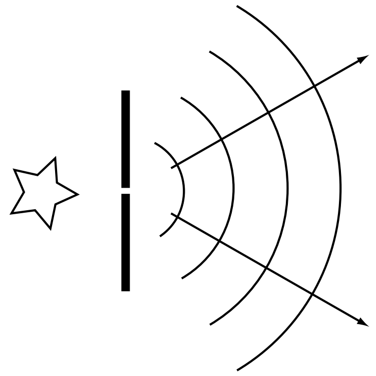
Each wavefront joins points in phase, for example the crest or trough of a wave. If the light source is monochromatic then the points on the wavefront are coherent, as they have the same wavelength and are in phase. If we can take two parts of a wavefront and combine them, then we will see interference effects. This is known as **interference by division of wavefront**. This is a different process to division of amplitude, in which a single wave was divided and re-combined. Here we are combining two separate waves. The two waves must be coherent to produce stable interference when they are combined.

An extended source acts like a collection of point sources, and cannot be used for a division of wavefront experiment. To overcome this problem, an extended source is often used behind a small aperture in a screen (see Figure 4.2). The size of the aperture must be of the same order of magnitude as the wavelength of the light. In this way the light appears to come from a point source.

Figure 4.2: Coherent beam produced by an extended (monochromatic) source



....

In the next Section, we will see how coherent waves produced in this manner can be combined to show interference effects. Note that if a coherent source such as a laser is used, we do not need to pass the beam through an aperture.

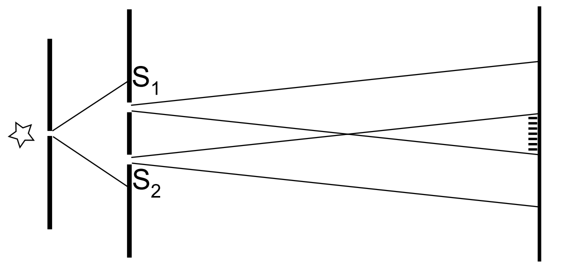
4.3 Young's slits experiment

Learning Objective

To describe and analyse the Young's slits experiment

This experiment is one of the earliest examples of interference by division of wavefront, first carried out by Thomas Young in 1801. The experimental arrangement is shown in Figure 4.3.

Figure 4.3: Young's slits experiment



....

Monochromatic light is passed through one narrow slit to give a coherent source, as described earlier. The division of wavefront takes place at the two slits S_1 and S_2.

These slits are typically less than 1 mm apart, and act as point sources. A screen is placed about 1 m from the slits. Where the two beams overlap, a symmetrical pattern of fringes is formed, with a bright fringe at the centre.

Figure 4.4: Young's slits interference fringes

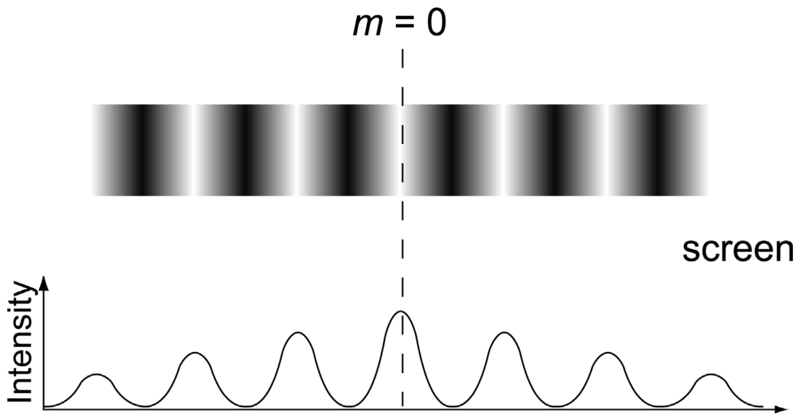


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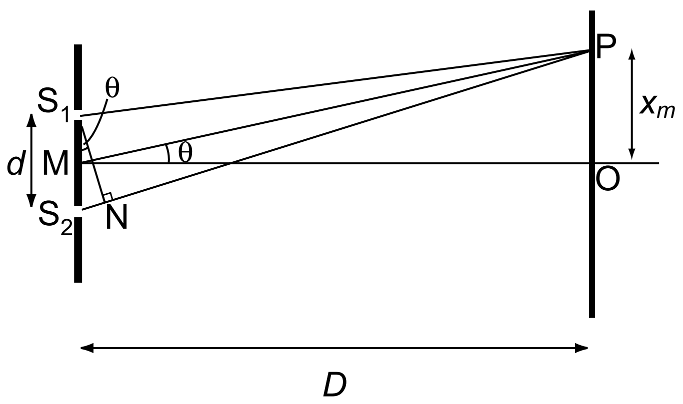
Figure 4.4 shows the fringe pattern seen on the screen. The lower part of Figure 4.4 shows a plot of irradiance across the screen. The brightest fringe occurs at the centre of the interference pattern as this point is equidistant from the slits, and so the waves arrive exactly in phase. The first dark fringes occur on either side of this when the optical path difference between the beams is exactly half a wavelength. This is followed by the next bright fringe, due to a path difference of exactly one wavelength, and so on.

In general, a bright fringe occurs when the path difference between the two beams is $m\lambda$, where m = 0, 1, 2... The central bright fringe corresponds to m = 0.

The fringe spacing can be analysed to determine the wavelength of the light.

Figure 4.5: Analysis of Young's slits experiment



....

In Figure 4.5, the slits are separated by a distance d, and M marks the midpoint between the slits S_1 and S_2. The screen is placed a distance D ($>> d$) from the slits, with O being the point directly opposite M where the m = 0 bright fringe occurs. The m^{th} bright fringe is located at the point P, a distance x_m from O.

The path difference between the two beams is $S_2P - S_1P = m\lambda$. If the length PN equal to S_1P is marked on S_2P, then the path difference is the distance $S_2N = m\lambda$.

Since PM is very much larger than S_1S_2, the line S_1N meets S_2P at approximately a right angle and S_1S_2N is a right-angled triangle. In this triangle

$$\sin\theta = \frac{S_2N}{S_1S_2} = \frac{m\lambda}{d}$$

We can also look at the right-angled triangle formed by MPO

$$\tan\theta = \frac{OP}{MO} = \frac{x_m}{D}$$

Because θ is very small, $\sin\theta \approx \tan\theta \approx \theta$. Therefore

$$\frac{x_m}{D} = \frac{m\lambda}{d}$$
$$\therefore x_m = \frac{m\lambda D}{d}$$

To find the separation Δx between fringes, we need to find the distance $x_{m+1} - x_m$ between the $(m+1)^{th}$ and m^{th} bright fringes

$$\begin{aligned}\Delta x &= x_{m+1} - x_m \\ \therefore \Delta x &= \frac{(m+1)\lambda D}{d} - \frac{m\lambda D}{d} \\ \therefore \Delta x &= \frac{\lambda D}{d}\end{aligned} \tag{4.1}$$

....

Rearranging Equation 4.1 in terms of λ

$$\lambda = \frac{\Delta x d}{D} \tag{4.2}$$

....

So a Young's slits experiment can be used to determine the wavelength of a monochromatic light source.

Carry out the following activity to see how the experimental parameters affect the appearance of the interference fringes.

15 min

Young's slits

At this stage there is an online activity.

This activity is a Young's slits demonstration in which the wavelength, slit separation and screen location can all be changed.

You should understand how each of the parameters: wavelength, slit separation and slit-to-screen distance affect the fringe pattern in a Young's slits experiment.

....

Example

A Young's slits experiment is set up with a slit separation of 0.400 mm. The fringes are viewed on a screen placed 1.00 m from the slits. The separation between the m = 0 and m = 10 bright fringes is 1.40 cm. What is the wavelength of the monochromatic light used?

We are given the separation for 10 fringes as 1.40 cm, so the fringe separation Δx = 0.140 cm. Converting all the distances involved into metres, we have Δx = 1.40 $\times$ 10^{-3} m, d = 4.00 $\times$ 10^{-4} m and D = 1.00 m. Using Equation 4.2

$$\begin{aligned} \lambda &= \frac{\Delta x d}{D} \\ \therefore \lambda &= \frac{1.40 \times 10^{-3} \times 4.00 \times 10^{-4}}{1.00} \\ \therefore \lambda &= 5.60 \times 10^{-7}\,\text{m} \\ &= 560\,\text{nm} \end{aligned}$$

....

There is one more problem to consider - what happens if a Young's slits experiment is performed with white light? What does the interference pattern look like then?

If white light is used, there is no difference to the m = 0 bright fringe. Since this appears at an equal distance from both slits, then constructive interference will occur whatever the wavelength. For the m = 1, 2, 3... fringes, Equation 4.1 tells us that the fringe separation $\Delta x \propto \lambda$, so the fringe separation is smallest for short wavelengths. Thus the violet end of the spectrum produces the m = 1 bright fringe closest to m = 0. The red end of the spectrum produces a bright fringe at a larger separation. At higher orders, the fringe patterns of different colours will overlap.

Quiz 1 Young's slits

20 min

First try the questions. If you get a question wrong or do not understand a question, there are Hints. The hints are in the same order as the questions. If you read the hint and still do not understand then ask your tutor. All references in the hints are to online materials.

Q1: The Young's slits experiment is an example of

a) interference by division of amplitude.
b) interference by division of wavefront.
c) the Doppler effect.
d) phase change.
e) refractive index.

. . . .

Q2: A Young's slits experiment is set up using slits 0.50 mm apart. The fringes are detected 1.2 m from the slits, and are found to have a fringe spacing of 0.80 mm.

What is the wavelength of the radiation used?

a) 113 nm
b) 128 nm
c) 200 nm
d) 333 nm
e) 720 nm

. . . .

Q3: What happens to the fringes in a Young's slits experiment if a red light source (λ = 640 nm) is replaced by a green light source (λ = 510 nm)?

a) The fringes disappear.
b) The fringes move closer together.
c) The fringes move further apart.
d) There is no change to the position of the fringes.
e) No fringes are formed in either case.

. . . .

Q4: A Young's slits experiment is carried out by passing coherent light of wavelength 480 nm through slits 0.50 mm apart, with a screen placed 2.5 m from the slits.

What is the spacing between bright fringes viewed on the screen?

a) 2.4 mm
b) 3.8 mm
c) 4.8 mm
d) 5.8 mm
e) 9.6 mm

. . . .

Q5: Using a similar set-up to the previous question, with a red light source at 650 nm, how far from the central bright fringe is the 5th bright fringe formed?

a) 0.00065 m
b) 0.0033 m
c) 0.0041 m
d) 0.0065 m
e) 0.016 m

....

4.4 Summary

Using a coherent source, interference by division of wavefront can be demonstrated. We have studied the Young's slits experiment as an example of this type of interference. The path difference between light from two slits determines whether constructive or destructive interference takes place where the beams overlap on a viewing screen.

By the end of this topic you should be able to:

- state what is meant by 'interference by division of wavefront';
- explain why the principle of division of a wavefront requires the use of a point or line source;
- derive the expression

$$\Delta x = \frac{\lambda D}{d}$$

  for the fringe spacing in the Young's slits experiment, and explain how altering the parameters λ, D and d affects the appearance of the interference pattern;
- carry out calculations using the expression

$$\Delta x = \frac{\lambda D}{d}$$

4.5 End of topic test

End of topic test

30 min

At this stage there is an end of topic test available online. If however you do not have access to the internet you may try the questions which follow.

The following data should be used when required:

speed of light in a vacuum *c*	3.00×10^{8} m s1
speed of sound	340 m s^{-1}
acceleration due to gravity *g*	9.8 m s^{-2}

Q6: A Young's slits experiment is carried out using monochromatic light of wavelength 530 nm. The slit separation is 4.45×10^{-4} m, and the screen is placed 1.00 m from the slits.

Calculate the spacing between adjacent bright fringes, in m.

. . . .

Q7: The wavelength of a monochromatic light source is determined by a Young's slits experiment. The slits are separated by 5.25×10^{-4} m, and located 2.50 m from the viewing screen.

If the fringe spacing is 2.48 mm, calculate the wavelength (in nm) of the light.

. . . .

Q8: In a Young's slits experiment, the fringes are viewed on a screen 1.15 m from the slits. The 8 th bright fringe is observed 12.8 mm from the central maximum. The slit separation is 0.400 mm.

Calculate the wavelength (in nm) of the monochromatic light source.

. . . .

Q9: Coherent light at wavelength 490 nm is passed through a pair of slits placed 1.15 mm apart. The resulting fringes are viewed on a screen placed 1.50 m from the slits.

Calculate how far from the central bright fringe the 5th bright fringe appears, giving your answer in mm.

. . . .

Q10: A Young's slits experiment was carried out using two monochromatic sources A and B. The longer wavelength source (A) had wavelength 618 nm. It was found that the 5th bright fringe from A occurred at the same distance from the central bright fringe as the 6th bright fringe from B.

Calculate the wavelength of B, in nm.

. . . .

Topic 5

Polarisation

Contents

Prerequisite knowledge

- *The relationship between the irradiance and amplitude of a wave (Waves topic 1).*
- *Snell's law of refraction.*

Learning Objectives

By the end of this topic you should be able to:

- *state the difference between polarised and unpolarised light;*
- *calculate the irradiance of light transmitted through a polariser-analyser pair;*
- *derive and apply Brewster's law;*
- *describe different methods of polarising a beam of light;*
- *describe some applications of polarised light.*

5.1 Introduction

One of the properties of a transverse wave is that it can be polarised. This means that all the oscillations of the wave are in the same plane. In this topic we will investigate the production and properties of polarised waves. Most of this topic will deal with light waves, and some of the applications of polarised light will be described at the end of the topic.

Light waves are electromagnetic waves, made up of orthogonal (perpendicular) oscillating electric and magnetic fields. When we talk about the oscillations of a light wave, we will be describing the oscillating electric field. For clarity, the magnetic fields will not be shown on any of the diagrams in this chapter - this is the normal practice when describing electromagnetic waves.

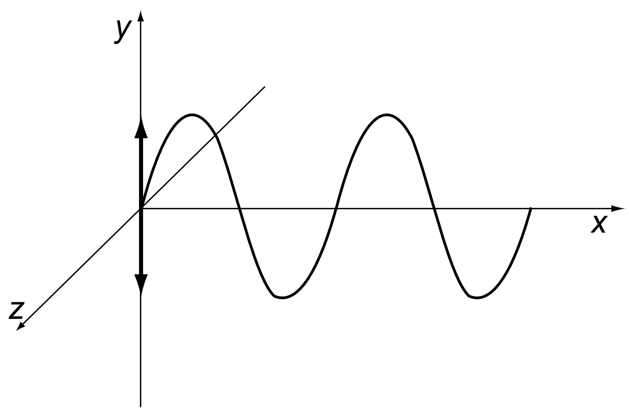
5.2 Polarised and unpolarised waves

Learning Objective

To describe the difference between polarised and unpolarised light

Let us consider a transverse wave travelling in the *x*-direction. Although we will be concentrating on light waves in this topic, it is useful to picture transverse waves travelling along a rope. Figure 5.1 shows transverse waves oscillating in the *y*-direction.

Figure 5.1: Transverse waves with oscillations in the y-direction



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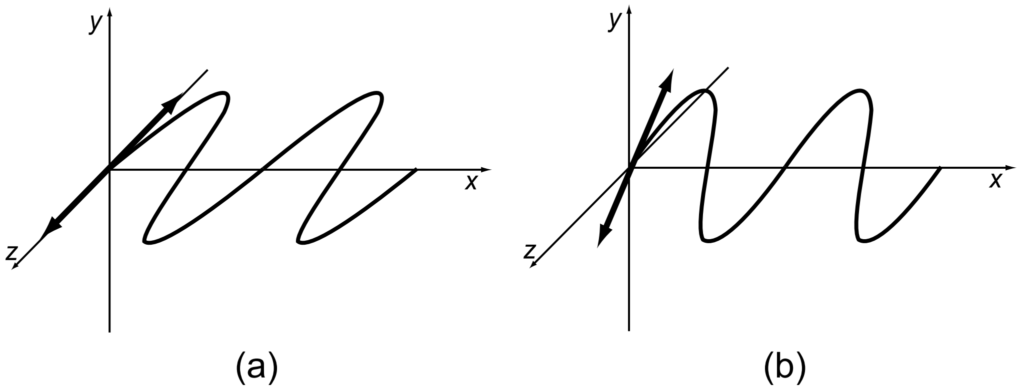
The oscillations are not constrained to the *y*-direction (the vertical plane). The wave can make horizontal oscillations in the *z*-direction, or at any angle ϕ in the *y*-*z* plane, so long as the oscillations are at right angles to the direction in which the wave is travelling (see Figure 5.2).

Figure 5.2: Transverse waves oscillating (a) in the z-direction, and (b) at an angle ϕ in the y-z plane.



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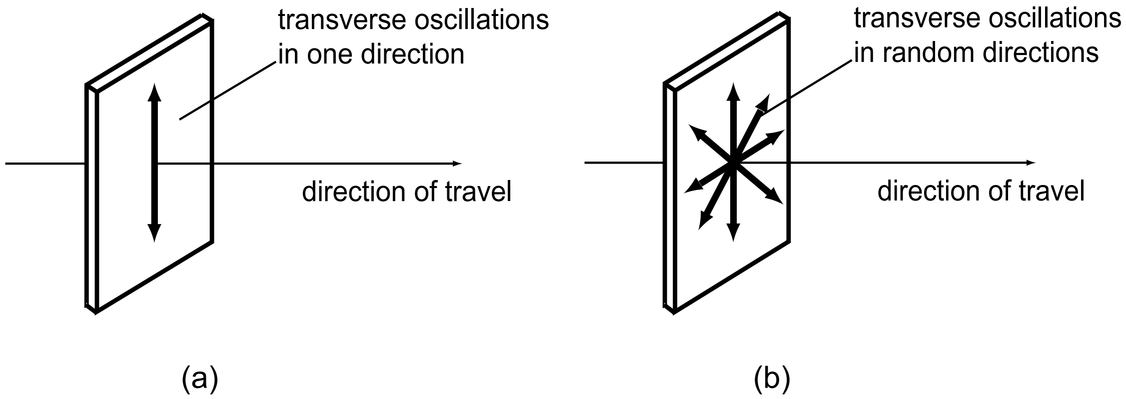
When all the oscillations occur in one plane, as shown in Figure 5.1 and Figure 5.2, the wave is said to be **polarised**. If oscillations are occurring in many or random directions, the wave is **unpolarised**. The difference between polarised and unpolarised waves is shown in Figure 5.3.

Figure 5.3: (a) Polarised, and (b) unpolarised waves



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Light waves produced by a filament bulb or strip light are **unpolarised**. In the next two Sections of this topic different methods of producing **polarised light** will be described. You should note that longitudinal waves cannot be polarised since the oscillations occur in the direction in which the wave is travelling. This means that sound waves, for example, cannot be **polarised**.

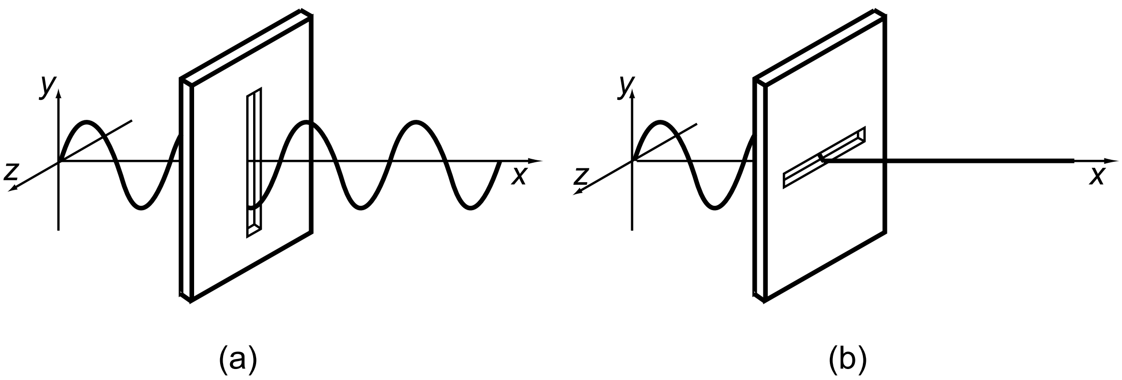
5.3 Polaroid and Malus' law

Learning Objective

To describe the action of a sheet of Polaroid on a beam of light

Before we look at how to polarise light waves, let us think again about a (polarised) transverse wave travelling along a rope in the *x*-direction with its transverse oscillations in the *y*-direction. In Figure 5.4 the rope passes through a board with a slit cut into it. Figure 5.4 (a) shows what happens if the slit is aligned parallel to the *y*-axis. The waves pass through, since the oscillations of the rope are parallel to the slit. In Figure 5.4 (b), the slit is aligned along the *z*-axis, perpendicular to the oscillations. As a result, the waves cannot be transmitted through the slit.

Figure 5.4: (a) Transverse waves passing through a slit parallel to its oscillations, and (b) a slit perpendicular to the oscillations blocking the transmission of the waves



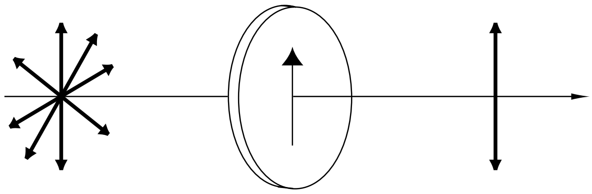
....

What would happen if the wave incident on the slit was oscillating in the *y*-*z* plane, making an angle of 45° with the *y*-axis, for instance? Under those conditions, the amplitude of the incident wave would need to be resolved into components parallel and perpendicular to the slit. The component parallel to the slit is transmitted, whilst the component perpendicular to the slit is blocked. The transmitted wave emerges polarised parallel to the slit. Later in this Section we will derive the equation used for calculating the irradiance of light transmitted through a polariser whose transmission axis is not parallel to the plane of polarisation of the light waves.

5.3.1 Polarisation of light waves using Polaroid

Although the mechanical analogy is helpful it cannot be carried over directly to the comparable situation involving light waves. Consider a sheet of Polaroid, a material consisting of long, thin polymer molecules (doped with iodine) that are aligned with each other. Because of the way a polarised light wave interacts with the molecules, the sheet of Polaroid only transmits the components of the light with the electric field vector perpendicular to the molecular alignment. The direction which passes the polarised light waves is called the **transmission axis**. The Polaroid sheet blocks the electric field component that is parallel to the molecular alignment.

Figure 5.5: Action of a sheet of Polaroid on unpolarised light



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In Figure 5.5, light from a filament bulb is unpolarised. This light is incident on a sheet of Polaroid whose transmission axis is vertical. The beam that emerges on the right of the diagram is polarised in the same direction as the transmission axis of the Polaroid. Remember, this means that the electric field vector of the electromagnetic wave is oscillating in the direction shown.

Polarised light

10 min

At this stage there is an online activity.

This activity shows an electromagnetic wave, namely a plane polarized wave, which propagates in positive *x* direction and explores the electric and magnetic fields of a polarised light wave.

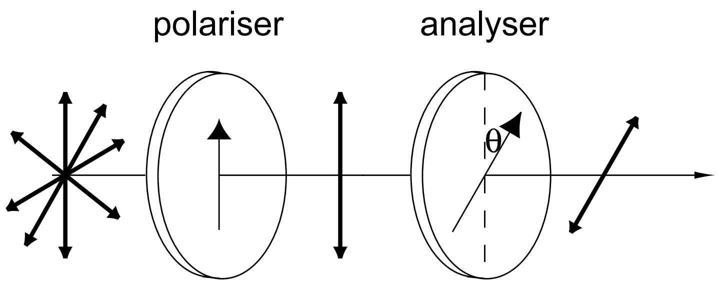
The animation shows the electric and magnetic fields of a polarised light wave. Make sure you understand how this animation relates to the diagrams in the text, where the magnetic field is not included.

. . . .

5.3.2 Malus' law

Earlier in this Section, the problem of a polariser acting on a polarised beam of light was introduced. We will now tackle this problem and calculate how much light is transmitted when the transmission axis of a Polaroid is at an angle to the plane of polarisation. The two cases illustrated in Figure 5.4 show what would happen if the transmission axis is parallel or perpendicular to the polarisation direction of the beam. In the former case all of the light is transmitted, in the latter case none of it is. Figure 5.6 shows what happens when the transmission axis of a sheet of Polaroid makes an angle θ with the plane of polarisation of an incident beam.

Figure 5.6: Polarised beam incident on a second sheet of Polaroid



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In Figure 5.6 an unpolarised beam of light is polarised by passing it through a sheet of Polaroid. The polarised beam is then passed through a second Polaroid sheet, often called the analyser. The transmission axis of the analyser makes an angle θ with the plane of polarisation of the incident beam. The beam that emerges from the analyser is polarised in the same direction as the transmission axis of the analyser.

5 min

Polariser and analyser

Learning Objective

To illustrate the passage of polarised light through two polarisers

Shows the passage of a polarised light beam through two polarisers

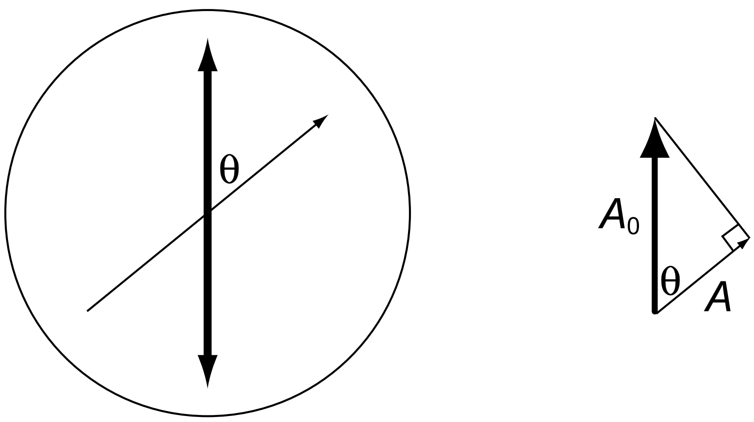
At this stage there is an online activity.

This activity shows how the irradiance of the transmitted beam depends on the angle between the transmission axes of the polariser and the analyser.

. . . .

A ‘head-on' view of the analyser will help us to find the irradiance of the transmitted beam (see Figure 5.7).

Figure 5.7: Vertically polarised beam incident on an analyser



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The incident beam has amplitude A_0. From Figure 5.7, the component of A_0 parallel to the transmission axis of the analyser is $A_0\cos\theta$. So the beam transmitted through the analyser has amplitude A, where

$$A = A_0 \cos\theta \tag{5.1}$$

The irradiance of a beam, measured in W m^{-2}, is proportional to the square of the amplitude. Thus the irradiance I_0 of the incident beam is proportional to A_0^2 and the irradiance I of the transmitted beam is proportional to A^2 (= $(A_0\cos\theta)^2$). From Equation 5.1

$$\begin{aligned} A &= A_0 \cos\theta \\ \therefore A^2 &= (A_0 \cos\theta)^2 \\ \therefore A^2 &= A_0^2 \cos^2\theta \\ \therefore I &= I_0 \cos^2\theta \end{aligned} \tag{5.2}$$

Equation 5.2 is known as **Malus' law**, and gives the irradiance of the beam transmitted through the analyser.

Example

A sheet of Polaroid is being used to reduce the irradiance of a beam of polarised light. What angle should the transmission axis of the Polaroid make with the plane of polarisation of the beam in order to reduce the irradiance of the beam by 50%?

We will use Malus' law to solve this problem, with I_0 as the irradiance of the incident beam and $I_0/2$ as the irradiance of the transmitted beam. Equation 5.2 then becomes

$$\begin{aligned} \frac{I_0}{2} &= I_0 \cos^2\theta \\ \therefore \cos^2\theta &= \frac{1}{2} \\ \therefore \cos\theta &= \sqrt{\frac{1}{2}} \\ \therefore \theta &= 45^\circ \end{aligned}$$

Quiz 1 Polarisation and Malus' law

20 min

First try the questions. If you get a question wrong or do not understand a question, there are Hints. The hints are in the same order as the questions. If you read the hint and still do not understand then ask your tutor. All references in the hints are to online materials.

Q1: Light waves can be polarised. This provides evidence that light waves are

a) coherent.
b) stationary waves.
c) monochromatic.
d) longitudinal waves.
e) transverse waves.

....

Q2: Unpolarised light passes through a sheet of Polaroid whose transmission axis is parallel to the *y*-axis. It then passes through a second Polaroid whose transmission axis is at 20° to the *y*-axis. At what angle is the plane of polarisation of the emergent beam?

a) Parallel to the *y*-axis.
b) At 10° to the *y*-axis.
c) At 20° to the *y*-axis.
d) At 70° to the *y*-axis.
e) Perpendicular to the *y*-axis.

....

Q3: Can sound waves be polarised?

a) Yes, any wave can be polarised.
b) No, because the oscillations are parallel to the direction of travel.
c) Yes, because the oscillations are perpendicular to the direction of travel.
d) No, because sound waves are not coherent.
e) Yes, because they are periodic waves.

....

Q4: A polarised beam of light is incident on a sheet of Polaroid. The angle between the plane of polarisation and the transmission axis is 30°. If the irradiance of the incident beam is 8.0×10^{-4} W m^{-2}, what is the irradiance of the transmitted beam?

a) 2.0×10^{-4} W m^{-2}
b) 4.0×10^{-4} W m^{-2}
c) 6.0×10^{-4} W m^{-2}
d) 6.9×10^{-4} W m^{-2}
e) 8.0×10^{-4} W m^{-2}

....

Q5: A polarised beam of light of irradiance 3.00 mW m^{-2} is incident on a sheet of Polaroid. What is the angle of the transmission axis relative to the incident beam's plane of polarisation if the transmitted beam has irradiance 1.00 mW m^{-2}?

a) 8.40°
b) 30.0°
c) 35.3°
d) 54.7°
e) 70.5°

....

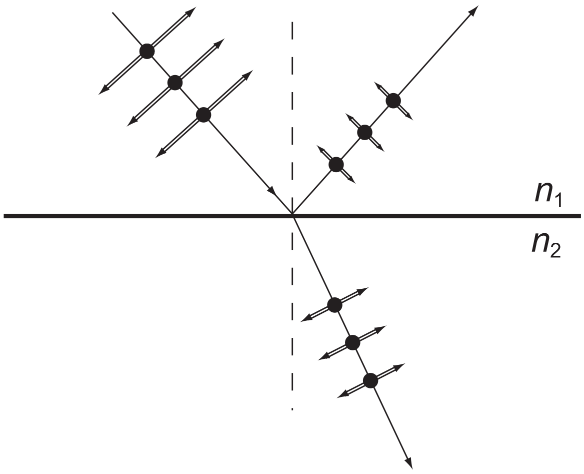
5.4 Polarisation by reflection

Learning Objective

To describe polarisation by reflection, and apply Brewster's law

Light reflected by the surface of an electrical insulator is partially, and sometimes fully, polarised. The degree of polarisation is determined by the angle of incidence of the beam and the refractive index of the reflecting material.

Figure 5.8: Polarisation by reflection



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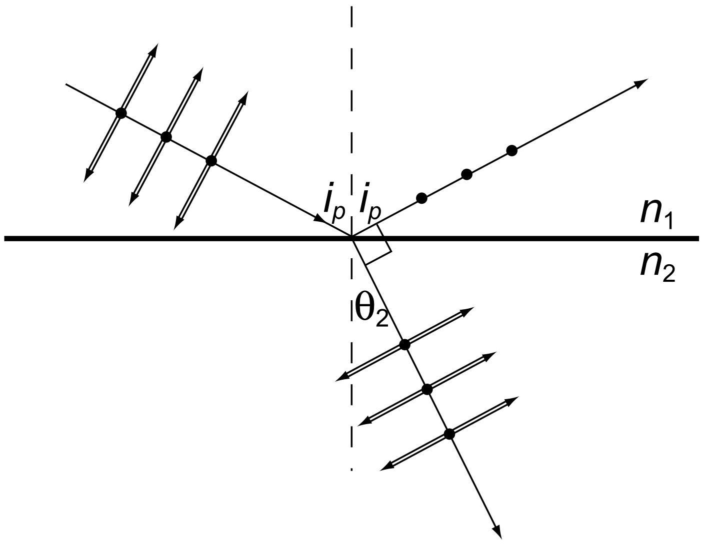
In Figure 5.8 the solid circles represent the components of the incident beam that are polarised parallel to the surface of the reflecting material. The double-headed arrows represent the components at right angles to those shown by the circles. The refracted (transmitted) beam contains both of these components, although the component in the plane of incidence is reduced. The refracted beam is therefore partially polarised, but the reflected beam can be completely polarised parallel to the reflecting surface and perpendicular to the direction in which the beam is travelling.

Usually the reflected beam is not completely polarised, and contains some of the ‘arrows' components. We shall look now at the special case in which the reflected beam does become completely polarised.

5.4.1 Brewster's law

The Scottish physicist Sir David Brewster discovered that for a certain angle of incidence, monochromatic light was 100% polarised upon reflection. The refracted beam was partially polarised, but the reflected beam was completely polarised parallel to the reflecting surface. Furthermore, he noticed that at this angle of incidence, the reflected and refracted beams were perpendicular, as shown in Figure 5.9.

Figure 5.9: Brewster's law



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Light is travelling in a medium with refractive index n_1, and being partially reflected at the boundary with a medium of refractive index n_2. The angles of incidence and reflection are i_p, the polarising angle. The angle of refraction is θ_2. Snell's law for the incident and refracted beams is

$$n_1 \sin i_p = n_2 \sin \theta_2$$

According to Brewster

$$\begin{aligned} i_p + \theta_2 &= 90^\circ \\ \therefore \theta_2 &= 90 - i_p \end{aligned}$$

We can substitute for sin θ_2 in the Snell's law equation

$$\begin{aligned} n_1 \sin i_p &= n_2 \sin \theta_2 \\ \therefore n_1 \sin i_p &= n_2 \sin (90 - i_p) \\ \therefore n_1 \sin i_p &= n_2 \cos i_p \\ \therefore \frac{\sin i_p}{\cos i_p} &= \frac{n_2}{n_1} \\ \therefore \tan i_p &= \frac{n_2}{n_1} \end{aligned} \tag{5.3}$$

....

This equation is known as **Brewster's Law**. Usually the incident beam is travelling in air, so $n_1 \approx 1.00$, and the equation becomes $\tan i_p = n_2$. The polarising angle is sometimes referred to as the Brewster angle of the material.

Example

What is the polarising angle for a beam of light travelling in air when it is reflected by a pool of water (n = 1.33)?

Using Brewster's law

$$\tan i_p = n_2$$
$$\therefore \tan i_p = 1.33$$
$$\therefore i_p = 53.1°$$

. . . .

The refractive index of a material varies slightly with the wavelength of incident light. The polarising angle therefore also depends on wavelength, so a beam of white light does not have a unique polarising angle.

Brewster's law

20 min

At this stage there is an online activity.

This activity investigates polarisation by reflection and Brewster's law.

The degree of polarisation of a reflected beam depends upon the angle of incidence.

. . . .

5.5 Applications of polarisation

Learning Objective

To describe some applications of polarised light

In addition to the applications, this section includes other methods of producing polarised light.

5.5.1 Other methods of producing polarised light

Two methods for polarising a beam of light have been discussed. You should be aware that other techniques can be used. **Birefringent** materials such as calcite (calcium carbonate) have different refractive indices for perpendicular polarisation components. An unpolarised beam incident on a calcite crystal will be split into two beams polarised at right angles to each other. **Dichroic** crystals act in the same way as Polaroid. Their crystal structure allows only light with electric field components parallel to the crystal axis to be transmitted.

5.5.2 Applications of polarisation

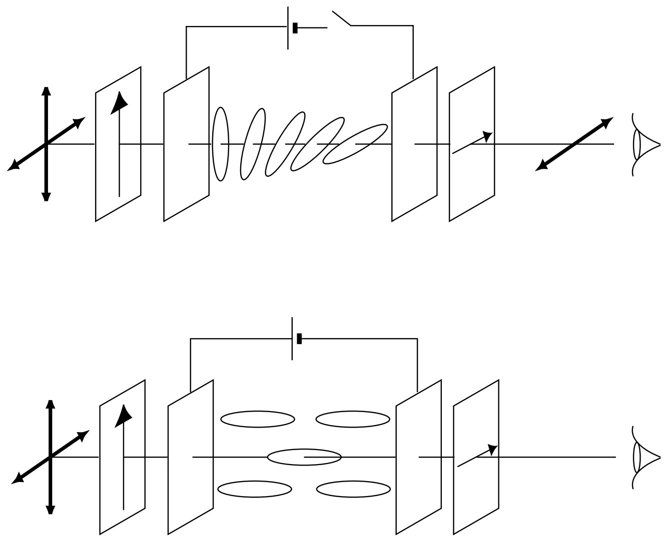
Polarised light can be used to measure strain in **photoelastic** materials, such as glass and celluloid. These are materials that become birefringent when placed under mechanical stress. One application of this effect is in stress analysis. A celluloid model of a machine part, for example, is placed between a crossed polariser and analyser. The model is then placed under stress to simulate working conditions. Bright and dark fringes appear, with the fringe concentration highest where the stress is greatest. This sort of analysis gives important information in the design of mechanical parts and structures.

Polaroid sunglasses and camera lens filters are often used to reduce glare. In the ideal case, light reflected from a horizontal surface will be polarised in a horizontal plane as described earlier, so a Polaroid with a vertical transmission axis should prevent transmission completely. In practice, reflected light is only partially polarised, and is not always being reflected from a horizontal surface, so glare is only partially reduced by Polaroid sunglasses and filters.

Optically active materials can change the plane of polarisation of a beam of light. This process comes about because of the molecular structure of these materials, and has been observed in crystalline materials such as quartz and organic (liquid) compounds such as sugar solutions. The degree of optical activity can be used to help determine the molecular structure of these compounds. In the technique known as **saccharimetry**, the angle of rotation of the plane of polarisation is used as a measure of the concentration of a sugar solution. Polarised light is passed through an empty tube, and an analyser on the other side of the tube is adjusted until no light is transmitted through it. The tube is then filled with the solution, and the analyser is adjusted until the transmission through it is again zero. The adjustment needed to return to zero transmission is the angle of rotation.

Perhaps the most common everyday use of optical activity is in **liquid crystal displays** (LCDs). A typical LCD on a digital watch or electronic calculator consists of a small cell of aligned crystals sandwiched between two transparent plates between a crossed polariser and analyser. This arrangement is shown schematically in Figure 5.10.

Figure 5.10: LCD display with no field applied (top) and with a field applied (bottom)



When no electric field is applied across the cell, the liquid crystal molecules are arranged in a helical twist. The polarised light entering the cell has its polarisation angle changed as it travels through the cell, and emerges polarised parallel to the transmission axis of the analyser. The cell appears light in colour, and is thus indistinguishable from the background. When a field is applied, the liquid crystal molecules align in the same direction (the direction of the electric field), and do not change the polarisation of the light. The emerging light remains polarised perpendicular to the analyser transmission axis. This light is not transmitted by the analyser, and so the cell looks dark. In this case, a black segment is seen against a lighter background.

Quiz 2 Brewster's law and applications of polarisation

20 min

First try the questions. If you get a question wrong or do not understand a question, there are Hints. The hints are in the same order as the questions. If you read the hint and still do not understand then ask your tutor. All references in the hints are to online materials.

Q6: What is the polarisation angle for monochromatic light travelling in air, incident on a sheet of glass of refractive index 1.52?

a) 0.026°
b) 33.3°
c) 41.1°
d) 48.8°
e) 56.7°

. . . .

Q7: Light reflected at the polarising angle is

a) polarised parallel to the reflecting surface.
b) polarised perpendicular to the reflecting surface.
c) polarised parallel and perpendicular to the reflecting surface.
d) polarised in the direction in which the wave is travelling.
e) completely unpolarised.

. . . .

Q8: It is found that a beam of light is 100% polarised when reflected from a smooth plastic table-top. If the angle of incidence is 61.0°, what is the refractive index of the plastic?

a) 1.00
b) 1.14
c) 1.31
d) 1.80
e) 2.06

. . . .

Q9: Which of the following sentences best describe a material that exhibits 'optical activity'?

a) The material changes its polarisation when under mechanical stress.
b) The material can rotate the plane of polarisation of a beam of light.
c) The material does not transmit polarised light.
d) The material reflects polarised light.
e) The material has different refractive indices for perpendicular polarisation components.

. . . .

Q10: Polaroid sunglasses are most effective at reducing glare when the transmission axis of the Polaroid is

a) vertical.
b) horizontal.
c) at the polarising angle.
d) at 45° to the horizontal.
e) at 30° to the horizontal.

. . . .

5.6 Summary

Transverse waves can be polarised, so that all the transverse oscillations of the wave take place in the same plane. In the case of light waves, this plane is taken to be the plane of the oscillating electric field. The orientation of the plane of polarisation can be determined by passing the beam through a Polaroid analyser. When the plane of polarisation is not parallel to the transmission axis of the analyser, Malus' law can be used to calculate the irradiance of the transmitted beam.

Light can be polarised by a number of different methods. The two methods that have been studied in detail in this topic are polarisation using a Polaroid sheet and polarisation by reflection. In the latter case, the incident beam must be reflected at the polarising angle (the Brewster angle) to be 100 % polarised.

Polarised light has many applications, chief among them being liquid crystal displays. Saccharimetry and photoelasticity are other applications in which polarised light is used to provide information about test materials.

By the end of this topic you should be able to:

- explain the difference between polarised and unpolarised waves, and state why only transverse waves can be polarised;
- state that light can be polarised by transmission through a Polaroid filter;
- apply Malus' law to calculate the transmission through a polariser - analyser pair, and state the conditions under which a polariser and analyser are non-transmitting and 100% transmitting;
- state that light can be polarised by reflection from any electrical insulator;
- explain what is meant by the polarising angle (also known as Brewster's angle);
- derive and use Brewster's law to find the polarising angle for any material;
- describe some of the applications of polarised light.

5.7 End of topic test

End of topic test

At this stage there is an end of topic test available online. If however you do not have access to the internet you may try the questions which follow.

30 min

The following data should be used when required:

speed of light in a vacuum *c*	3.00×10^8 m s1
speed of sound	340 m s^{-1}
acceleration due to gravity *g*	9.8 m s^{-2}

Q11: An unpolarised beam of light is incident on a Polaroid filter. The emerging beam then travels through a second Polaroid filter. The first filter has its transmission axis at 20°to the vertical, the second is at 58° to the vertical.

State the angle of polarisation of the beam emerging from the second Polaroid, giving your answer in degrees (°) to the vertical.

....

Q12: A beam of polarised light of irradiance 4.1 W m^{-2} is incident on a sheet of Polaroid.

If the transmission axis of the Polaroid makes an angle 28° with the plane of polarisation of the incident beam, calculate the irradiance in W m^{-2} of the transmitted beam

....

Q13: Three Polaroid filters are lined up along the x -axis. The first has its transmission axis aligned in the y -direction, the second has its axis at 45° in the y - z plane, and the third has its axis in the z -direction. A beam of light travelling in the x -direction has initial irradiance 9.6 W m^{-2} , and is polarised in the y -direction. The beam passes through each filter as it travels along the x -axis.

1. Calculate the irradiance in W m^{-2} of the transmitted beam emerging from the third filter.
2. In which direction is the emerging beam polarised?
    a) In the z -direction.
    b) In the y -direction.
    c) In the x -direction.
    d) At 45° in the y - z plane.

....

Q14: Calculate the angle (in degrees °) required between the incident plane of polarisation and the transmission axis of a sheet of Polaroid to reduce the irradiance of a beam of polarised light from 7.7 W m^{-2} to 1.9 W m^{-2} .

....

Q15: The polarising angle for a particular glass is 58.2°.

Calculate the refractive index of the glass.

....

Q16: Calculate the polarising angle (in degrees °) for a beam of light travelling in water (n_w = 1.33) incident on a block of leaded glass (n_g = 1.54).

....

Q17: Light travelling in air is partially reflected from a glass block. The reflected light is found to be 100% polarised when the angle of incidence is 57.7°.

1. State the magnitude of the angle between the reflected and refracted beams, in degrees.
2. Calculate the refractive index of the block.

....

Q18: A beam of light is 100% polarised by reflection from the surface of a swimming pool. The reflected beam is then passed through a Polaroid. The transmission axis of the Polaroid makes an angle of 32.5° with the normal to the plane of polarisation of the reflected beam.

If the reflected beam has irradiance 6.25 W m^{-2}, calculate the irradiance (in W m^{-2}) of the beam transmitted by the Polaroid.

. . . .

Topic 6

Waves end-of-unit assessment

Contents

End-of-unit assessment

30 min

At this stage there is an end of unit test available online. If however you do not have access to the internet you may try the questions which follow.

The following data should be used when required:

speed of light in a vacuum *c*	3.00×10^8 m s1
speed of sound	340 m s^{-1}
acceleration due to gravity *g*	9.8 m s^{-2}

Q1: An unpolarised beam of light is incident on a Polaroid filter. The emerging beam then travels through a second Polaroid filter. The first filter has its transmission axis at 22° to the vertical, the second is at 48° to the vertical.

State the angle of polarisation of the beam emerging from the second Polaroid, giving your answer in degrees (°) to the vertical.

....

Q2: A beam of polarised light of irradiance 4.8 W m^{-2} is incident on a sheet of Polaroid.

If the transmission axis of the Polaroid makes an angle 40° with the plane of polarisation of the incident beam, calculate the irradiance in W m^{-2} of the transmitted beam

....

Q3: Three Polaroid filters are lined up along the x -axis. The first has its transmission axis aligned in the y -direction, the second has its axis at 45° in the y - z plane, and the third has its axis in the z -direction. A beam of light travelling in the x -direction has initial irradiance 6 W m^{-2} , and is polarised in the y -direction. The beam passes through each filter as it travels along the x -axis.

1. Calculate the irradiance in W m^{-2} of the transmitted beam emerging from the third filter.
2. In which direction is the emerging beam polarised?
   a) In the y -direction.
   b) In the z -direction.
   c) At 45° in the y - z plane.
   d) In the x -direction.

....

Q4: Calculate the angle (in degrees °) required between the incident plane of polarisation and the transmission axis of a sheet of Polaroid to reduce the irradiance of a beam of polarised light from 9.7 W m^{-2} to 3.9 W m^{-2} .

....

Q5: The polarising angle for a particular glass is 57.3° .

Calculate the refractive index of the glass.

....

Q6: Calculate the polarising angle (in degrees °) for a beam of light travelling in water (n_w = 1.33) incident on a block of leaded glass (n_g = 1.54).

....

Q7: Light travelling in air is partially reflected from a glass block. The reflected light is found to be 100% polarised when the angle of incidence is 57.2°.

1. State the magnitude of the angle between the reflected and refracted beams, in degrees.
2. Calculate the refractive index of the block.

. . . .

Q8: A beam of light is 100% polarised by reflection from the surface of a swimming pool. The reflected beam is then passed through a Polaroid. The transmission axis of the Polaroid makes an angle of 46.5° with the normal to the plane of polarisation of the reflected beam.

If the reflected beam has irradiance 6.25 W m^{-2} , calculate the irradiance (in W m^{-2}) of the beam transmitted by the Polaroid.

. . . .

Glossary

Amplitude

The maximum displacement of the medium from its mean position, measured in metres.

Birefringence

The property of some materials to split an incident beam into two beams polarised at right angles to each other.

Bloomed lens

A lens that has been given a thin coating to make it anti-reflecting at certain wavelengths.

Blue-shift

Doppler-shifting of a light wave towards the blue end of the spectrum (apparent frequency higher than emitted frequency) owing to relative motion of the source towards the observer.

Brewster's Law

A beam of light travelling in a medium of refractive index n_1 will be 100% polarised by reflection from a medium of refractive index n_2 if the angle of incidence i_p obeys the relationship $\tan i_p = {}^{n_2}/_{n_1}$

Coherent waves

Two or more waves that have the same frequency and wavelength, and similar amplitudes, and that have a constant phase relationship.

Dichroism

The property of some materials to absorb light waves oscillating in one plane, but transmit light waves oscillating in the perpendicular plane.

Doppler effect

The apparent change in frequency of a wave caused by relative motion between the source and observer.

Frequency

The number of complete cycles of the wave performed in a given time, usually per second. Frequency is measured in hertz (Hz).

Interference by division of amplitude

A wave can be split into two or more individual waves, for example by partial reflection at the surface of a glass slide, producing two coherent waves. Interference by division of amplitude takes place when the two waves are recombined.

Interference by division of wavefront

All points along a wavefront are coherent. Interference by division of amplitude takes place when waves from two such points are superposed.

Irradiance

The rate at which energy is being transmitted per unit area, measured in W m^{-2} or J s^{-1} m^{-2}.

Liquid crystal displays

A display unit in which electronically-controlled liquid crystals are enclosed between crossed polarisers. If no signal is supplied to the unit, it transmits light. When a signal is applied, the liquid crystal molecules take up a different alignment and the cell does not transmit light, hence appearing black.

Malus' law

If a polarised beam of light, irradiance I_0, is incident on a sheet of Polaroid, with an angle θ between the plane of polarisation and the transmission axis, the transmitted beam has irradiance $I_0\cos^2\theta$.

Optical activity

The effect of some materials of rotating the plane of polarisation of a beam of light.

Optical path difference

The optical path between two points is equal to the distance between the points multiplied by the refractive index. An optical path difference will exist between two rays travelling between two points along different paths if they travel through different distances or through media with different refractive indices.

Periodic time

The time taken to complete one cycle of the wave, measured in seconds.

Phase

A way of describing how far through a cycle a wave is.

Phase difference

If two waves that are overlapping at a point in space have their maximum and minimum values occurring at the same times, then they are in phase. If the maximum of one does not occur at the same time as the maximum of the other, there is a phase difference between them.

Photoelasticity

The effect of a material becoming birefringent when placed under a mechanical stress.

Polarisation

The alignment of all the oscillations of a transverse wave in one direction.

Polarised light

Light in which all the electric field oscillations are in the same direction.

Principle of superposition

This principle states that the total disturbance at a point due to the presence of two or more waves is equal to the algebraic sum of the disturbances that each of the individual waves would have produced.

Red-shift

Doppler-shifting of a light wave towards the red end of the spectrum (apparent frequency lower than emitted frequency) owing to relative motion of the source away from the observer.

Saccharimetry

A technique that uses the optical activity of a sugar solution to measure its concentration.

Speed

The speed of a wave is the distance travelled by a wave per unit time, measured in m s^{-1}.

Stationary wave

A wave in which the points of zero and maximum displacement do not move through the medium (also called a standing wave).

Transmission axis

The transmission axis of a sheet of Polaroid is the direction in which transmitted light is polarised.

Travelling wave

A periodic disturbance in which energy is transferred from one place to another by the vibrations.

Unpolarised light

Light in which the electric field oscillations occur in random directions.

Wavelength

The distance between successive points of equal phase in a wave, measured in metres.

Hints for activities

Topic 1: Introduction to Waves

Quiz 1 Properties of waves

Hint 1: See the section titled Definitions.

Hint 2: Use $v = f\lambda$ twice.

Hint 3: See the figure The electromagnetic spectrum in the section titled Definitions.

Hint 4: Use v = f λ .

Hint 5: See the section titled Definitions.

Quiz 2 Travelling waves

Hint 1: Substitute the values in the equation.

Hint 2: Compare the options with the general expression for a travelling wave

$$y = A \sin 2\pi(ft - \frac{x}{\lambda})$$

Hint 3: Substitute the values in the equation.

Hint 4: Compare the equation with the general expression for a travelling wave

$$y = A \sin 2\pi(ft - \frac{x}{\lambda})$$

Hint 5: First work out the values of *f* and λ by comparing the equation with the general expression for a travelling wave

$$y = A \sin 2\pi(ft - \frac{x}{\lambda})$$

Quiz 3 Stationary waves

Hint 1: How does the frequency of the third harmonic compare with the fundamental frequency?

Hint 2: See the section titled Stationary waves.

Hint 3: How many wavelengths are there on the string when the fundamental note is played?

Hint 4: See the section titled Stationary waves.

Hint 5: To find out how to calculate the wavelength see the activity titled Longitudinal stationary waves. Then use $v = f\lambda$.

Topic 2: The principle of superposition and the Doppler effect

Quiz 1 Superposition

Hint 1: The amplitude of the disturbance at this point is equal to the difference between amplitudes of the two waves.

Hint 2: Consider the phase of the waves arriving at the listener from the two loudspeakers.

Hint 3: Divide the wavelength into the distance.

Hint 4: See the section titled Fourier Series.

Hint 5: See the section titled superposition of two waves.

Quiz 2 Doppler effect

Hint 1: See the section titled The Doppler effect with a moving source.

Hint 2: See the section titled The Doppler effect with a moving source for the correct relationship.

Hint 3: See the section titled The Doppler effect with a moving observer.

Hint 4: The apparent frequency is higher if the source and observer are getting closer together.

Hint 5: See the section titled The Doppler effect with a moving source for the correct relationship.

Topic 3: Interference by division of amplitude

Quiz 1 Coherence and optical paths

Hint 1: See the section titled Coherence.

Hint 2: Divide the wavelength into the distance.

Hint 3: The optical path length in the glass is equal to *(actual path length in the glass x refractive index*).

Hint 4: Be careful to work out the optical path **difference**.

Hint 5: The optical path **difference** is equal to 4λ; remember that the ray in air travelled the thickness of the plastic.

Quiz 2 Thin film interference

Hint 1: See the section titled Optical path difference.

Hint 2: See the section titled Thin film interference.

Hint 3: Find the values of λ that give constructive interference.

Hint 4: See the end of the section titled Thin film interference.

Hint 5: See the end of the section titled Thin film interference for the correct relationship.

Wedge fringes

Hint 1: Make sure you are measuring all the distances on the same scale - use m throughout.

Quiz 3 Wedge fringes

Hint 1: See the relationship derived in the section titled Wedge fringes.

Hint 2: Consider the expression for constructive interference derived in the section titled Wedge fringes.

Hint 3: See the relationship derived at the end of the section titled Wedge fringes.

Hint 4: See the relationship derived in the section titled Wedge fringes. Find the thickness of the air wedge for $m = 1$.

Hint 5: Consider the effect on the optical path and the condition for constructive interference.

Topic 4: Interference by division of wavefront

Quiz 1 Young's slits

Hint 1: See the title of this topic.

Hint 2: Apply the relationship derived in the section titled Young's slit experiment.

Hint 3: Consider the relationship for Δx derived in the section titled Young's slit experiment.

Hint 4: This is a straight application of the relationship derived in the section titled Young's slit experiment.

Hint 5: Work out 5 times Δx.

Topic 5: Polarisation

Quiz 1 Polarisation and Malus' law

Hint 1: See the introduction to this topic.

Hint 2: The plane of polarisation of the emergent beam is the same as the transmission axis of the last polarising filter through which it travelled.

Hint 3: Only transverse waves can be polarised.

Hint 4: This is a straight application of Malus' Law.

Hint 5: This is a straight application of Malus' Law.

Quiz 2 Brewster's law and applications of polarisation

Hint 1: This is a straight application of Brewster's Law.

Hint 2: See the section titled Brewster's Law

Hint 3: This is a straight application of Brewster's Law.

Hint 4: See the section titled Applications of polarisation.

Hint 5: See the section titled Applications of polarisation.

Answers to questions and activities

1 Introduction to Waves

Quiz 1 Properties of waves (page 5)

Q1: e) amplitude

Q2: b) 4.3 $\times$ 10^{14} - 7.5 $\times$ 10^{14} Hz

Q3: a) X-rays, infrared, microwaves.

Q4: c) 4.74$\times$$10^5$ GHz

Q5: d) 180 W m^{-2}

Quiz 2 Travelling waves (page 9)

Q6: c) 0.25 m

Q7: b) $y = 2 \sin 2\pi \left(20t - x\right)$

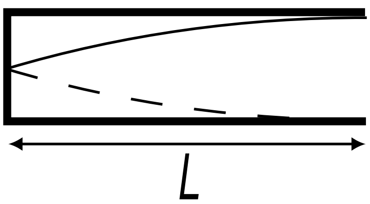
Q8: e) 4 m

Q9: e) 12 Hz

Q10: b) 1.25 m s^{-1}

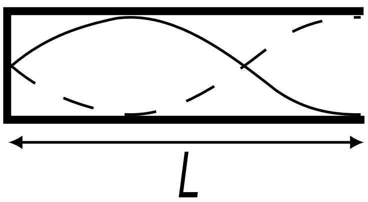
Longitudinal stationary waves (page 14)

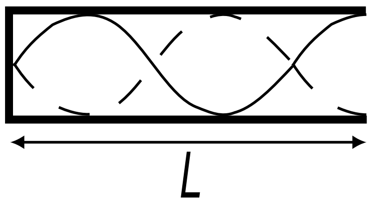
a) (i)



The fundamental has wavelength $4L$. Remember that λ is twice the distance between adjacent nodes, and so four times the distance between adjacent node and antinode.

a) (ii)

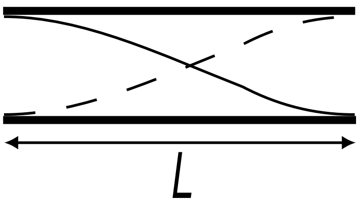




The wavelengths of the harmonics of a tube or pipe with one end open are $4L$/3, $4L$/5...

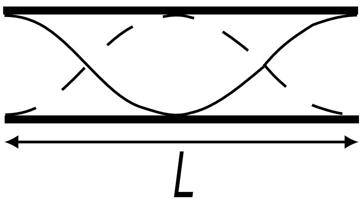
Note that only odd harmonics are possible for a tube with one end open.

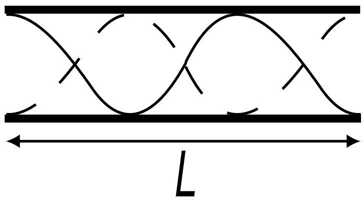
b) (i)



The fundamental has wavelength $2L$. Remember that λ is twice the distance between adjacent nodes or antinodes.

b) (ii)





The wavelengths of the harmonics of a tube or pipe with both ends open are L, $2L$/3...

Quiz 3 Stationary waves (page 14)

Q11: c) 143 Hz

Q12: b) Every point between adjacent nodes of a stationary wave oscillates in phase.

Q13: e) 3.00 m

Q14: c) 4

Q15: d) 408 Hz

End of topic test (page 16)

Q16: 5.39 $\times$ 10^{14}

Q17: 9.72 $\times$ 10^{-6}

Q18: 434

Q19: 4.2

Q20: 0.54

Q21: 0.34

Q22: 0.583

Q23: 14

2 The principle of superposition and the Doppler effect

Quiz 1 Superposition (page 24)

Q1: d) 3.0 cm

Q2: a) a loud signal, owing to constructive interference?

Q3: c) 22.5 wavelengths

Q4: b) any periodic wave is a superposition of harmonic sine and cosine waves.

Q5: e) a multiple of λ.

Quiz 2 Doppler effect (page 30)

Q6: b) frequency.

Q7: c) 495 Hz

Q8: d) 530 Hz

Q9: a) higher when the man is walking towards her.

Q10: b) 10.6 m s^{-1}

Doppler effect in light (page 32)

Since we have an observer moving towards a stationary source, we will need to use Equation 2.6

$$f' = f\frac{(v + v_o)}{v}$$

Substituting for $f = {}^{v}/_{\lambda}$

$$\frac{v}{\lambda'} = \frac{v}{\lambda} \times \frac{(v + v_o)}{v}$$

The speed of the waves is c, the speed of light

$$\begin{aligned}
\frac{c}{\lambda'} &= \frac{c}{\lambda} \times \frac{(c + v_o)}{c} \\
\therefore \frac{c}{\lambda'} &= \frac{1}{\lambda} \times (c + v_o) \\
\therefore \frac{c\lambda}{\lambda'} &= (c + v_o) \\
\therefore v_o &= \frac{c\lambda}{\lambda'} - c \\
\therefore v_o &= c\left(\frac{\lambda}{\lambda'} - 1\right)
\end{aligned}$$

Now, put in the values of c = 3.0 $\times$ 10^{8} m s^{-1}, λ = 6.5 $\times$ 10^{-7} m and λ' = 5.4 $\times$ 10^{-7} m

$$v_o = 3.0 \times 10^8 \times \left(\frac{6.5 \times 10^{-7}}{5.4 \times 10^{-7}} - 1\right)$$
$$\therefore v_o = 3.0 \times 10^8 \times 0.204$$
$$\therefore v_o = 6.1 \times 10^7 \text{ m s}^{-1}$$

The car must have been travelling at around 6.1 $\times$ 10^{7} m s^{-1}!

End of topic test (page 34)

Q11: 3.6 cm

Q12: 0.41 m

Q13: 94 Hz

Q14: 513 Hz

Q15: 11.1 m s^{-1}

Q16: 14.8 m s^{-1}

Q17:

1. 14.6 m s^{-1}
2. 499 Hz

3 Interference by division of amplitude

Quiz 1 Coherence and optical paths (page 41)

Q1: d) their phase difference is constant.

Q2: e) 9 wavelengths

Q3: c) 0.180 m

Q4: a) 8.75×10^{-3} m

Q5: b) 0.0682 m

Quiz 2 Thin film interference (page 49)

Q6: c) their optical path difference is an integer number of wavelengths.

Q7: b) 2.09×10^{-7} m

Q8: c) 1120 nm and 373 nm

Q9: d) The coating is only anti-reflecting for the green part of the visible spectrum.

Q10: a) 9.06×10^{-8} m

Wedge fringes (page 53)

Use Equation 3.8, and make sure you have converted all the lengths into metres.

1. In the first case, the wavelength is 6.33×10^{-7} m.

$$\begin{aligned}\Delta x &= \frac{\lambda l}{2y}\\ \therefore \Delta x &= \frac{6.33\times10^{-7}\times0.08}{2\times2\times10^{-5}}\\ \therefore \Delta x &= \frac{5.064\times10^{-8}}{4\times10^{-5}}\\ \therefore \Delta x &= 1.3\times10^{-3}\text{m} = 1.3\,\text{mm}\end{aligned}$$

2. Now, the argon laser has wavelength 5.12×10^{-7} m.

$$\begin{aligned}\Delta x &= \frac{\lambda l}{2y}\\ \therefore \Delta x &= \frac{5.12\times10^{-7}\times0.08}{2\times2\times10^{-5}}\\ \therefore \Delta x &= \frac{4.096\times10^{-8}}{4\times10^{-5}}\\ \therefore \Delta x &= 1.0\times10^{-3}\text{m} = 1.0\,\text{mm}\end{aligned}$$

Quiz 3 Wedge fringes (page 53)

Q11: d) 480 nm

Q12: a) Blue

Q13: d) 3.15×10^{-3} m

Q14: b) 162 nm

Q15: b) The fringes would move closer together.

End of topic test (page 56)

Q16: 35.7

Q17: 16.1

Q18: 1.2×10^{-7}

Q19: 732 and 366

Q20: 547

Q21: 2.19×10^{-6}

Q22: 3.15×10^{-3}

Q23: 555

4 Interference by division of wavefront

Quiz 1 Young's slits (page 65)

Q1: b) interference by division of wavefront.

Q2: d) 333 nm

Q3: b) The fringes move closer together.

Q4: a) 2.4 mm

Q5: e) 0.016 m

End of topic test (page 67)

Q6: 1.19×10^{-3}

Q7: 521

Q8: 557

Q9: 3.2

Q10: 515

5 Polarisation

Quiz 1 Polarisation and Malus' law (page 75)

Q1: e) transverse waves.

Q2: c) At 20° to the *y*-axis.

Q3: b) No, because the oscillations are parallel to the direction of travel.

Q4: c) 6.0×10^{-4} W m^{-2}

Q5: d) 54.7°

Quiz 2 Brewster's law and applications of polarisation (page 81)

Q6: e) 56.7°

Q7: a) polarised parallel to the reflecting surface.

Q8: d) 1.80

Q9: b) The material can rotate the plane of polarisation of a beam of light.

Q10: a) vertical.

End of topic test (page 83)

Q11: 58

Q12: 3.2

Q13:

1. 2.4
2. a

Q14: 60

Q15: 1.61

Q16: 49.2

Q17:

1. 90
2. 1.58

Q18: 1.8

6 Waves end-of-unit assessment

End-of-unit assessment (page 88)

Q1: 48

Q2: 2.8

Q3:

1. 1.5
2. b

Q4: 51

Q5: 1.56

Q6: 49.2

Q7:

1. 90
2. 1.55

Q8: 3.29