CfE Advanced Higher Physics

Rotational Motion and Gravitation

Derivation of equations of motion

This is the really fun bit to start with! You do need to know this derivation since it is in the LOs

FACT: Acceleration is the rate of change of velocity.

$$\frac{dv}{dt} = a \qquad \text{eqn.1}$$

Consider an object accelerating from rest.

At t = 0 v = u and s =0 where s, u, v, a and t have the usual meanings.

To find an expression for velocity we must integrate eqn.1.

$$\int \frac{dv}{dt}.dt = \int a.dt$$

$$v = at + C$$

from initial conditions when t = 0 v = u so C = u

now have $v = u + at$ [A]

FACT: Velocity is the rate of change of displacement.

$$v = \frac{ds}{dt} = u + at \qquad \text{eqn.2}$$

To find an expression for displacement we must integrate eqn.2.

$$\int \frac{ds}{dt}.dt = \int u.dt + \int at.dt$$

$$s = ut + \tfrac{1}{2}at^2 + C$$

from initial conditions when t = 0 s = 0 so C = 0

now have $s = ut + \tfrac{1}{2}at^2$ [B]

To obtain the 'third' equation of motion

Square eqn. [A]

$$v^2 = (u + at)^2$$

$$v^2 = u^2 + 2uat + a^2t^2 = u^2 + 2a[ut + \tfrac{1}{2}at^2]$$

$$v^2 = u^2 + 2as \quad [C]$$

<u>Angular Motion</u>

For this part of the course you need to learn a new language, angular motion.
The equations of motion for angular are the same as those for linear motion from Higher, we just say them differently. Comprendez vous?

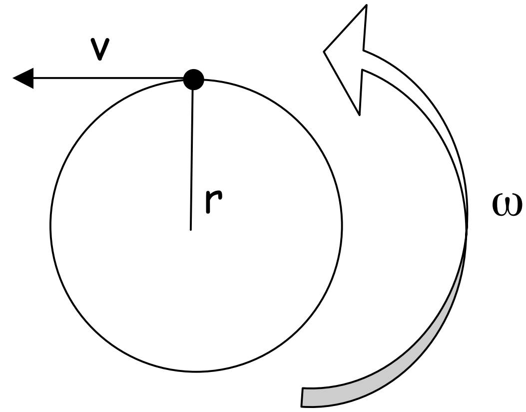
Vocabulary

linear		angular	
displacement	s	angular displacement	θ
initial velocity	u	initial ang. velocity	ω_0
final velocity	v	final ang. velocity	ω
acceleration	a	angular acceleration	α

$v = u + at$	$\omega = \omega_0 + \alpha t$
$s = ut + \frac{1}{2}at^2$	$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 = u^2 + 2as$	$\omega^2 = \omega_0^2 + 2\alpha\theta$

As you can see the equations are identical in how the terms relate to each other, though $\theta\omega_0\omega\alpha t$ doesn't roll off the tongue quite so readily as suvat!!

Rotational Motion



Consider a point on the circumference of a circle. It will make one complete rotation in time T. (A capital is used to denote this time as it is a specific time value known as the period.)

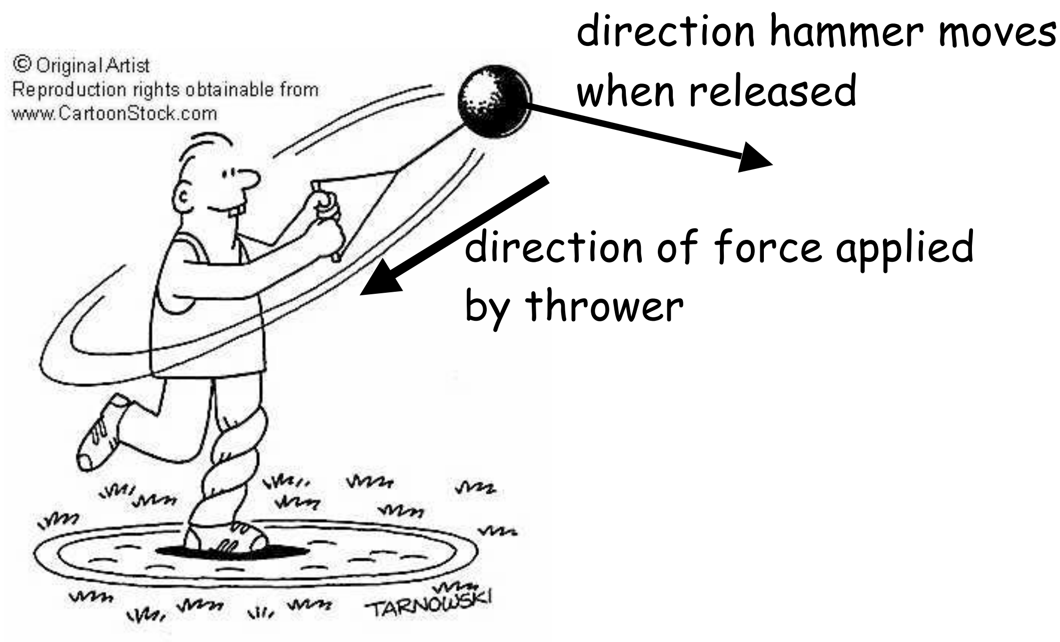
The speed of the point is $v = \frac{d}{t} = \frac{\text{circumference}}{\text{period}} = \frac{2\Pi r}{T}$

The angular velocity of the point is $\omega = \frac{\theta}{T} = \frac{2\Pi}{T}$

Equating both relationships gives $v = \omega r$

We have a situation where an object on the circumference of the circle, moving with constant speed as above is also accelerating. This is due to its continual change in direction and hence subsequent change in velocity.

If there is an acceleration there must also be an unbalanced force acting on the object. In the case of circular motion this force must act towards the centre of the circle.

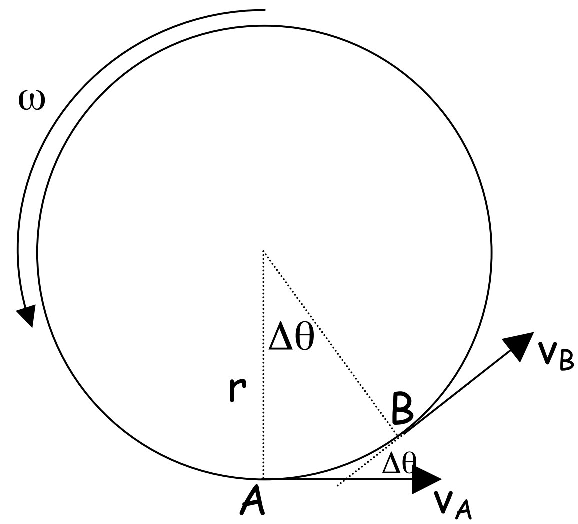


The magnitude of the linear and angular accelerations are related by the following equation

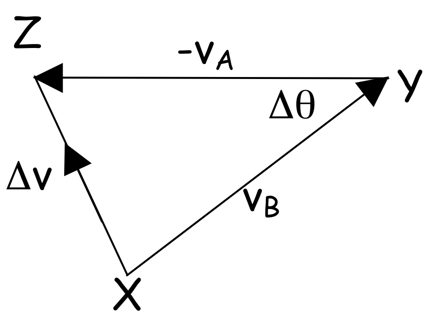
$$a = \alpha r$$

The magnitude of the central force acting on the object will depend on the mass of the object, the radius of the orbit and the speed of the object.

Radial Acceleration.



Consider an object moving in a circular path radius, r.
Two tangential vectors representing velocity are shown at A and B.
Drawn as a vector diagram the resultant of the velocities is shown in the diagram below.



When the time interval between A and B is very small Δθ will be very small and angle ZXY will be almost 90°. This means that XZ is towards the centre of the circle. [Radius is perpendicular to the tangent]

$$a = \frac{\Delta v}{\Delta t}$$

if $\Delta\theta$ is small, then $\Delta v = v\Delta\theta$ if θ is measured in radians

so $$a = \frac{v\Delta\theta}{\Delta t}$$

as Δt approaches zero $a = \frac{vd\theta}{dt} = v\omega$

Making use of the fact that v = ωr, we can substitute to obtain

$$a = \omega^2 r = \frac{v^2}{r}$$

If the above expressions represent the radial acceleration then the central force producing it can be determined by using the 'rude equation', $F_U = ma$.

$$F = m\omega^2 r = \frac{mv^2}{r}$$

This radial force is called the **centripetal force**; it is always present whenever any object is moving in a circular orbit.

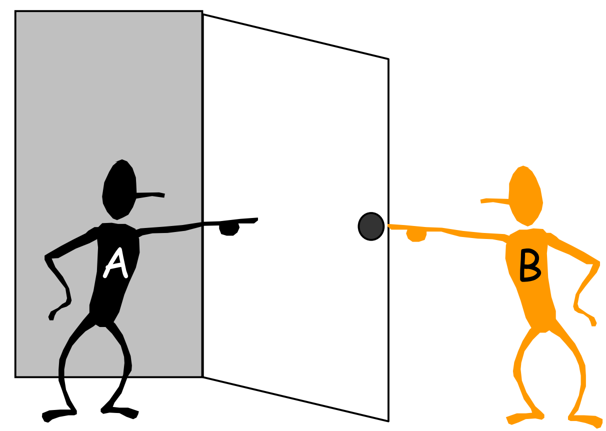
The force itself is normally produced by gravitational [satellite motion], electrostatic [electron orbit], magnetic [mass spectrometer], tension [hammer thrower], friction [car travelling around a corner with no slipping] and normal reaction forces [standing on the surface of the Earth without flying off!!].

Rotational Dynamics

Again other than language this topic area is essentially the same as in Higher.

Force vs Torque: In a rotational situation the magnitude of the force applied is not the only factor that needs to be taken into account.

Simple experiment: Apparatus, two people and a door.
Procedure: one person pushes close to the door hinge, one person pushes close to the edge of the door with the same magnitude of force.
Observe, discuss and explain.



The distance the force is applied from the pivot determines its effect. The larger the distance is the greater the effect. This is called the **moment of a force**.

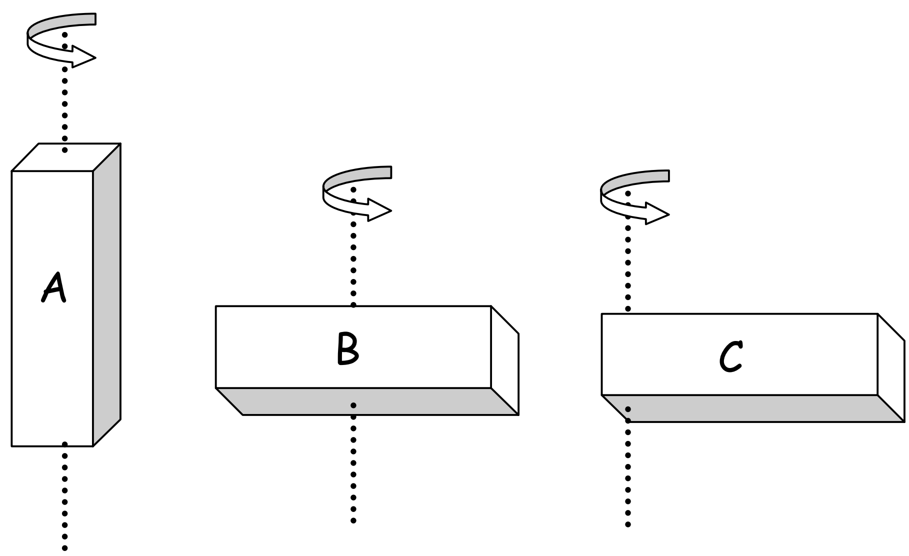
As always we can show this as a numerical value called the **torque**.

$$T = Fr$$

Torque is measured in Nm.

Angular Acceleration

It should come as no surprise that an unbalanced torque will produce an angular acceleration in the same way that an unbalanced force will produce a linear acceleration. But, yes there's always a but, there is also a rotational equivalent for mass.
A single object may react differently to an applied torque depending on how it rotates.



Consider the three identical blocks A, B and C above.
Which of the three would be hardest to rotate?
The answer depends on how the mass is distributed around the axis of rotation. The greater the distance the mass is from the axis the greater the torque will be required to produce a particular angular acceleration.

This distribution of mass is called the **moment of inertia**. For a single object, of mass m a distance of r from the axis of rotation, its moment of inertia is given as

$$I = mr^2$$

The units of moment of inertia are kgm^2.

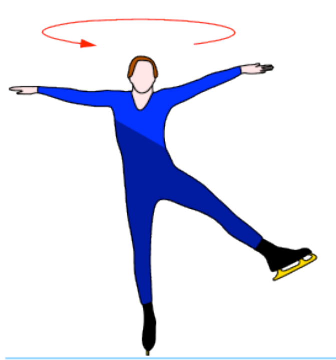
Vocabulary

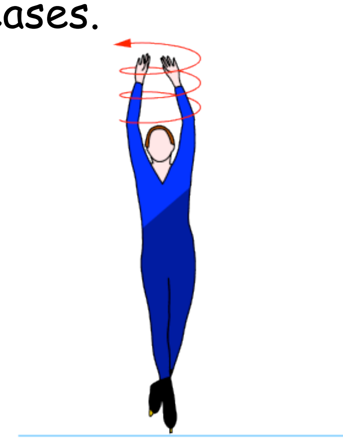
linear		angular	
mass	m	moment of inertia	I
force	F	torque	T

The related equations and principles follow from this.

$F_u = ma$	$T = I\alpha$
$E_K = \frac{1}{2}mv^2$	$E_{Krot} = \frac{1}{2}I\omega^2$
cons. of momentum	cons. of angular momentum
$m_1v_1 = m_2v_2$	$I\omega_1 = I\omega_2$

Conservation of angular momentum; You can see this in action when ice skaters spin then pull their arms in and spin faster. Their moment of inertia has been reduced and so their angular velocity increases.



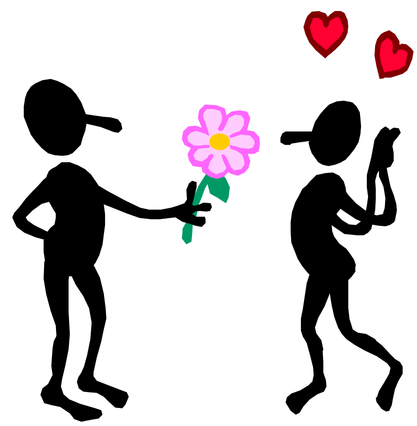


Gravitation

Inverse square law of gravitation

$$F = \frac{Gm_1m_2}{r^2}$$

G is the universal gravitational constant and has a value of 6.67 x 10-11 Nm^2/kg^2.



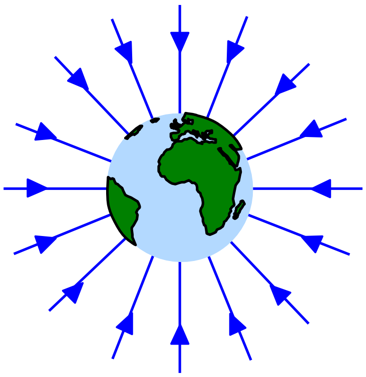
This force acts between any two objects which have mass. In fact you are **always** attracted to the person sitting next to you!! [Scary thought] It is however a strictly gravitational attraction.

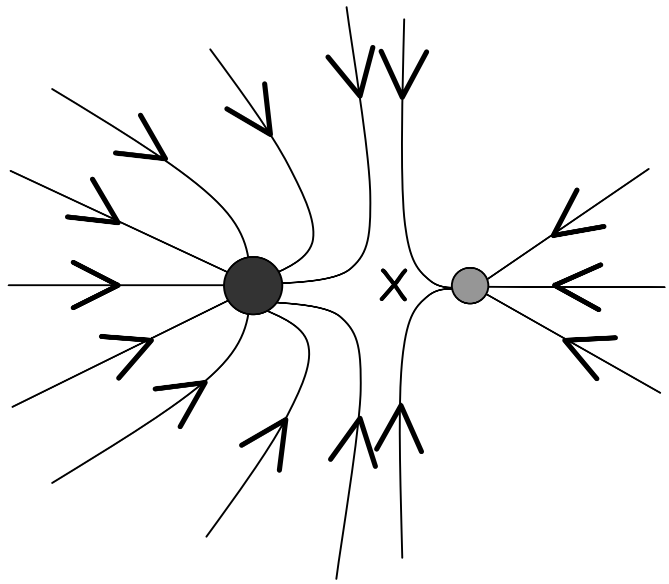
Gravitational field strength was introduced in Standard Grade and is defined as.

> The force acting per unit mass on
> an object in the field.

A gravitational field is a model by which the effects of gravitation can be explained. The force acting on a mass in the field is always attractive. We can represent the strength and shape of the field by drawing field lines. [Not unlike the patterns produced by filings around a magnet]

Gravitational field line patterns for single and dual planet systems.





There is a point between any two planet system where the net gravitational field will be zero. In reality there should be nowhere in the universe since any point will have some gravitational effect from every object in the universe.

Gravitational Potential

Up till now we have calculated a change in potential energy of an object by considering a change of height and the mass of the object in question. At Advanced Higher we will consider potential energies of satellites which are in orbit hundreds of km above the surface of the Earth. This leads to a problem since the Earth's gravitational field changes with distance from the centre of the earth.

It is possible to calculate the gravitational potential at a point in space some distance from the Earth.

$$V = -\frac{GM}{r}$$

where M = mass of Earth
r = distance from **centre** of Earth

We can use this equation based on two factors:
1. The gravitational potential at a point is defined as the work done in bringing an object from infinity to that point.
2. The gravitational potential at infinity is zero.

The gravitational potential is **always** negative. This is due to the fact that gravitation is an attractive force and the field does work on the object bringing it closer to Earth.

To find the gravitational potential energy of the object we simply multiply the potential by the mass of the object, m.

$$E_P = -\frac{GMm}{r}$$

Conservative Field

This is where David Cameron has a picnic. Not really!!
A conservative field is one where the work done against the field in moving an object between two points is independent of the path taken.
A gravitational field is a conservative field.

Escape velocity

The velocity required for an object to move to infinity.
This is relatively easy to calculate.

Step 1: Calculate the energy of the object on the surface of the planet, radius R and mass M.

$$E_P = -\frac{GMm}{R}$$

Step 2: When the object reaches infinity it will have an E_P of 0J.

We must supply kinetic energy sufficient to make the total energy equal to 0J. At infinity the E_K of the object will be zero.

$$E_K + E_P = 0$$

$$\tfrac{1}{2}mv^2 + \left(-\frac{GMm}{R}\right) = 0$$

$$v = \sqrt{\frac{2GM}{R}}$$

Black Holes

A black hole is an object where the mass/radius ratio is such that the escape velocity is greater than 3×10^8 m/s. This means that nothing can escape the surface of the object since the maximum allowable velocity is 3×10^8 m/s.

If light cannot escape it must mean that photons are affected by gravity. This was proposed by Einstein in his General Theory of relativity in 1915. It was confirmed by observation in 1919.