The Scattering of α and β Particles by Matter and the Structure of the Atom

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§ 1

It is well known that the α and β particles suffer deflexions from their rectilinear paths by encounters with atoms of matter. This scattering is far more marked for the β than for the α particle on account of the much smaller momentum and energy of the former particle. There seems to be no doubt that such swiftly moving particles pass through the atoms in their path, and that the deflexions observed are due to the strong electric field traversed within the atomic system. It has generally been supposed that the scattering of a pencil of α or β rays in passing through a thin plate of matter is the result of a multitude of small scattering by the atoms of matter traversed. The observations, however, of Geiger and Marsden[2] on the scattering of α rays indicate that some of the α particles must suffer a deflexions of more than a right angle at a single encounter. They found, for example, that a small fraction of the incident α particles, about 1 in 20,000 turned through an average angle of 90 degrees in passing through a layer of gold–foil about .00004 cm. thick, which was equivalent in stopping power of the α particle to 1.6 millimetres of air. Geiger[3] showed later that the most

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[2]Proc. Roy. Soc., LXXXII, p. 495 (1909)
[3]Proc. Roy. Soc. LXXXIII. p. 492. (1910).

probable angle of deflexions for a pencil of α particles traversing a gold–foil of this thickness was about $0.87°$. A simple calculation based on the theory of probability shows that the chance of an α particle being deflected through 90 degrees is vanishingly small. In addition, it will be seen later that the distribution of the α particles for various angles of large deflexion does not follow the probability law to be expected if such deflexions are made up of a large number of small deviations. It seems reasonable to suppose that the deflexion through a large angle is due to a single atomic encounter, for the chance of a second encounter of a kind to produce a large deflexion must in most cases be exceedingly small. A simple calculation shows that the atom must be a seat of an intense electric field in order to produce such a large deflexion at a single encounter.

Recently Sir J.J. Thomson[4] has put forward a theory to explain the scattering of electrified in passing through small thickness of matter. The atom is supposed to consist of a number N of negatively charged corpuscles, accompanied by an equal quantity of positive electricity uniformly distributed throughout a sphere. The deflexion of a negatively electrified particle in passing through the atom is ascribed to two causes – (1) the repulsion of the corpuscles distribution through the atom, and (2) the attraction of the positive electricity in the atom. The deflexion of the particle in passing through the atom is supposed to be small, while the average deflexion after a large number m of encounters was taken as $\sqrt{m \cdot \theta}$, where θ is the average deflexion due to a single atom. It was shown that the number N of the electrons within the atom could be deduced from observations of the scattering of electrified particles. The accuracy of this theory of compound scattering was examined experimentally by Crowther[5] in a later paper. His result apparently confirmed the main conclusions of the theory, and he deduced, on the assumption that the positive electricity was continuous, that the number of electrons in an atom was about three times its atomic weight.

The theory of Sir J.J. Thomson is based on the assumption that the scattering due to a single encounter is small, and the particular structure assumed for the atom does not admit of a very large deflexion of an α particle in traversing a single unless it be supposed that the diameter of the sphere of positive electricity is minute compared with the diameter of the sphere of influence of the atom.

Since the α and β particles traverse the atom, it should be possible

[4] Camb. Lit. & Phil. Soc. XV. pt. 5 (1910).
[5] Crowther, Proc. Roy. Soc. LXXXIV. p.226 (1910).

from a close study of the nature of the deflexion to form some idea of the constitution of the atom to produce the effects observed. In fact, the scattering of high–speed charged particles by the atoms of matter is one of the most promising methods of attack of this problem. The development of the scintillation method of counting single α particles affords unusual advantages of investigation, and the researches of H. Geiger by this method have already added much to our knowledge of the scattering of α rays by matter.

§ 2

We shall first examine theoretically the single encounters [6] with an atom of simple structure, which is able to produce large deflexions of an α particle, and then compare the deductions from the theory with the experimental data available.

Consider an atom which contains a charge $\pm Ne$ at its centre surrounded by a sphere of electrification containing a charge $\mp Ne$ supposed uniformly distributed throughout a sphere of radius is the fundamental unit of charge, which in this paper is taken as 4.65×10^{-10} E.S. unit. We shall suppose that for distances less than 10^{-12} cm. the central charge and also the charge on the α particle may be supposed to be concentrated at a point. It will be shown that the main deductions from the theory are independent of whether the central charge is supposed to be positive or negative. For convenience, the sign will be assumed to be positive. The question of the stability of the atom proposed need not be considered at this stage, for this will obviously depend upon the minute structure of the atom, and on the motion of the constituent charged parts.

In order to from some idea of the forces required to deflect an α particle through a large angle, consider an atom containing a positive charge Ne at its centre, and surrounded by a distribution of negative electricity Ne uniformly distributed within a sphere of radius R. The electric force X and the potential V at a distance r from the centre of an atom for a point inside the atom, are given by

$$X = Ne \cdot \left(\frac{1}{r^2} - \frac{r}{R^3} \right)$$

[6] The deviation of a particle throughout a considerable angle from an encounter with a single atom will in this paper be called “single” scattering. The deviation of a particle resulting from a multitude of small deviations will be termed “compound” scattering

$$V = Ne \cdot \left(\frac{1}{r} - \frac{3}{2R} + \frac{r^2}{2R^3} \right)$$

Suppose an α particle of mass m velocity u and charge E shot directly towards the centre of the atom. It will be brought to rest at a distance b from the centre given by

$$1/2mu^2 = NeE \cdot \left(\frac{1}{b} - \frac{3}{2R} + \frac{b^2}{2R^3} \right).$$

It will be seen that b is an important quantity in later calculations. Assuming that the central charge is $100e$. it can be calculated that the value of b for an α particle of velocity 2.09×10^9 cms. per second is about 3.4×10^{-12} cm. In this calculation b is supposed to be very small compared with R. Since R is supposed to be of the order of the radius of the atom, viz. 10^{-8} cm., it is obvious that the α particle before being turned back penetrates so close to the central charge, that the field due to the uniform distribution of negative electricity may be neglected. In general, a simple calculation shows that for all deflexions greater than a degree, we may without sensible error suppose the deflexion due to the field of the central charge alone. Possible single deviations due to the negative electricity, if distributed in the form of corpuscles, are not taken into account at this stage of the theory. It will be shown later that its effect is in general small compared with that due to the central field.

Consider the passage of a positive electrified particle close to the centre of an atom. Supposing that the velocity of the particle is not appreciably charged by its passage through the atom, the path of the particle under the influence of a repulsive force varying inversely as the square of the distance will be an hyperbola with the centre of the atom S as the external focus. Suppose the particle to enter the atom in the direction PO [Fig. 44–1], and that the direction on motion on escaping the atom is OP'. OP and OP' make equal angles with the line SA, where A is the apse of the hyperbola. $p = SN =$ perpendicular distance from centre on direction of initial motion of particle.

Let angle $POA = \theta$.

Let $V =$ velocity of particle on entering the atom, ν its velocity at A, then from consideration of angular momentum.

$$pV = SA \cdot v$$

From conservation of energy

$$1/2mV^2 = 1/2mv^2 + \frac{NeE}{SA},$$

$$v^2 = V^2(1 - \frac{b}{SA})$$

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Рис. 1:

Since the eccentricity is see θ

$$SA = SO + OA = p \;\; \mathrm{cosec}\theta(1 + \cos\theta) = p\cot\theta/2$$
$$p^2 = SA(SA - b) = p\cot\theta/2(p\cot\theta/2 - b),$$
$$b = 2p\cot\theta.$$

The angle of deviation ϕ of the particle is $\pi - 2\theta$ and

$$\cot\phi/2 = \frac{2p^*}{b} \tag{1}$$ [7]

This gives the angle of deviation of the particle in terms of b, and the perpendicular distance of the direction of projection from the centre of the atom.

For illustration, the angle of deviation ϕ for different values of p/b are shown in the following table:–

p/b	10	5	2	1	.5	.25	.125
ϕ	5°.7	11°.4	28°	53°	90°	127°	152°

§ 3 *Probability of Single Deflexion Through any Angle*

Suppose a pencil of electrified particles to fall normally on a thin screen of matter of thickness t. With the exception of the few particles which are scattered through a large angle, the particles are supposed to pass nearly normally through the plate with only a small change of velocity. Let $n =$ number of atom in unit volume of material. Then the number of collisions of the particle with the atom of radius R is $\pi R^2 nt$ in the thickness t.

The probability m of entering an atom within a distance p of its centre is given by

$$m = \pi p^2 nt.$$

Chance dm of striking within radii p and $p + dp$ is given by

$$dm = 2\pi p \cdot n \cdot t\ dp = \frac{\pi}{4} ntb^2 \cot\phi/\mathrm{cosec}^2\phi/2 d\phi \tag{2}$$

since

$$\cot\phi/2 = 2p/b.$$

The value of dm gives the *fraction* of the total number of particles which are deviated between the angles ϕ and $\phi + d\phi$.

The fraction ρ of the total number of particles which are deflected through an angle greater than ϕ is given by

$$\rho = \frac{\pi}{4} ntb^2 \cot^2\phi/2 \tag{3}$$

The fraction ρ which is deflected between the angles ϕ_1 and ϕ_2 is given by

$$\rho = \frac{\pi}{4} ntb^2 (\cot^2\frac{\phi_1}{2} - \cot^2\frac{\phi_2}{2}) \tag{4}$$

It is convenient to express the equation (2) in another from for comparison with experiment. In the case of the α rays, the number of scintillation appearing on a *constant* area of a zinc sulphide screen are counted for different angles with the direction of incidence of the particles. Let $r =$ distance from point of incidence of α rays on scattering material, then if Q be the total number of particles falling on the scattering material, the number y of α particles falling on until area which are deflected through an angle ϕ is given by

$$y = \frac{Qdm}{2\pi r^2 \sin\phi \cdot d\phi} = \frac{ntb^2 \cdot Q \cdot \operatorname{cosec}^4 \phi/2}{16r^2} \tag{5}$$

Since

$$b = \frac{2NeE}{mu^2},$$

we see from this equation that a number of α particles (scintillations) per unit area of zinc sulphide screen at a given distance r from the point of incidence of the rays is proportional to

(1) $\operatorname{cosec}^4 \phi/2$ or $1/\phi^4$ if ϕ be small;

(2) thickness of scattering material t provided this is small;

(3) magnitude of central charge Ne;

(4) and is inversely proportional to $(mu^2)^2$, or to the fourth power of velocity if m be constant.

In these calculations, it is assumed that the α particles scattered through a large angle suffer only large deflexion. For this to hold, it is essential that the thickness of the scattering material should be so small that the chance of a second encounter involving another large deflexion is very small. If, for example, the probability of a single deflexion ϕ in passing through a thickness t is 1/1000, the probability, of two successive deflexions each of value ϕ is $1/10^6$, and is negligibly small.

The angular distribution of the α particles scattered from a thin metal sheet affords one the simplest methods of testing the general correctness of this theory of single scattering. This has been done recently for α rays by Dr. Geiger,[8] who found that the distribution for particles deflected between 30 degrees and 150 degrees from a thin gold–foil was substantial agreement with the theory. A more detailed account of these and other experiments to test the validity of the theory will be published later.

[8] Manch. Lit. & Phil. Soc. 1910.

§ 4 *Alteration of Velocity in an Atomic Encounter*

It has so far been assumed that an α or β particles does not suffer an appreciable change of velocity as the result of a single atomic encounter resulting in large deflexion of the particle. The effect of such an encounter in altering the velocity of the particle can be calculated on certain assumptions. It is supposed that only two systems are involved, viz., the swiftly moving particle and the atom which it traverses supposed initially at rest. It is supposed that the principle of conservation of momentum and of energy applies and that there is no appreciable loss of energy or momentum by radiation.

Let m be mass of the particle

ν_1 = velocity of approach,

ν_2 = velocity of recession,

M = mass of atom,

V = velocity communicated to atom as result of encounter.

Let OA [Fig. 44–2] represent in magnitude and direction the momentum mv_1 of the entering particle, and OB the momentum of the receding particle which has been turned through an angle $AOB = \phi$. Then BA represents in magnitude and direction the momentum MV of the recoiling atom.

$$(MV)^2 = (mv_1)^2 + (mv_2)^2 - 2m2v_1v_2\cos\phi. \tag{1}$$

By the conservation of energy

$$MV^2 = mv_1^2 - mv_2^2 \tag{2}$$

Suppose $M/m = K$ and $v_2 = \rho v_1$, where ρ is < 1.
From (1) and (2),

$$(K+1)\rho^2 - 2\rho\cos\phi = K - 1,$$

or

$$\rho = \frac{\cos\phi}{K+1} + \frac{1}{K+1}\cdot\sqrt{K^2 - \sin^2\phi}.$$

Consider the case of an α particle of atomic weight 4, deflected through an angle of 90 degrees by an encounter with an atom of gold of atomic weight 197.

Рис. 2:

Since $K = 49$ nearly,

$$\rho = \sqrt{\frac{K-1}{K+1}} = .979,$$

or the velocity of the particle is reduced only about 2 per cent. by the encounter.

In the case of aluminium $K = 27/4$ and for $\phi = 90^\circ$, $\rho = .86$,

It is seen that the reduction of velocity of the α particle becomes marked on this theory for encounters with the lighter atoms. since the range of an α particle in air or other matter is approximately proportional to the cube of the velocity, it follows that an α particle of range 7 cms. has its range reduced to 4.5 cms. after incurring a single deviation of 90 degrees in traversing an aluminium atom. This is of magnitude to be easily detected experimentally. Since the value of k is very large for an encounter of a β particle with an atom, the reduction of velocity on this formula is very small.

Some very interesting cases of the theory arise in considering the changes of velocity and the distribution of scattered particles when the α particle encounters a light atom, for example a hydrogen or helium atom. A discussion of these and similar cases is reserved until the question has been examined experimentally.

§ 5 *Comparison of single and compound scattering*

Before comparing the results of theory with experiment, it is desirable to consider the relative importance of single and compound scattering in determining the distribution of the scattering particles. Since the atom is supposed to consist of a central charge surrounded by a uniform distribution of the opposite sign through a sphere of radius R, the chance of encounters with the atom involving small deflexions is very great compared with the chance of a single large deflexion.

This question of compound scattering has been examined by Sir J.J. Thomson in the paper previously discussed (§ 1). In the notation of this paper, the average deflexion ϕ_1 due to the field of the sphere of positive electricity of radius R and quantity Ne was found by him to be

$$\phi_1 = \frac{\pi}{4} \cdot \frac{NeE}{mu^2} \cdot \frac{1}{R}.$$

The average deflexion ϕ_2 due to the N negative corpuscles supposed distributed uniformly throughout the sphere was found to be

$$\phi_2 = \frac{16eE}{5mu^2} \cdot \frac{1}{R} \cdot \sqrt{\frac{3N}{2}}.$$

The mean deflexion due to both positive and negative electricity was taken as

$$(\phi_1^2 + \phi_2^2)^{1/2}.$$

In a similar way, it is not difficult to calculate the average deflexion due to the atom with a central charge discussed in this paper.

Since the radial electric field X at any distance r from the centre is given by

$$X = Ne \cdot \left(\frac{1}{r^2} - \frac{r}{R^3}\right),$$

it is not difficult to show that the deflexion (supposed small) of an electrified particle due to this field is given by

$$\theta = \frac{b}{p} \cdot \left(1 - \frac{p^2}{R^2}\right)^{3/2},$$

where p is the perpendicular from the centre on the path of the particle and b has the same value as before. It is seen that the value of θ increases with diminution of p and becomes great for small values of ϕ.

Since we have already seen that the deflexions become very large for a particle passing near the centre of the atom, it is obviously not correct to find the average values by assuming θ is small.

Taking R of the order 10^{-8} cm, the value of p for a large deflexion is for α and β particles of the order 10^{-11} cm. Since the chance of an encounter involving a large deflexion is small compared with the chance of small deflexions, a simple consideration shows that the average small deflexion is practically unaltered if the large deflexions are omitted. This is equivalent to integrating over that part of the cross section of the atom where the deflexions are small and neglecting the small central area. It can in this way be simply shown that the average small deflexion is given by

$$\phi_1 = \frac{3\pi}{8} \cdot \frac{b}{R}.$$

This value of ϕ_1 for the atom a concentrated central charge is three times the magnitude of the average deflexion for the same value of Ne in the type of atom examined by Sir J.J. Thomson. Combining the deflexions due to the electric field and to the corpuscles, the average deflexion is

$$(\phi_1^2 + \phi_2^2)^{1/2} \quad \text{or} \quad \frac{b}{2R} \cdot \left(5.54 + \frac{15.4}{N}\right)^{1/2}.$$

It will be seen later that the value of N is nearly proportional to the atomic weight, and is about 100 for gold. The effect due to scattering of the individual corpuscles expressed by the second term of the equation is consequently small for heavy atoms compared with that due to the distributed electric field.

Neglecting the second term, the average deflexion per atom is $3\pi b/8R$. We are now in a position to consider the relative effects on the distribution of particles due to single and to compound scattering. Following J.J. Thomson's argument, the average deflecion θ_t after passing through a thickness t of matter is proportional to the square of the number of encounters and is given by

$$\theta_t = \frac{3\pi b}{8R} \cdot \sqrt{\pi R^2 nt} = \frac{3\pi b}{8} \cdot \sqrt{\pi nt},$$

where n as before is equal to the number of atoms per unit volume.

The probability p_1 for compound scattering that the deflexion of the particle is greater than ϕ is equal $e^{-\phi/\theta_t^2}$. Consequently

$$\phi^2 = -\frac{9\pi^3}{64} \cdot b^2 nt \log p_1.$$

Next suppose that single scattering alone is operative. We have seen (§ 3) that the probability p_2 of deflexion greater than ϕ is given by

$$p_2 = \frac{\pi}{4} \cdot b^2 \cdot n \cdot t\cot^2\phi/2.$$

By comparing these two equations

$$p_2 \log p_1 = -0.181\phi^2 \text{ctg}^2\phi/2,$$

ϕ is sufficiently small that

$$\tan \phi/2 = \phi/2,$$

$$p_2 \log p_1 = -0.72.$$

If we suppose $p = 0.5$, than $p_1 = 0.24$. If $p_2 = 0.1$, $p_1 = 0.0004$.

It is evident from this comparison, that the probability for any given deflexion is always greater for single than for compound scattering. The difference is especially marked when only a small fraction of the particles are scattered through any given angle. It follows from this result that the distribution of particles due to encounters with the atoms is for small thickness mainly governed by single scattering. No doubt compound scattering produces some effect in equalizing the distribution of the scattered particles; but its effect becomes relatively smaller, the smaller the fraction of the particles scattered through a given angle.

§ 6 *Comparison of Theory with Experiments*

On the present theory, the value of the central charge Ne is an important constant, and it is desirable to determine its value for different atoms. This can be most simply done by determining the small fraction of α or β particles of known velocity falling on a thin metal screen, which are scattered between ϕ and $\phi + d\phi$ where is the angle of deflexion. The influence of compound scattering should be small when this fraction is small.

Experiments in these directions are in progress, but it is desirable at this stage to discuss in the light of the present theory the data already published on scattering of α and β particles.

The following points will be discussed: –

(a) The "diffuse deflexion" of α particles, i.e. the scattering of α particles through large angles (Geiger and Marsden).

(b) The variation of diffuse reflexion with atomic weight of the radiator (Geiger and Marsden).

(c) The average scattering of a pencil of α rays transmitted through a thin metal plate (Geiger).

(d) The experiments of Crowther on the scattering of β rays different velocities by various metals.

(a)In the paper of Geiger and Marsden on the diffuse reflexion of α particles falling on various substances it was shown that about 1/8000 of the α particles from radium C falling on a thick plate of platinum are scattered back in the direction of the incidence. This fraction is deduced on the assumption that the α particles are uniformly scattered in all directions, the observations being made for a deflexion of about 90 degrees. The form of experiment is not very suited for accurate calculation, but from the data available it can be shown that the scattering observed is about that to be expected on the theory if the atom of platinum has a central charge of about $100e$.

(b) In their experiments on this subject, Geiger and Marsden gave the relative number of α particles diffusely reflected from thick layers of different metals, under similar conditions. The number obtained by them are given in [Table 1] below, where z represents the relative number of scattered particles, measured by the number of scintillations per minute on a zinc sulphide screen.

Table 1.

Metal.	Atomic weight, A.	z.	$z/A^{3/2}$
Lead	207	62	208
Gold	197	67	242
Platinum	195	63	232
Tin	119	34	226
Silver	108	27	241
Copper	64	14.5	225
Iron	56	10.2	250
Aluminium	27	3.4	243
		Average	233

On the theory of single scattering, the fraction of the total number of α particles scattered through any given angle in passing through a thickness t is proportional to $n.\ A^2\ t$, assuming that the central charge is proportional to the atomic weight A. In the present case, the thickness of matter from which the scattered α particles are able to emerge and affect the zinc sulphide screen depends on the metal. Since Bragg has shown that the stopping power of an atom for an α particle is proportional to the square root of its atomic weight, the value of nt for different elements is proportional to $1/\sqrt{A}$. In this case t represents the greatest depth from which the scattered α particles emerge. The number z of α particles scattered back from a thick layer is consequently proportional to $A^{3/2}$ or $z/A^{3/2}$ should be a constant.

To compare this deduction with experiment, the relative values of the latter quotient are given in the last column. Considering the difficulty of the experiments, the agreement between theory and experiment is reasonably good. [9]

The single large scattering of α particles will obviously affect to some extent the shape of the Bragg ionization curve for a pencil of α rays. This effect of large scattering should be marked when the α rays have traversed screens of metals of high atomic weight, but should be small for atoms of light atomic weight.

(c) Geiger made a careful determination of the scattering of α particles passing through thin metal foils, by the scintillation method, and deduced

[9] The effect of charge of velocity in an atomic encounter is neglected in this calculation.

the most probable angle through which the α particles are deflected in passing through known thicknesses kind of matter.

A narrow pencil of homogeneous α rays was used as a source. After passing through the scattering foil, the total number of α particles deflected through different angles was directly measured. The angle for which the number of scattered particles was a maximum was taken as the most probable angle. The variation of the most probable angle with thickness of matter was determined, but calculation from these data is some what complicated by the variation of velocity of the α particles in their passage through the scattering material. A consideration of the curve of distribution of the α particles given in the paper . . . shows that an angle through which half the particles are scattered is about 20 per cent greater than the most probable angle.

We have already seen that compound scattering may become important when about half the particles are scattered through a given angle, and it is difficult to disentangle in such cases the relative effect due to the two kinds of scattering. An a approximate estimate can be made in the following way: From (§ 5) the relation between the probabilities p_1 and p_2 for compound and single scattering respectively is given by

$$p_2 \ln p_1 = -0.721.$$

The probability q of the combined effects may as a first approximation be taken as

$$q = (p_1^2 + p_2^2)^{1/2}.$$

If q = 0.5, it follows that

$$p_1 = 0.2 \quad \text{and} \quad p_2 = 0.46.$$

We have seen that the probability p_2 of a single deflexion greater than ϕ is given by

$$p_2 = \frac{\pi}{4} \cdot n \cdot t \cdot b^2 \ \cot^2 \ \phi/2.$$

Since in the experiments considered ϕ comparatively small

$$\frac{\phi\sqrt{p_2}}{\sqrt{\pi n t}} = b = \frac{2NeE}{mu^2}.$$

Geiger found that the most probable angle of scattering of the α rays in passing through a thickness of gold equivalent in stopping power to about .76 cm. of air was 1 degree 40′. The angle ϕ through which half the α particles are turned thus corresponds to 2 degrees nearly.

$$t = 0.00017cm.; \quad n = 6 \cdot \times 10^{22};$$

$$u \text{ (average value)} = 1.8 \times 10^9.$$

$$e/m = 1.5 \times 10^{14}. \text{ E.S. units}; \quad e = 4.65 \times 10^{-10}$$

Taking the probability of single scattering 0.46 and substituting the above values in the formula, the value of N for gold comes out to be 97.

For a thickness of gold equivalent in stopping power to 2.12 cms. of air, Geiger found the most probable angle to be $3°40'$. In this case $t = .00047, \phi = 4° \cdot 4$, and average $u = 1.7 \times 10^0$, and N comes out to be 114.

Geiger showed that the most probable angle of deflexion for an atom was nearly proportional to its atomic weigh. It consequently follows that the value of N for different atoms should be nearly proportional to their atomic weights, at any rate for atomic weight between gold and aluminium.

Since the atomic weight of platinum is nearly equal to that of gold, it follows from these considerations that the magnitude of the diffuse reflexion of α particles through more than 90 degrees from gold and the magnitude of the average small angle scattering of a pencil of rays in passing through gold–foil are both explained on the hypothesis of single scattering by supposing the atom of gold has a central charge of about $100e$.

(d) *Experiments of Crowther on scattering of β rays.* —

We shall now consider how far the experimental results of Crowther on scattering of β particles of different velocities by various materials can be explained on the general theory of single scattering. On this theory, the fraction of β particles p turned through an angle greater than ϕ is given by

$$p = \frac{\pi}{4} \cdot n \cdot t \cdot b^2 \cot^2 \phi/2.$$

In most of Crowther's experiments ϕ is sufficiently small that $\tan\phi/2$ may be put equal to $\phi/2$ without much error. Consequently

$$\phi^2 = 2\pi n \cdot t \cdot b^2 \qquad \text{if} \quad p = 1/2.$$

On the theory of compound scattering, we have already seen that the chance p_1 that the deflexion of the particles is greater than ϕ is given by

$$\phi^2 / \log p_1 = -\frac{9\pi^3}{64} n \cdot t \cdot b^2.$$

Since in the experiments of Crowther the thickness t of matter was determined for which $p_1 = 1/2$,

$$\phi^2 = 0.96 \cdot \pi n \cdot t \cdot b^2.$$

For a probability of 1/2, the theories of single and compound scattering are thus identical in general from, but differ by a numerical constant. It is thus clear that the main relations on the theory of compound scattering of Sir J.J. Thomson, which were verified experimentally by Crowther, hold equally well on the theory of single scattering.

For example, if t_m be the thickness for which half the particles are scattered through an angle ϕ, Crowther showed that $\phi/\sqrt{t_m}$ and also $mu^2/E\cdot\sqrt{t_m}$ were constants for a given material when ϕ was fixed. These relations hold also on the theory of single scattering. Notwithstanding this apparent similarity in form, the two theories are fundamentally different. In one case, the effects observed are due to cumulative effects of small deflexions, while in the other the large deflexions are supposed to result from a single encounter. The distribution of scattered particles is entirely different on the two theories when the probability of deflexion greater than ϕ is small.

We have already seen that the distribution of scattered α particles at various angles has been found by Geiger to be in substantial agreement with the theory of single scattering, but cannot be explained on the theory of compound scattering alone. Since there is every reason to believe that the laws of scattering of α and β particles are very similar, the law of distribution of scattered β particles should be the same as for α particles for small thickness of matter. Since the value of mu^2/E for the β particles is in most cases much smaller than the corresponding value for the α particles, the chance of large single deflexions for β particles in passing through a given thickness of matter is much greater than for α particles. Since on the theory of single scattering the fraction of the number of particles which are deflected through a given angle is proportional to kt, where t is the thickness supposed small and k a constant, the number of particles which are undeflected through this angle is proportional to $1-kt$. From considerations based on the theory of compound scattering, Sir J.J. Thomson deduced that the probability of deflexion less than ϕ is proportional to $1 - e^{-\mu/t}$ where μ is constant for any given value of ϕ.

The correctness of this latter formula was tested by Crowther by measuring electrically the fraction I/I_0 of the scattered β particles which passed through a circular opening subtending an angle of 36° with the scattering material. If

$$I/I_0 = 1 - e^{-\mu/t},$$

the value of I should decrease very slowly at first with increase of t. Crowther, using aluminium as scattering material, states that the variation of I/I_0 was in good accord with this theory for small values of t. On the other hand, if single scattering be present, as it undoubtedly is for α rays, the curve showing the relation between I/I_0 and t should be nearly linear in the initial stages. The experiments of Madsen[10] on scattering of β rays, although not made with quite so small a thickness of aluminium as that used by Crowther, certainly support such a conclusion. Considering the importance of the point at issue, further experiments on this question are desirable.

From the table given by Crowther of the value $\phi/\sqrt{t_m}$. for different elements for β rays of velocity 2.68×10^{10} cms. per second, the values of the central charge Ne can be calculated on the theory of single scattering. It is supposed, as in the case of the α rays, that for the given value of $\phi/\sqrt{t_m}$ the fraction of the β particles deflected by single scattering through an angle greater than ϕ is 0.46 instead of 0.5. The values of N calculated from Crowther's data are given below.

Table 2.

Element	Atomic weight	$\varphi/\sqrt{t_m}$	N
Aluminium	27	4.25	22
Copper	63.2	10.0	42
Silver	108	15.4	78
Platinum	194	29.0	138

It will be remembered that the values of N for gold deduced from scattering of the α rays were in two calculations 97 and 114. These numbers are somewhat smaller than the values given above for platinum (viz. 138), whose atomic weight is not very different from gold. Taking into account the uncertainties involved in the calculation from the experimental data, the agreement is sufficiently close to indicate that the same general laws of scattering hold for the α and β particles, notwithstanding the wide differences in the relative velocity and mass of these particles.

As in the case of the α rays, the value of N should be most simply determined for any given element by measuring the small fraction of the incident β particles scattering through a large angle. In this way, possible errors due to small scattering will be avoided.

[10] Phil. Mag. XVIII. p. 909 (1909).

The scattering data for the β rays, as well as for the α rays. indicate that the central charge in an atom is approximately proportional to its atomic weight. This falls in with the experimental deductions of Schmidt[11]. In his theory of absorption of β rays, he supposed that in traversing a thin sheet of matter, a small fraction α of the particles are stopped, and a small fraction β are reflected or scattered back in the direction of incidence. From comparison of the absorption curves of different elements, he deduced that the value of the constant β for different elements is proportional to nA^2 where n is the number of atoms per unit volume and A the atomic weight of the element. This is exactly the relation to be expected on the theory of single scattering if the central charge on an atom is proportional to its atomic weight.

§ 7 *General Considerations*

In comparing the theory outlines in this paper with the experimental result, it has been supposed that the atom consists of a central charge supposed concentrated at a point, and that the large single deflexions of the α and β particles are mainly due to their passage through the strong central field. The effect of the equal and opposite compensating charge supposed distributed uniformly throughout a sphere has been neglected. Some of the evidence in support of these assumptions will now be briefly considered. For concreteness, consider the passage of a high speed α particle through an atom having a positive central charge Ne, and surrounded by a compensating charge of N electrons. Remembering that the mass, momentum, and kinetic energy of the α particle are very large compared with the corresponding values for an electron in rapid motion, it does not seem possible from dynamic considerations that an α particles can be deflected through a large angle by a close approach to an electron, even if the latter be in rapid motion and constrained by strong electrical forces. It seem reasonable to suppose that the chance of single deflexions through a large due to this cause, if not zero, must be exceedingly small compared with that due to the central charge.

It is of interest to examine how far the experimental evidence throws light on the question of the extent of the distribution of the central charge. Suppose, for example, the central charge to be composed of N unit charges distributed over such a volume that the large single deflexions are mainly due to the constituent and not to the external field produced by the distribution. It has shown that the fraction of the α particles scattered through a large

[11] Annal. d. Phys., IV. 23, p. 671 (1907)

angle is proportional to $(NeE)^2$, where Ne is the central charge concentrated at a point and E the charge on the deflected particle. If, however, this charge is distributed in single units, the fraction of the α particles scattered through a given angle is proportional to Ne^2 instead of N^2e^2. In this calculation, the influence of mass of the constituent particles has been neglected, and account has only been taken of its electric field. Since it has been shown that the value of the central point charge for gold must be about 100, the value of the distributed charge requires to produce the same proportion of single deflexions through a large angle should be at least 10,000. Under these conditions the mass of the constituent particle would be small compared with that of the α particle, and the difficulty arises of the production of large single deflexions at all. In addition, with such a large distributed charge, the effect of compound scattering is relatively more important than that of single scattering. For example, the probable small angle of deflexion of a pencil of α particles passing through a thin gold–foil would be much greater than that experimentally observed by geiger. The large and small angle scattering could not then be explained by the assumption of a central charge of the same value. Considering the evidence as a whole, it seems simplest to suppose that the atom contains a central charge distributed through a very small volume, and that the large single deflexions are due to the central charge as a whole, and not to its constituents. At the same time, the experimental evidence is not precise enough to negative the possibility that a small fraction of the positive charge may be carried by satellites extending some distance from the centre. Evidence on this point could be obtained by examining whether the same central charge is required to explain the single deflexions of α and β particles; for the α particle must approach much closer to the centre of the atom than the β particle of average speed to suffer the same large deflexion.

The general data available indicate that the value of this central charge for different atoms is approximately proportional to their atomic weights, at any rate atoms heavier than aluminium. It will be of great interest to examine experimentally whether such a simple relation holds also for the lighter atoms. In cases where the mass of the deflecting atom (for example, hydrogen, helium, lithium) is not very different from that of the α particle, the general theory of single scattering will require modification, for it is necessary to take into account the movements of the atom itself (see § 4).

It is interest to note that Nagaoka[12] has mathematically considered the properties of a "Saturnian" atom which he supposed to consist of a central attracting mass surrounded by rings of rotating electrons. He showed that

[12] Nagaoka, Phil. Mag. VII. p. 445 (1904).

such a system was stable if the attractive force was large. From the point of view considered in this paper, the chance of large deflexion would practically be unaltered, whether the atom is considered to be a disk or a sphere. It may be remarked that the approximate value found for the central charge of the atom of gold ($100e$) is about that to be expected if the atom of gold consisted of 49 atoms of helium, each carrying a charge $2e$. This may be only a coincidence, but it is certainly suggestive in view of the expulsion of helium atoms carrying two unit charge from radioactive matter.

The deductions from the theory so far considered are independent of the sigh of the central charge, and it has not so far been found possible to obtain definite evidence to determine whether it be positive or negative. It may be possible to settle the question of sign by consideration of the difference of the laws of absorption of the β particle to be expected on the two hypotheses, for the effect of radiation in reducing the velocity of the β particle should be far more marked with a possible than with a negative centre. If the central charge be positive, it is easily seen that a positively charge mass, if released from the centre of a heavy atom, would acquire a great velocity in moving through the electric field. It may be possible in this way to account for the high velocity of expulsion of α particles without supposing that they are initially in rapid motion within the atom.

Further consideration of the application of this theory to these and other questions will be reserved for a later paper, when the main deductions of the theory have been tested experimentally. Experiments in this directions are already in progress by Geiger and Marsden.

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The preceding paper by Rutherford sets forth his theory of the scattering of α particles by atoms composed of a small, centrally located, positively charged nucleus surrounded by a sphere of equal but uniformly distributed negative charge whose effect on the scattering of the particles is negligible. The orders of magnitude envisioned were, roughly, for the nuclear radius about 10^{-12} cm, and for the whole atom about 10^{-8} cm. If one imagines that discrete negative electrons were present instead of a distribution of negative charge, then the nuclear atom would be mostly empty space.

Rutherford's scattering formula (see the previous paper, equation [5]) predicted that the fraction of the α particles scattered by a thin foil should be proportional to: (1) the inverse $\sin^4 \phi/2$, where ϕ is the angle through which the α particle is deflected by its encounter with an atom through which the α particle is deflected by its encounter with an atom of the foil; (2) the thickness of the scattering foil, provided this is small ; (3) the square of the nuclear charge Ne; (4) the inverse fourth power of the velocity ν of the bombarding α particles.

Fig. 44–3 in Geiger and Marsden's paper, which follows, shows the very simple apparatus that they set up to examine the theoretical predictions enumerated above. Essentially the device consists of a scattering foil F, upon which α particles from the source R, drawn into a stream of parallel trajectories by a diaphragm at D, strike at right angles. The box B, which carries the viewing microscope M, rotates around the axis of the foil F by means of the ground–glass joint C. A cap over the end of vibrated they radiated energy until they came to rest again. This picture is in complete accord with the classical electromagnetic theory of the electron as developed by Maxwell and Lorentz, and for this reason the Thomson model was favorably received by Thompson's contemporaries.

Thomson's purpose in developing this model was to explain the "scattering of electrified particles in passing through small thickness of matter". In scattering experiments, the crucial criterion for the atom model is the angle through which a charged particle is deflected from its original direction of motion as it passes through a metal foil used as the scatterer. Thomson assumed that the angle of deviation suffered by the charged particle was always caused by a large number of collision with many atoms. Any single collision played only a minimal role in the total deviation, which was a cumulative affect. It can be shown that on the basis of the Thomson model the total deviation is not the average deviation produced in a single collision multiplied by the number of collisions; rather, the multiplicand is the *square root* of this sum of collisions. Thus, if each collision resulted on the average in a deviation of 1 degree, . 100 collisions would give rise to a net deviation of only 10 degrees.

Rutherford pointed out the importance of this fact by calling attention to the observations of Geiger and Marsden. They had found, in their experiments with α particles passing through a layer of gold foil about 0.00006 cm thick, that they could be scattered through an angle of 90 degrees or more. If only small deviations occurred in each encounter, the α particle would have had to undergo 10,000 of the lesser collisions to produce such a large total deviation. This was highly improbable, as Rutherford pointed out, because of the extreme thinness of the gold foil. Rutherford contended that such large deviations must have been caused, therefore, by single direct collisions. He then produced to analyze the theory of single collisions on the basis of a model of the atom that is radically different from the Thomson model.

In this Rutherford model the positive electricity is not distributed over a large volume but instead is concentrated in a very small nucleus at the center of the atom. As Rutherford points out in the paper that follows, the actual analysis is the same whether one assumes that the positive charge is concentrated at the nucleus and the electrons are on the outside, or vice versa.

A model of this sort cannot be in static equilibrium, since the electrons would all be dragged into the nucleus if they were not moving in stable orbits around this nucleus. yet this kind of dynamical equilibrium is in serious contradiction with classical electrodynamics. Rutherford was aware of this, but chose to ignore the difficulty for the time being . He stated that the "question of the stability of the atom proposed need not be considered at this stage . . ."

By very simple but elegant arguments and with the most elementary mathematics, Rutherford showed that his model of the atom gives rise to the kind of deviations during single collisions that Geiger and Marsden had observed. The paper itself is exemplary in its simplicity, yet so profound that none could doubt that Rutherford's ideas must serve as the basis of a new and correct picture of the structure of matter.

We should note, however, that the picture of the atom that Rutherford drew was still very tentative and vague. He speculates that not all of the positive charge is in the nucleus; "a small fraction of the positive charge may be carried by satellites extending some distance from the center." Although the values he obtained for the charge on the nuclei of different metals are all too large, for example, 100 for gold, he correctly concludes that the nuclear charge should be " approximately proportional" to the atomic weight of the atom. But Rutherford was not sure that this would hold for the light elements and indicated that for such elements his simple theory of atomic collisions is no longer applicable.

Although throughout most of the paper he does not specifically mention the planetary theory of the atom, it is clear from his reference to the work of Nagaoka that Rutherford had this planetary model in mind, and it is here that we have the starting point of modern atomic theory.

It has often happened in the past that authors in referring to this paper of Rutherford's speak of his experiments on the scattering of α particles; however, the experiments used by Rutherford were not own but those of

Geiger and Marsden. Rutherford's great contribution lay in showing that the Thomson model of the atom cannot possible explain the large number of large–angle scatterings, whereas the nuclear model can.