SOLUTIONS

Uncertainties

1 Total mass = 112 + 252 + 151 = 515 kg

Thus the uncertainty in the sum is given by: $\sqrt{1^2 + 2^2 + 1^2} = 2.4$ kg

Thus total mass = (515 ± 2) kg

Notice that simply adding the uncertainties in the masses is over pessimistic, giving 4 kg. The square root of the sum of the squares is better statistically in that uncertainties of this nature will sometimes cancel each other).

Note: the uncertainty is given as ± 2, not ± 2.4. Giving an excessive number of figures must be avoided. In general the uncertainty is given to one figure unless the leading digit is one, see question 4 below.
Remember to give the value to the same number of decimal places as the uncertainty, see question 3 below where the 2.75 becomes 2.8.

2 (a) & (b) uncertainty in the sum and difference = $\sqrt{(0.2)^2 + (0.2)^2}$
= 0.3 mm (one sig. fig.)

$$\text{Thus \% uncertainty in sum} = \frac{0.3}{226.6} \times 100 = 0.1\ \%$$

$$\text{Thus \% uncertainty in difference} = \frac{0.3}{2.4} \times 100 = 13\ \%$$

(c) Usually the difference in two readings is needed when using the travelling microscope. Great care has to be taken when measuring very small distances, even with an "accurate" instrument large uncertainties can be incurred.

3 (a) Volume = L x B x H = 0.050 x 0.100 x 0.040 = 2.00×10^{-4} m^3

$$\text{density} = \frac{\text{mass}}{\text{volume}} = \frac{0.560}{2.0 \times 10^{-4}} = 2.8 \times 10^3 \text{ kg m}^{-3}$$

(b) $\%\text{ uncertainty in mass} = \dfrac{0.002}{0.560} \times 100 = 0.4\ \%$

% uncertainty in length = 2 %, % uncertainty in breadth = 1 %,
% uncertainty in height = 3 %

Thus % uncertainty in volume = $\sqrt{2^2 + 1^2 + 3^2} = 3.7\ \%$ or 4%

The dominant uncertainty is in the volume. Thus the % uncertainty in density will be 4 %.

density = $2.8 \times 10^3 \pm 4\ \%$ kg m^{-3}
= $(2.8 \pm 0.1) \times 10^3$ kg m^{-3}

4 % uncertainty in radius, r = $\dfrac{0.1}{1.2} \times 100 = 8\ \%$

The volume depends on the cube of r:

the % uncertainty in the volume = 3 x 8 % = 24 %

volume = $\dfrac{4}{3}\pi (1.2 \times 10^{-2})^3$ m^3 = 7.24×10^{-6} m^3

volume = $(7.2 \pm 1.7) \times 10^{-6}$ m^3

5 % uncertainty in mass $= \frac{0.01}{4.04} \times 100 = 0.25\ \%$

mean diameter = 0.248 m

random uncertainty in mean $= \frac{0.255 - 0.243}{8} = 0.0015$

$= 0.002$ m

% uncertainty in mean diameter $= \frac{0.002}{0.248} \times 100 = 0.8\ \%$

The % uncertainty will be the same for the radius.

Thus moment of inertia of the disc $= \frac{1}{2} M R^2 = 0.5 \times 4.04 \times (0.124)^2$

$= 0.0311\ \text{kg m}^2$

The dominant uncertainty here is in the radius, which is squared:

total % uncertainty $= 2 \times 0.8\ \% = 1.6\ \%$

Thus moment of inertia $= 0.0311 \pm 1.6\ \%$

$= (0.0311 \pm 0.0005)\ \text{kg m}^2$

6 refractive index, $n = \frac{\sin \theta_1}{\sin \theta_2} = \frac{\sin 46}{\sin 28} = \frac{0.7193}{0.4695} = 1.53$

The easiest way to work out the uncertainty in a sine value is to work out the maximum and minimum values. Find the difference between these values and halve it.

Thus for θ_1 : $\sin 47° = 0.7314$ $\sin 45° = 0.7071$

range = 0.0243 uncertainty $= \frac{0.0243}{2} = 0.0122$

Thus % uncertainty in $\sin \theta_1 = \frac{0.0122}{0.7193} \times 100 = 1.7\ \%$.

For θ_2 : $\sin 29° = 0.4848$ $\sin 27° = 0.4540$

range = 0.0308 uncertainty $= \frac{0.0308}{2} = 0.0154$

Thus % uncertainty in $\sin \theta_2 = \frac{0.0154}{0.4695} \times 100 = 3.3\ \%$.

To find the overall uncertainty in refractive index these two uncertainties have to be combined.

% uncertainty in refractive index, $n = \sqrt{1.7^2 + 3.3^2} = \sqrt{13.8}$

$= 3.7\ \%$ or $4\ \%$

Final value : $n = 1.53 \pm 4\ \%$

$= 1.53 \pm 0.06$