Higher Physics

Our Dynamic Universe

Revision

Vector and Scalar Quantities

Physical quantities can be divided into two groups:

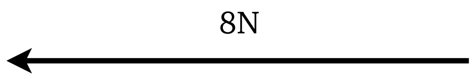
- Scalar quantities are completely described by stating their magnitude.
- Vector quantities are completely described by stating their magnitude and direction.

For example — speed is a scalar (i.e. the man was walking at 4 m s^{-1}), but force is a vector (i.e. the block was pushed with a force of 4 N to the right).

Vectors are represented in diagrams by drawing arrows. The length of the arrow gives the magnitude of the vector and the direction of the arrow gives the direction of the vector.

A force of 4 N to the right can be represented like this:

A force of 8 N to the left can be represented like this:



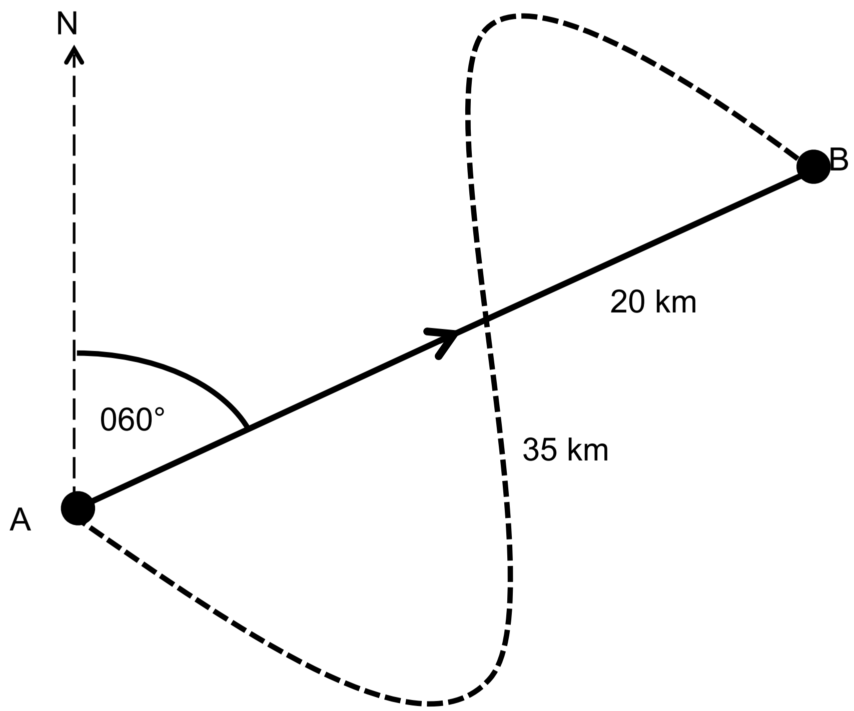
Vectors (Magnitude and direction)	Scalars (Magnitude only)
Velocity	Speed
Acceleration	Time
Force	Mass
Momentum	Energy
Displacement	Distance

Distance and Displacement

When fully describing the motion of an object it is important to not only know the distance it has travelled but also in which direction it has travelled. Distance is a measure of how far an object has travelled, and only has a magnitude (or size) and is therefore a scalar quantity.

Displacement measures how far an object moves from the starting point to the finishing point of its journey and the direction in which it has travelled. As displacement has a magnitude and a direction it is a vector quantity.

Example:



The distance travelled in the journey from A to B is 35 km.

The displacement from A to B is 20 km at 060.

Speed and Velocity

Speed is the rate of change of distance, i.e. the distance travelled in a unit of time. As distance and time are both scalar quantities, speed is also a scalar.

$$\bar{v} = \frac{d}{t}$$

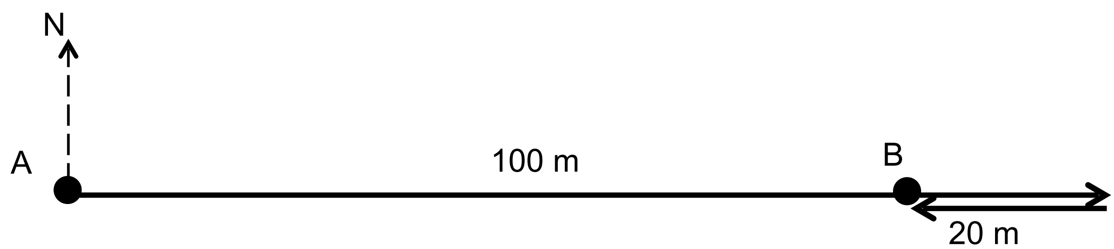
Velocity is the rate of change of displacement, i.e. the displacement per unit time. As displacement is a vector, velocity is also a vector and has a magnitude and a direction, the same direction as the displacement.

$$\underline{\bar{v}} = \frac{\underline{s}}{t}$$

Example: A runner sprints 100 m East along a straight track in 12 s and then takes a further 13 s to jog 20 m back towards the starting point.

1. What distance does she run during the 25 s?
2. What is her displacement from her starting point after the 25 s?
3. What is her speed?
4. What is her velocity?

Solutions:



d = 100 + 20

d = 120m

s = 100 + (− 20)

s = 80 m at 090°

$$\bar{v} = \frac{d}{t}$$

$$\bar{v} = \frac{120}{25}$$

$\bar{v} = 4.8\ \text{m s}^{-1}$

$$\bar{v} = \frac{s}{t}$$

$$\bar{v} = \frac{80}{25}$$

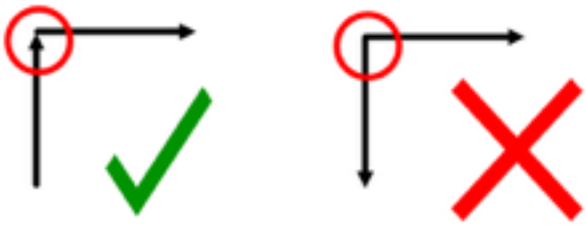
$\bar{v} = 3.2\ \text{m s}^{-1}$ at 090°

Adding Vectors in Two Dimensions

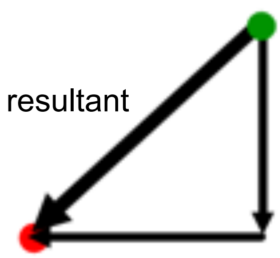
Vectors, acting in different directions, can be added together. When adding vectors you are looking for the resultant. The resultant is a vector which could replace all the vectors already present (components) with a single vector. For example; you can find resultant displacements, velocities and forces.

When working with any kind of vector it is always best to draw a diagram. Vector diagrams can be constructed using the following rules. Vectors should be drawn as an arrow representing the direction with its length representing the magnitude.

1. Vector arrows must be added tip to tail. Individual vectors can be placed anywhere to allow them to be tip to tail, as long as you maintain their direction.

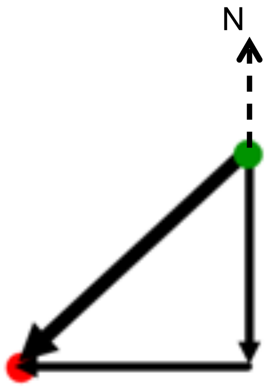


2. Mark the start and end points and draw an arrow between them; this is your resultant vector.



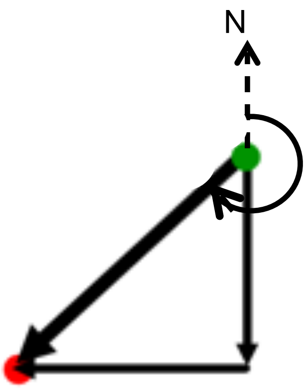
3. Once you have your vector — go back to the starting point and determine the direction of the resultant from some known reference direction.

For example, if North is known to be up the page then draw a small arrow pointing North.



With your pencil, follow from the North line clock-wise until you meet the resultant vector.

This is the bearing.

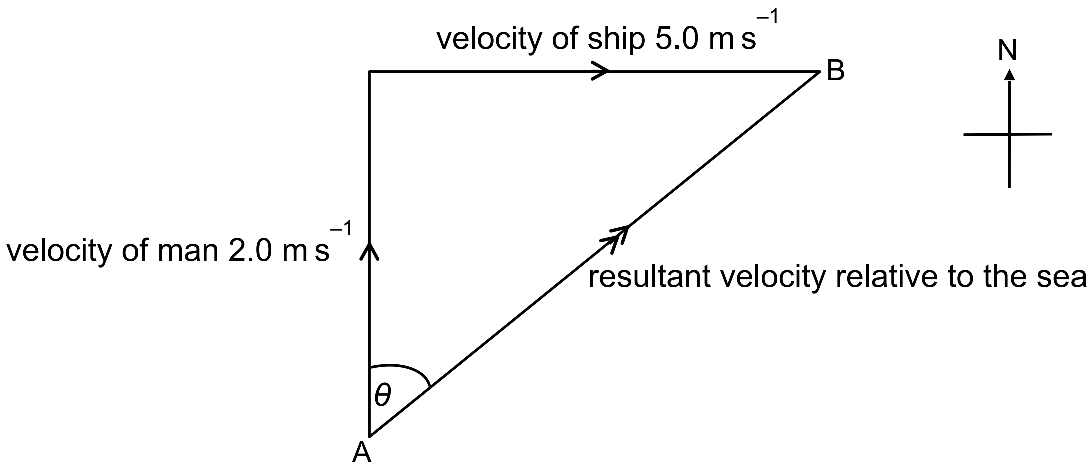


The bearing must be given in three figures.

State the answer separately from the diagram, using the appropriate symbols, i.e. s, v or F. For simple problems involving right angle triangles trigonometry and Pythagoras can be used. A sketch drawn roughly to scale is useful to help visualise the problem.

In Higher, some vector diagrams will not just be right-angled triangles. Regardless of the number of vectors being added they should always be drawn with the vectors tip to tail and the resultant drawn from the start to end of the diagram.

Example: A man walks at 2.0 $m\,s^{-1}$ due North across the deck of a ship that is sailing due east at 5.0 $m\,s^{-1}$. Calculate his resultant velocity relative to the sea.



Solution:

Magnitude of resultant velocity (v):

$$v^2 = x^2 + y^2$$

$$= 5^2 + 2^2$$

$$= 25 + 4$$

$$= 29$$

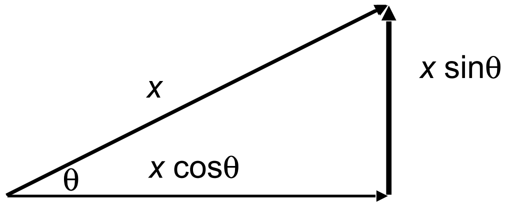
$$v = 5.4\ m\,s^{-1}$$

Direction of resultant velocity (θ):

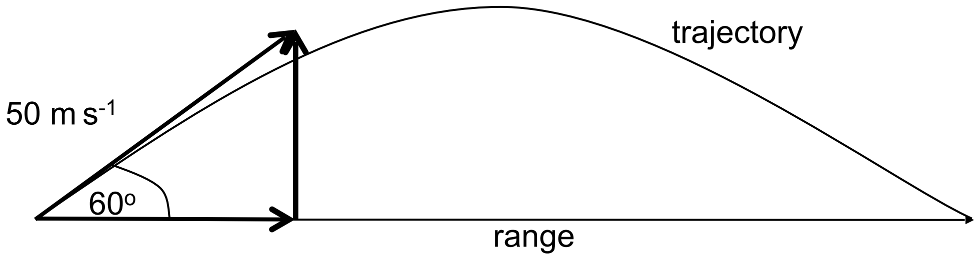
$$v = 5.4\ m\,s^{-1} \text{ at a bearing of } 068°$$

Components of Vectors

We have seen how to find the resultant vector by adding the components. You can also find the components of a vector from the resultant.



Example: A projectile is fired at a velocity of 50 m s^{-1} at an angle of 60° to the horizontal as shown below.



Calculate:

1. The horizontal component of the velocity of the projectile,
2. The vertical component of the velocity of the projectile.

Solution:

1.

$$v_h = 50\cos 60$$

$$v_h = 25 \text{ m s}^{-1}$$

2.

$$v_v = 50\sin 60$$

$$v_v = 43 \text{ m s}^{-1}$$

A Sign Convention for Direction

Direction matters, but it is not always appropriate to define it in a three figure bearing, so a system of defining direction is needed. Normally, we use the concept of positive and negative directions.

One common (but by no means the only) sign or direction convention is:

Upwards — positive

Downwards —

Right — positive

Left — negative

The magnitudes of vectors are assigned a + or – to show their relative direction.

This direction convention is essential when using equations of motion and other vector equations.

Hint: Use a direction convention which simplifies the problem being solved. For example, if an object only ever falls downwards then it is simplest to take down as the positive direction.

Acceleration

Acceleration is the rate of change of velocity, i.e. the change in velocity per unit time, as shown on a velocity-time graph. As velocity is a vector quantity, acceleration is also a vector quantity with a magnitude and direction.

Final velocity measured in meters per second

Initial velocity measured in meters per second (m s^{-1})

Change in velocity, or 'delta v', measured in (m s^{-1})

$$a = \frac{v - u}{t} \qquad a = \frac{\Delta v}{t}$$

Acceleration measured in meters per second squared (m s^{-2})

Time measured in seconds (s)

If the motion of the object is in a straight line it is usual to drop the vector signs and to take to the right or upwards as positive, and to the left or downwards as negative.

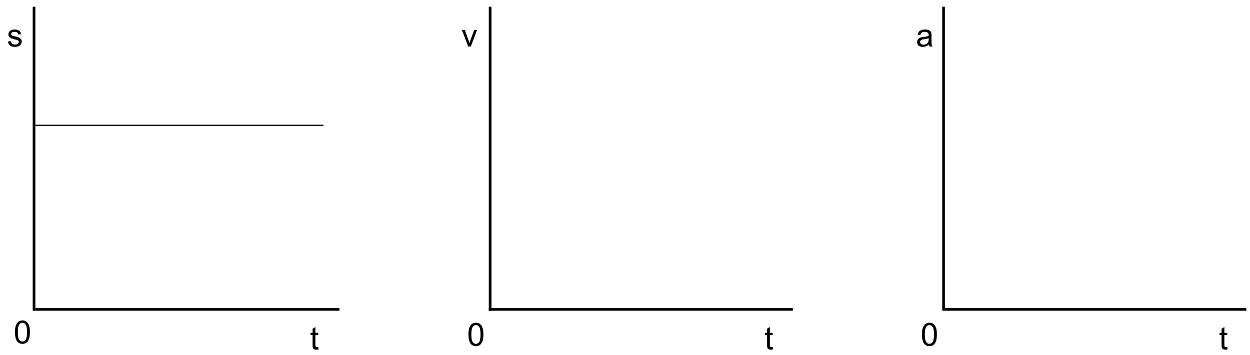
The speed of an object does not have to change for it to be accelerating! An object turning in a circle, such as the Moon orbiting the Earth, can move at constant speed but because its direction is continually changing its velocity must be changing.

Graphs of Motion

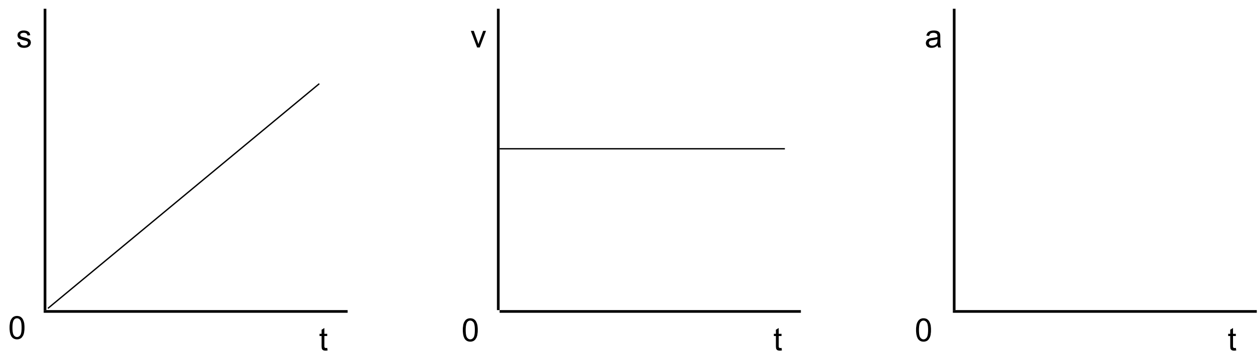
At Higher there are three graphs of motion:

- Displacement/time graphs
- Velocity/time graphs
- Acceleration/time graphs

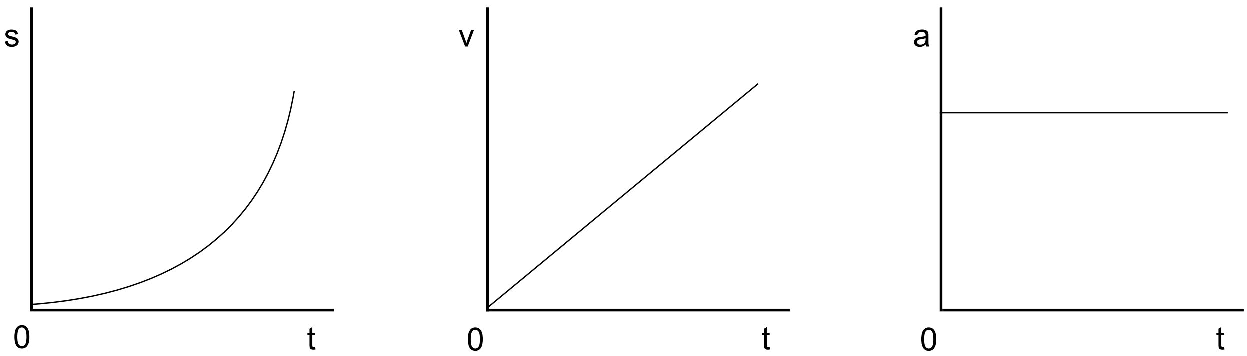
Constant displacement



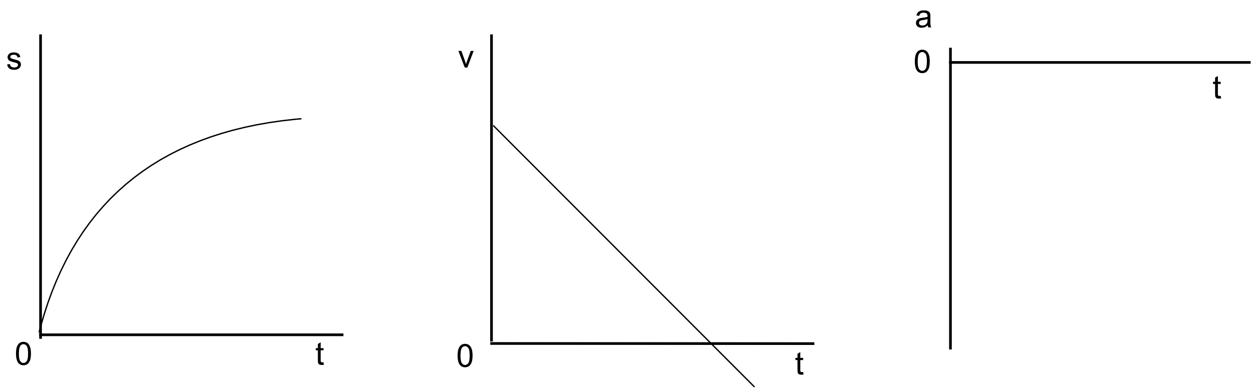
Constant velocity



Constant positive acceleration



Constant negative acceleration (deceleration)



Interpreting Graphs of Motion

Graph Type	Gradient equal to	Area equal to
Displacement/time	Velocity	–
Velocity/time	Acceleration	Displacement
Acceleration/time	Rate of change of acceleration (not needed in school physics!)	Velocity

Example 1: An object thrown upwards.

Example 2: A bouncing ball — ideal case (no energy lost)

Equations of Motion

There are five equations of motion that can help you to solve problems involving a constant acceleration. Two you are familiar with from National 5:

$$s = v\,t$$

Displacement measured in meters (m)

Velocity measured in meters per second (m s^{-1})

Time measured in seconds (s)

$$v = u + at$$

Final speed measured in meters per second (m s^{-1})

Initial speed measured in meters per second (m s^{-1})

Acceleration measured in meters per second squared (m s^{-2})

Time measured in seconds (s)

In addition there are three others (that can be derived from the two above):

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$s = \frac{1}{2}(u + v)t$$

A good technique to tackle questions involving accelerating motion is to list all of the quantities (s, u, v, a and t) for **both** the horizontal and vertical directions:

$s_h =$	$s_v =$
$u_h =$	$u_v =$
$v_h =$	$v_v =$
$a_h =$	$a_v =$
$t_h =$	$t_v =$

Sometimes not all of the data is given but appropriate assumptions can be made, including:

$a_v = 9.8\ \text{m s}^{-2}$ downwards for an object in free fall

$a_h = 0$ for projectiles, assuming no air resistance

$u_v = 0$ for an object being dropped

$v_v = 0$ at the highest point for a projectile

$s_v = 0$ when an object returns to its starting height

Force, Energy and Power

Newton's First Law

> Every body persists in its state of being at rest or of moving uniformly straight forward, except insofar as it is compelled to change its state by force impressed — Isaac Newton, *Philosophiæ Naturalis Principia Mathematica*, 1687.

Or, in today's language, an object will travel at the same speed, in a straight line, unless a force acts on the object. Newton's first law means that an object will never speed up, slow down or change direction unless there is an **unbalanced** force acting on it. This law is sometimes known as the *law of inertia*. It is a somewhat counter-intuitive law as we are used to friction acting on everything. So a car travelling along a road will slow to a stop if the engine was turned off. We tend to forget that there *is* an unbalanced force acting on the car, friction. In space, where there is no friction, Newton's first law is clearly evident — spacecraft need to use rockets or a planet to slow down. Newton's first law is also important when you consider going around a corner. To change direction, even if you have the same speed, need an unbalanced force.

Newton's Second Law

> The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed — Isaac Newton *Philosophiæ Naturalis Principia Mathematica*, 1687.

In more modern language what Newton stated in his second law is that an object will accelerate in the same direction as the unbalanced force acting on it. He also stated that the magnitude of the acceleration was proportional to the magnitude of the unbalanced force.

Newton's Second Law — formula

The unbalanced force acting on an object is equal to the mass times the acceleration of the object. This is the mathematical way of representing Newton's Second Law. It appears in the formula book and looks like this:

$$F = ma$$

Unbalanced force measured in newtons (N)

Mass measured in kilograms (kg)

Acceleration measured in meters per second squared ($\mathrm{m\,s^{-2}}$)

Newton’s Third Law

> To every action there is always an equal and opposite reaction — Isaac Newton *Philosophiæ Naturalis Principia Mathematica*, 1687.

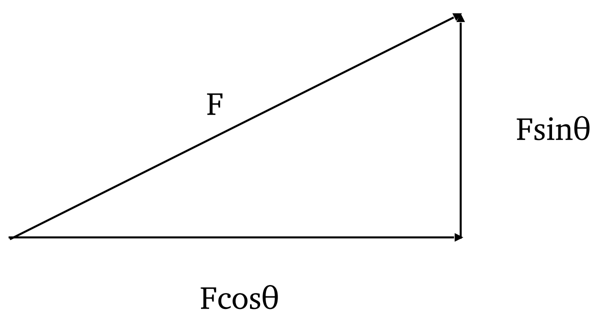
Newton’s third law simply means that if you push something, the something pushes back with the same force. This is why you do not fall through your chair. But it also explains how rockets and jet engines generate thrust. Imagine sitting on a chair with wheels that is on a surface with very low friction. You have a bucket of tennis balls. When you throw a tennis ball you exert a force on the ball (to throw it). However the ball exerts the same size of force on **you**. This force will actually move you backwards. Rockets use the same principle but use a stream of extremely hot gasses instead of tennis balls.

Vector Nature of Force

Force is a vector quantity with magnitude and direction.

Two or more forces can be added using vector addition, (tip to tail), to give a resultant.

A force, at an angle, can also be split into two components, (rectangular components).

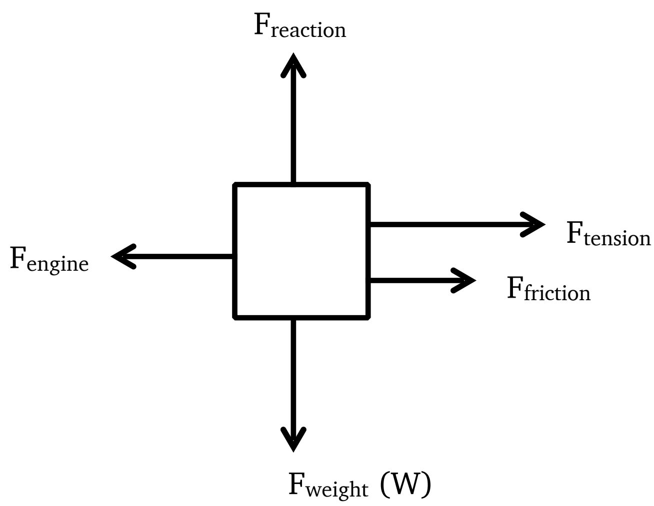


Free Body Diagrams

When analysing the forces acting upon an object it is always helpful to construct a free-body diagram. This displays all the forces acting upon that object at one point in time and their directions. This can help you see which forces are balanced and help you construct a vector diagram if needed.

In a free-body diagram all objects can be represented by a simple box. Individual forces should be drawn as vector arrows, starting at the box and pointing in the correct direction.

Example 1: Free-body diagram for a car pulling a trailer



Tension

Tension is a pulling force applied by a string or cable on another object

Friction

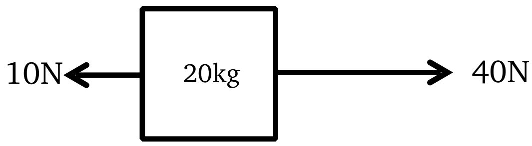
The force of friction always acts **against** the direction of motion. Frictional forces tend increase with velocity (or the velocity squared) but at Higher we can assume friction has a constant value.

Example 2: The effect of friction

A man pushes a shopping trolley of mass 20 kg along a flat surface with a force of 40 N. The force of friction acting on the trolley is 10N.

Calculate the acceleration of the shopping trolley.

Solution:



F = 40 – 10

F = 30N to the right

m = 20kg

F = ma

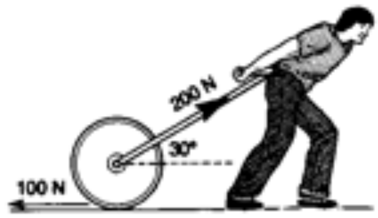
30 = 20a

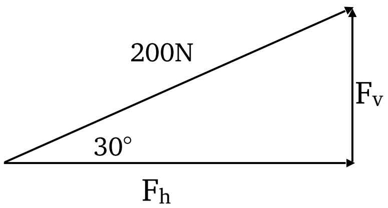
$a = 1.5 \text{ m s}^{-2}$

Example 3: A man pulls a garden roller of mass 100 kg with a force of 200 N acting at 30° to the horizontal.

If there is a friction force of 100 N between the roller and the ground, what is the acceleration of the roller?

Solution:

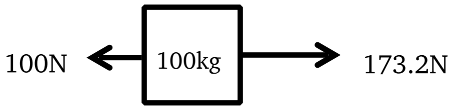




$F_h = F\cos\theta$

$F_h = 200\cos30$

$F_h = 173.2N$



$F = ma$

$73.2 = 100a$

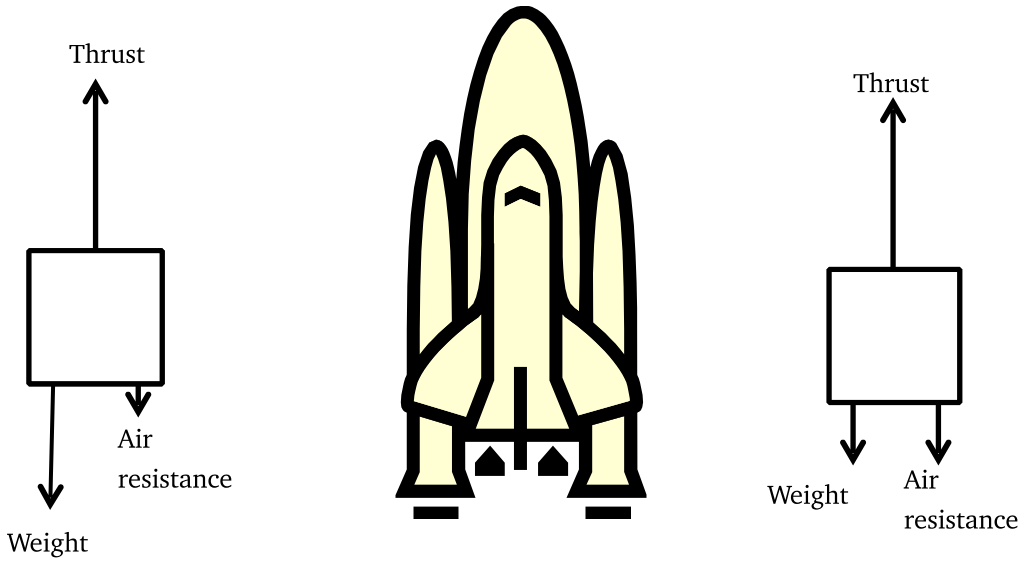
$a = 0.73 \text{ m s}^{-2}$

Rockets

Here are the free-body diagrams for a rocket, at two different points in time, during a flight.

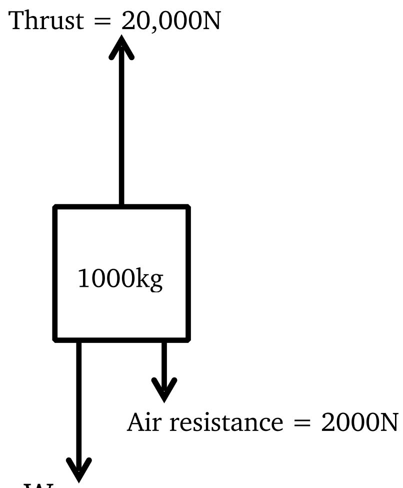
1. Take off: unbalanced force upwards due to large force of thrust, rocket accelerates upwards

2. As the rocket rises, mass is lost as fuel is used up. Therefore the weight of the rocket decreases. The unbalanced force upwards increases, therefore acceleration increases.



Example 1: A guided missile of mass 1000kg is fired vertically into the air. The missile's rocket motors provide 20,000N of thrust and there is a drag force of 2000N. Calculate the acceleration of the rocket.

Solution:



$W = mg$

$W = 1000 \times 9.8$

$W = 9800N$

$F = 20000 - (9800 + 2000)$

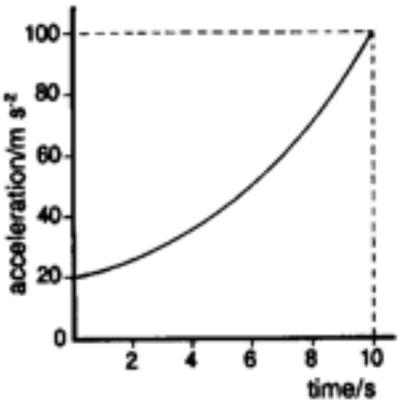
$F = 8200N$

$F = ma$

$8200 = 1000a$

$a = 8.2 \text{ m s}^{-2}$

Example 2: The graph shows how the acceleration of a rocket, in outer space, changed during a 10s period.



The rocket motors produced a constant thrust of 4×10^3N. How much fuel was used by the rocket during the 10 s?

Solution:

$t = 0s, a_1 = 20 \text{ m s}^{-2}$

$t = 10s, a_2 = 100 \text{ m s}^{-2}$

$F = m_1a_1$

$4\times10^3 = m_1 \times 20$

$m_1 = 200kg$

$F = m_2a_2$

$4\times10^3 = m_2 \times 100$

$m_2 = 40kg$

$\Delta m = m_1 - m_2$

$\Delta m = 200 - 40$

$\Delta m = 160kg$

Weight

The weight of an object is the **force due to gravity** on the object. The gravitational field strength (g) is the force due to gravity, per unit mass (1 kg).

$$W = mg$$

Weight measured in newtons (N)

Mass measured in kilograms (kg)

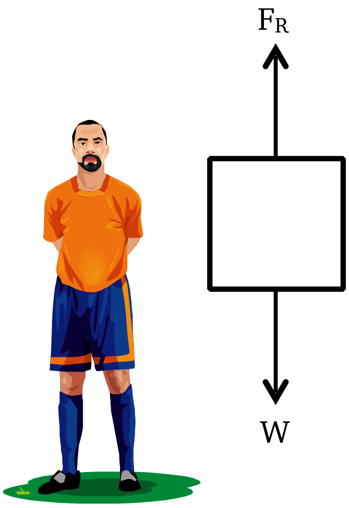
Gravitational field strength measured in newtons per kilogram (Nkg^{-1})

The acceleration due to gravity is equal to the gravitational field strength but has units of $m\,s^{-2}$ rather than $N\,kg^{-1}$. On Earth the gravitational field strength and the acceleration due to gravity have a magnitude of $9.8\ m\,s^{-2}$.

Weight and Lifts

When you are standing still there are two forces acting on you.

Note: The free-body diagram is the same if you are hanging down, rather than standing on a surface. F_R would simply become a force of tension (T)



Reaction force F_R: The force of the ground upon you, in reaction to the force of your weight on the ground (3rd Law).

Weight
$W = mg$

When you stand on a set of scales you can observe the size of the reaction force, as the spring in the scales must now provide it. F_R is equal in magnitude, and opposite in direction, to your weight. Can you explain why this must be the case?

If you are accelerating vertically, however, the forces acting on you are unbalanced. Accelerating up or down in a lift is an example of this situation.

The reading on the scales always shows the force acting upwards (the reaction force F_R), on your body. The reading on the scales in this situation is called your apparent weight. **Your apparent weight is equal to the reaction force.**

For all lift questions, we will assume up to be the positive direction:

Upwards acceleration

Downwards acceleration

But how do we know what direction the force is acting? There are four possible motions the lift can undergo, apart from constant speed or being at rest. Using the concept of positive and negative vector motion we can assign these motions an overall sign and this will tell us if the force is acting up or down.

Motion	**Direction of travel**	**Direction of acceleration**	**Sensation felt in lift**
After pressing ‘up’	upwards	upwards	Heavier
Going up, approaching your floor	upwards	downwards	Lighter
After pressing ‘down'	downwards	downwards	Lighter
Going down, approaching your floor	downwards	upwards	Heavier

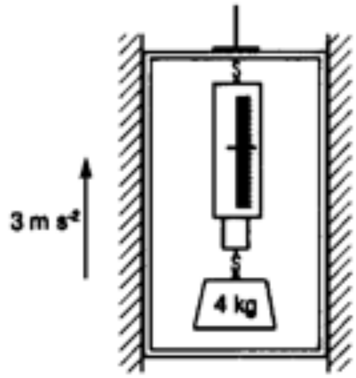
Example 1: A person of mass 80kg enters a lift which accelerates downwards towards a lower floor with an initial acceleration of 1.5 $m\,s^{-2}$.

Calculate the apparent weight of the person.

Solution:

Example 2: A package of 4kg is hung from a spring balance attached to the roof of a lift which is accelerating upwards at 3 m s^{-2}.

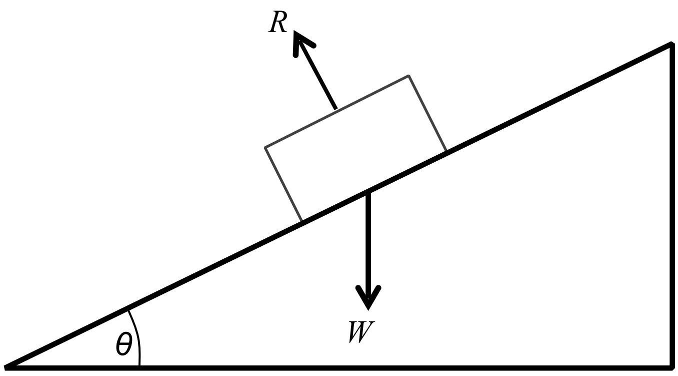
Calculate the reading on the balance.



Solution:

Weight on a Slope

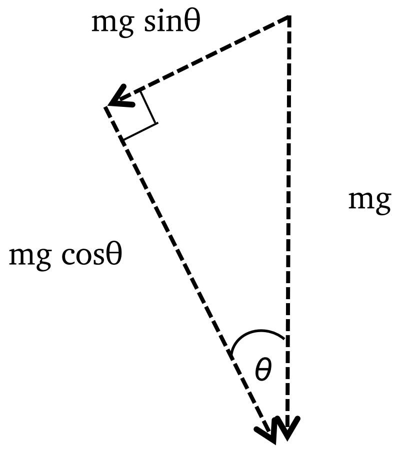
Frictionless slope



R is the reaction force at the normal to the slope.

The resultant is equal to the component of the weight perpendicular to the slope.

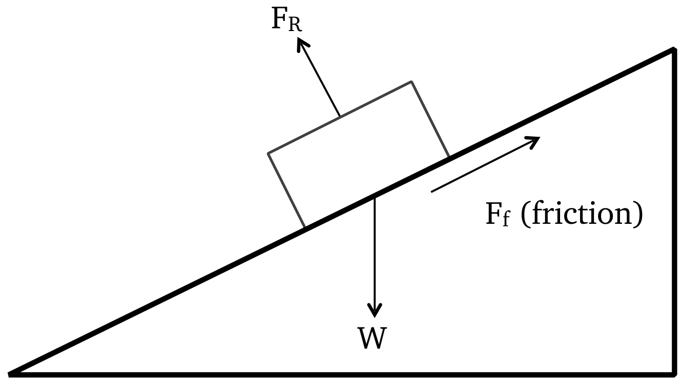
The component of weight which is parallel to the slope accelerates the object down the slope.



$W_{parallel} = mg\sin\theta$

$W_{perpendicular} = mg\cos\theta$

Real slope (with friction)



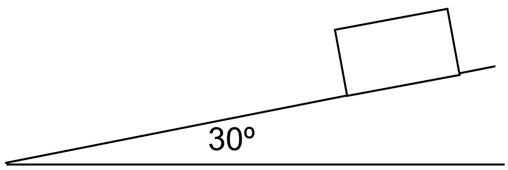
In reality there will be some friction between the object and the slope.

For the box to accelerate down the slope there must be an unbalanced force acting downwards, parallel to the slope.

$W_{parallel} = mg\sin\theta$

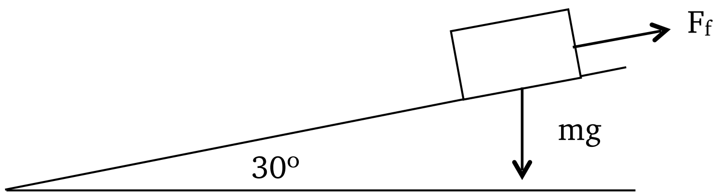
$W_{perpendicular} = mg\cos\theta$

Example 1: A 2.0kg block of wood is placed on a slope as shown.



The block remains stationary. What is the size and direction of the frictional force on the block?

Solution:



Object at rest — forces must be balanced

$F_f = W_{parallel}$

$F_f = mg\sin\theta$

$F_f = (2 \times 9.8) \sin 30$

$F_f = 9.8\text{N}$ – up the slope

Example 2: A car of mass 1,000kg is parked on a hill. The slope of the hill is 20° to the horizontal. The brakes on the car fail. The car runs down the hill for a distance of 75m until it crashes into a hedge. The average force of friction on the car as it runs down the hill is 250N.

a) Calculate the component of the weight acting down the slope.

b) Find the acceleration of the car.

c) Calculate the speed of the car just before it hits the hedge.

Conservation of Energy

One of the fundamental principles of Physics is that of conservation of energy.

Energy cannot be created or destroyed, only converted from one form to another.

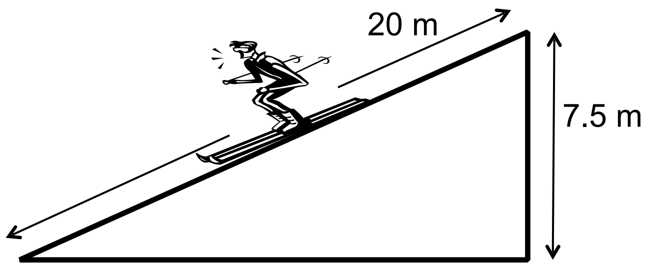
Work is done when converting from one form of energy to another. Power is a measure of the rate at which the energy is converted.

There are a number of equations for the different forms of energy:

$E_w = Fd$	Work Done
$E_k = ½mv^2$	Kinetic Energy
$E_p = mgh$	Gravitational Potential Energy
$E_h = cm\Delta T$	Heat Energy (with no phase change)
$E_h = ml$	Heat Energy (during phase change)
$E = Pt$	Energy, Power, time formula

All forms of energy can be converted into any other form, so each of these equations can be equated to any other.

Example: A skier of mass 60 kg slides from rest down a slope of length 20 m. The initial height of the skier was 7.5 m above the bottom of the ramp and the final speed of the skier at the bottom of the ramp was 13 m s^{-1}.



Calculate:

a) The work done against friction as the skier slides down the slope.

b) The average force of friction acting on the skier.

Collisions, Explosions and Impulse

Momentum

Momentum is the measure of an object's motion and is the product of mass and velocity.

$$p = mv$$

Momentum measured in kilogram metres per second (kg m s^{-1})

Mass measured in kilograms (kg)

Velocity measured in metres per second (m s^{-1})

As velocity is a vector, **so is momentum.** Therefore momentum has a direction and we must apply the convention of positive and negative directions.

An object can have a large momentum for two reasons, a large mass or a large velocity:

Newton's Third Law and Momentum

Consider a collision between two objects. We know that the forces acting on the two objects will be equal in magnitude but opposite in direction thanks to Newton's Third Law. The forces act on the objects for the same amount of time, the time of the collision.

So:

$F_1 = -F_2$

Therefore:

$m_1a_1 = -m_2a_2$ (from Newton's Second Law)

$m_1(v_1 - u_1)/t_1 = -m_2(v_2 - u_2)/t_2$

Since $t_1 = t_2$:

$m_1v_1 - m_1u_1 = -m_2v_2 + m_2u_2$

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

Total momentum before = Total momentum after

Conservation of Momentum

The law of conservation of linear momentum can be applied to the interaction (collision) of two objects, in the absence of net external forces. At Higher we will only consider collisions in one dimension.

total momentum before = total momentum after

Example: A trolley of mass 4.0kg is travelling with a speed of 3 m s^{-1}. The trolley collides with a stationary trolley of equal mass and they move off together. Calculate the velocity of the trolleys immediately after the collision.

Solution:

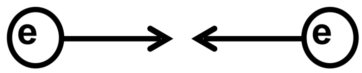
Kinetic Energy in Collisions

When two objects collide their momentum is always conserved but, depending on the type of collision, their kinetic energy may or may not be. Take the two examples below:

If you were to witness this car crash you would hear it happen. There would also be heat energy at the point of contact between the cars.

These two forms of energy will have come from the kinetic energy of the cars, converted during the collision.

Here, kinetic energy is **not** conserved as it is lost to sound and heat. This is an **inelastic** collision.



When these two electrons collide they will not actually come into contact with each other, as their electrostatic repulsion will keep them apart while they interact.

There is no mechanism here to convert their kinetic energy into another form and so it is **conserved** throughout the collision. This is an **elastic** collision.

Example: A car of mass 2,000kg is travelling at 15 m s^{-1}. Another car, of mass 1,500kg and travelling at 25 m s^{-1} collides with it head on. They lock together on impact and move off together.

a) Determine the speed and direction of the cars after the impact.

b) Is the collision elastic or inelastic? Justify your answer.

Explosions

In a simple explosion two objects start together at rest then move off in opposite directions. Momentum must still be conserved, as the total momentum before is zero, the total momentum after must also be zero.

Example: An early Stark Industries Jericho missile is launched vertically and when it reaches its maximum height it explodes into two individual warheads.

Both warheads have a mass of 1500kg and one moves off horizontally, with a velocity of 2.5 km s^{-1} (Mach 9) at a bearing of 090°.

Calculate the velocity of the other warhead.

Solution:

Interactions summary

Type of interaction	Momentum	Kinetic energy
Inelastic collision	Conserved	Not conserved - lost
Elastic collision	Conserved	Conserved
Explosion	conserved	Not conserved - gained

Impulse

Impulse is the force multiplied by time and is measured in Ns. Impulse is also equal to the change in momentum, measured in kg m s^{-1}. Impulse is calculated using vectors, so it is also a vector quantity. Applying the convention of positive and negative directions is very important. This means you can calculate the impulse using:

$$Ft = mv - mu$$

Impulse measured in kilogram metres per second (kg m s^{-1}) or Newton seconds (N s)

Momentum after

Momentum before

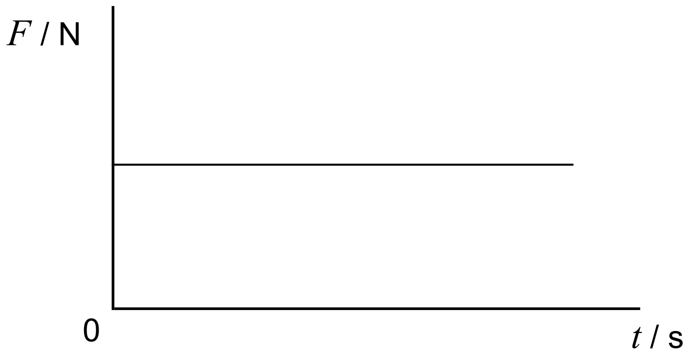
A change in momentum depends on:

- The size of the force
- The time the force acts

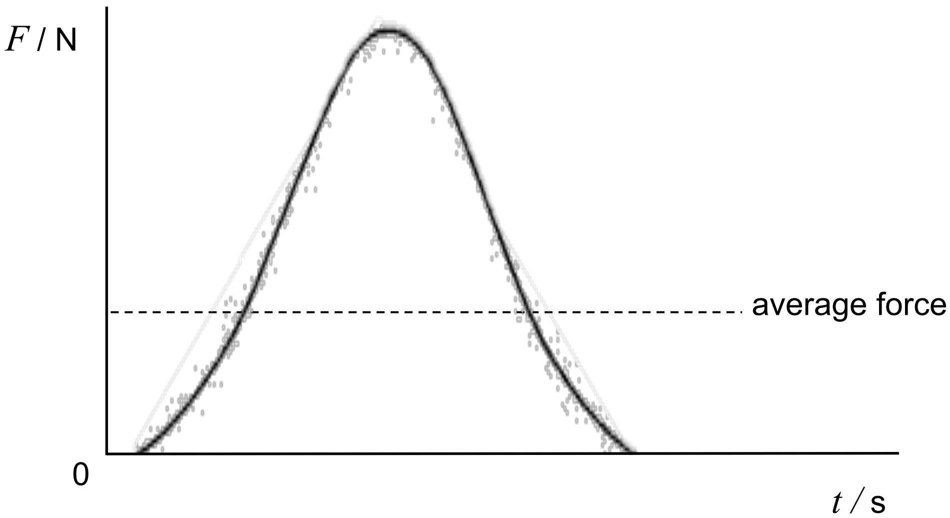
Example: A force of 100N is applied to a ball of mass 150g for a time of 0.020s. Calculate the final velocity of the ball.

Impulse Graphs

Impulse = Ft = area under F against t graph



In reality, the force applied is not usually constant.

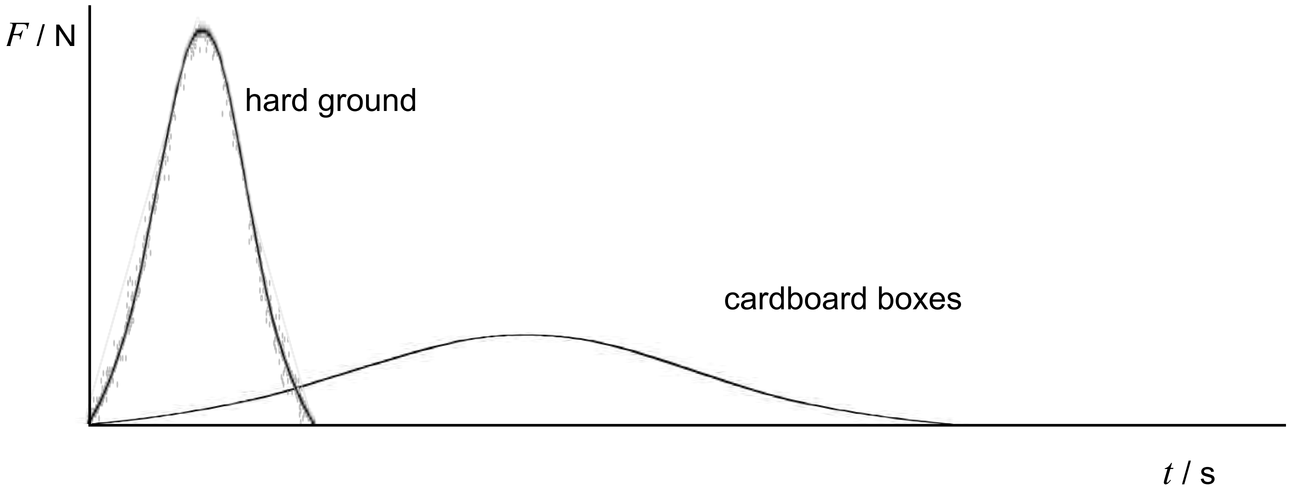


In May 2012, the first ever sky dive without a parachute was successfully attempted by stuntman, Gary Connery. He used a ‘wing-suit’ while in the air, which slows descent by allowing the wearer to glide, previously however, a parachute has always been deployed before landing.

He jumped out of a plane at 2,400ft (730m) and reached a terminal velocity of 80mph.

To allow him to safely survive the drop he landed on a 100m ‘runway’ of 18,600 cardboard boxes.

The impulse graphs below show the difference between landing on the boxes and landing on hard ground.



In both scenarios the change in momentum must be the same as the skydiver must go from 80mph to 0mph. This means that the area under the two curves must be equal.

The key difference is the period of time the impact happens over. This therefore affects the force of the impact. When a hard surface brings something to a stop it applies a large force over a very small period of time.

When a softer or more flexible surface is used, it can apply a smaller average force spread out over a much longer period of time, as the material deforms. This produces the same overall change in momentum but applies a much less damaging force to the object, in this case a person.

Gravitation

Projectiles

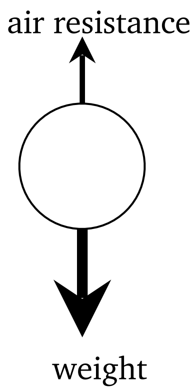
A projectile is any object, which, once projected, continues its motion by its own inertia and is influenced only by the downward force of gravity.

There are many examples of this condition: any object thrown vertically upwards; dropped vertically downwards or thrown at any angle through the air.

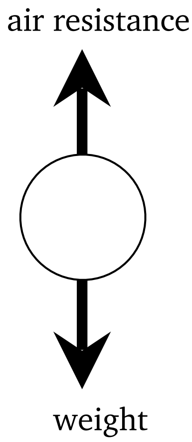
Free fall

In reality, when objects move through the air they have more than a single force acting on them.

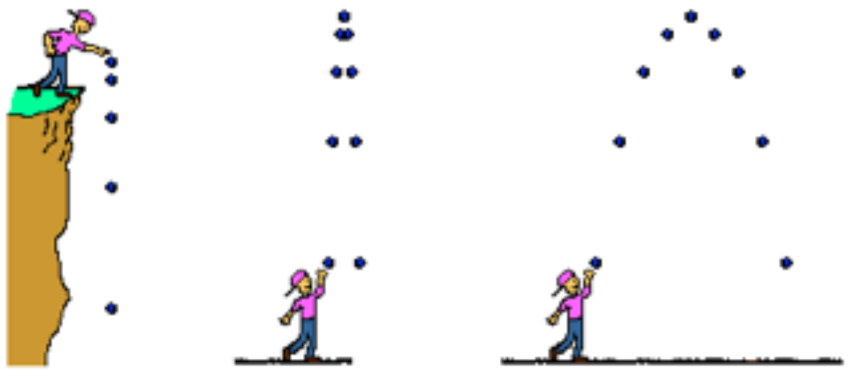
When an object is allowed to fall towards the Earth it will accelerate because of the force acting on it due to gravity, its weight. This will not be the only force acting on it though. There will be an upwards force due to air resistance. Air resistance increases with speed; you may notice this if you increase your speed when cycling.



If an object is allowed to fall through a large enough distance then the air resistance force may increase to become the same magnitude as weight of the object. The forces are now balanced and the object will fall with constant velocity, known as terminal velocity.

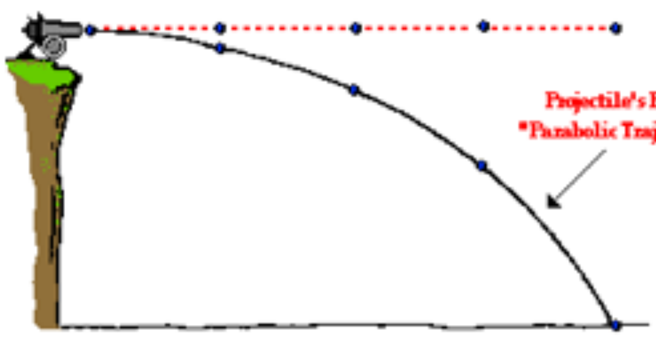


Most projectiles have both horizontal and vertical components of motion. As there is only a single force, gravity, acting in a single direction, only one of the components is being acted upon by the force. The two components are not undergoing the same kind of motion and must be treated separately.

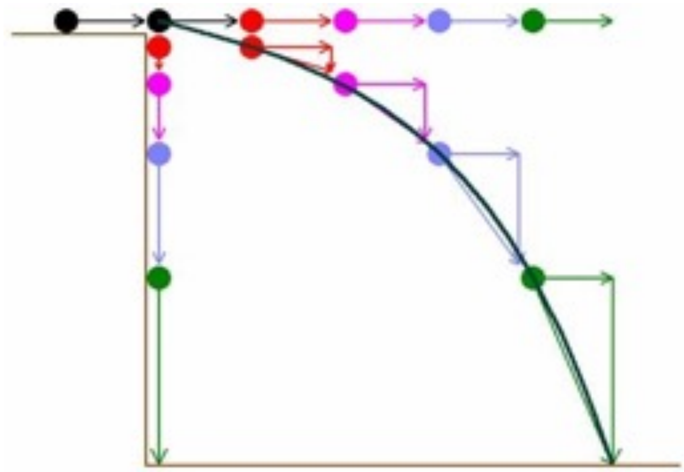


Projectiles fired horizontally

Here is a classic horizontal projectile scenario, from the time of Newton. In projectile motion we ignore all air resistance, or any force other than gravity.



Analysis of this projectile shows the two different components of motion.



Horizontally: There are no forces acting on the cannonball and therefore the horizontal velocity is constant.

Vertically: The force due to gravity is constant in the vertical plane and so the cannonball undergoes constant acceleration.

The combination of these two motions causes the curved path of a projectile.

Direction of motion	Forces	Velocity	Acceleration
Horizontal	Air resistance is negligible so no forces	Constant	Zero
Vertical	Air resistance is negligible so only the force of gravity	Changing at a constant rate	Constant or uniform acceleration of $9{\cdot}8\ \text{m s}^{-1}$

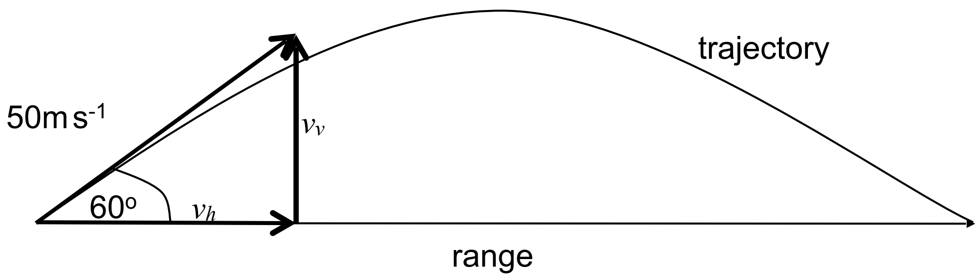
Example: The cannonball is projected horizontally from the cliff with a velocity of 100 $m\,s^{-1}$. The cliff is 20m high. Determine:

a) The vertical speed of the cannonball, just before it hits the water;

b) If the cannonball will hit a ship that is 200 m from the base of the cliff.

Projectiles at an Angle

Projectiles at an angle are an application for our knowledge of splitting vectors into their horizontal and vertical components.



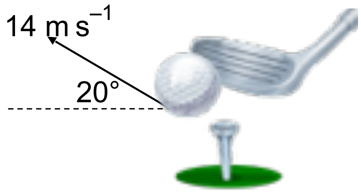
There is still only the single force of gravity acting on the projectile, so horizontal and vertical motions must still be treated very separately. This means that the velocity at an angle must be split into its vertical and horizontal components before any further consideration of the projectile.

You will never use the velocity at an angle (here 50 m s^{-1}) directly in any calculation!

For projectiles fired at an angle above a horizontal surface:

- The path of the projectile is symmetrical, in the horizontal plane, about the highest point.
- The time of flight = 2 × the time to highest point.
- The vertical velocity at the highest point is zero.

Example: A golfer hits a stationary ball and it leaves his club with a velocity of 14 m s–1 at an angle of 20° above the horizontal.



Calculate:

The horizontal component of the velocity of the ball.

The vertical component of the velocity of the ball.

The time for the ball to reach its maximum height.

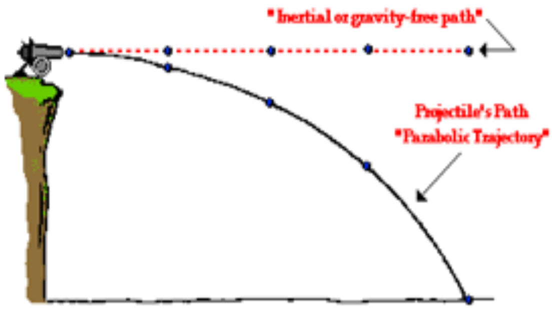
The total time of flight of the ball.

How far down the fairway does the ball lands

Orbits and Newton's Thought Experiment

Newton died in 1727, 230 years before the launch of Sputnik 1, the first man-made object to orbit Earth, in 1957. However, like all good Physicists, he did have a great imagination and conducted thought experiments. Einstein was also well known for conducting thought experiments on scenarios which, at the time, could not be empirically tested.

Newton considered the example of the cannon firing horizontally off a cliff. He knew that, as the Earth is a sphere, the ground curves away from the projectile as it falls.



If we give the projectile a greater horizontal velocity, it will travel a greater distance before reaching the ground. If that ground is also curving down and away from the projectile, it would take even longer for the projectile to land.

Newton knew that there must be a horizontal launch velocity you could give a projectile which meant the Earth would curve away from the projectile at the same rate that gravity accelerated it towards the ground. This is known as **escape velocity**. At this velocity the object will never return to Earth without the presence of an external force. Escape velocity on Earth is roughly 7.8 km s^{-1}.

As of May 2012, we have 544 satellites in orbit around Earth and one space station. Their orbits are at different radii, different orbital periods and different velocities — these are all linked which you can study at Advanced Higher (or in Kerbal Space Program)

Satellites in Low Earth Orbit (LEO), including the ISS, have a period of approximately 90 minutes. Many communication satellites, for telecommunications and television, are in **geostationary orbit**. This orbit is at a greater radius, with a period of 24 hours. This allows the satellites to stay above the same point on Earth at all times and provide consistent communication across the globe.

Though high orbits have been revolutionary for achieving successful global communications, they do have their drawbacks. They can lead to early failure of electronic components as they are not protected by the Earth's magnetic field and are exposed to very high levels of solar radiation and charge build-up. They also require a great deal of energy to achieve the altitudes required and very powerful amplifiers to ensure successful transmission back to Earth.

To avoid these orbits many organisations choose to use a 'constellation' of satellites in LEO These have their own issues, as gases from the upper atmosphere cause drag which can degrade the orbit.

Newton's Universal Law of Gravitation

Newton's Law of gravitation states that the gravitational attraction between two objects is directly proportional to the mass of each object and is inversely proportional to the square of their distance apart. Gravitational force is always attractive, unlike electrostatic or magnetic forces.

Universal constant of gravitation = $6{\cdot}67\times10^{-11}\,m^3kg^{-1}s^{-2}$.

$$F = \frac{Gm_1m_2}{r^2}$$

Force in newtons (N)

Distance between m_1 and m_2 in metres (m)

Example: Consider a folder, of mass 0.3kg and a pen, of mass 0.05kg, sitting on a desk, 0.25m apart. Calculate the magnitude of the gravitational force between the two masses. Assume they can be approximated to spherical objects.

We do not notice the gravitational force between everyday objects because it is so small, in fact it is the weakest of the four fundamental forces of our universe. This is just as well, or you would have to fight against gravity every time you walk past a large building! Gravity only becomes really apparent when very large masses are involved, like planets.

Gravitational Field Strength

A gravitational field is a region where gravitational forces exist. The gravitational field strength is the force on a 1kg mass placed in the field.

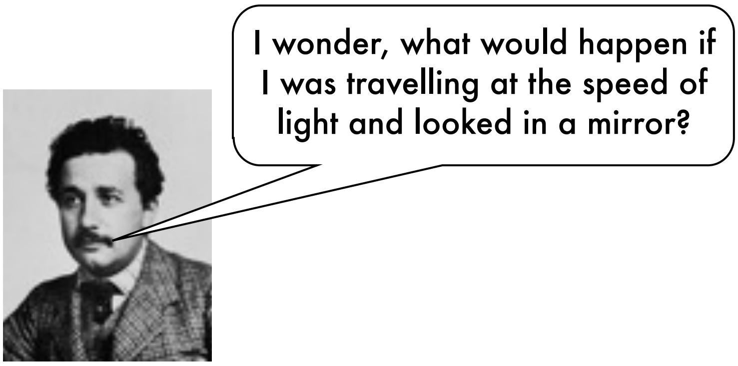
On Earth the gravitational field strength is the force of attraction between 1kg of mass and the Earth. Remember the law of gravitation is a force of attraction between two masses, this means the 1kg mass is attracting the Earth towards it, as well as the other way round.

Example 1: Show, using the universal law of gravitation, that the gravitational field strength on Earth is $9{\cdot}8Nkg^{-1}$. Hint: Put $m_1 = 5.97219 \times 10^{24}$ and $m_2 = 1kg$.

Example 2: Calculate the gravitational field strength on the surface of the Moon and calculate the gravitational force between the Moon and the Earth.

Special Relativity

Einstein originally proposed his theory of special relativity in 1905 and it is often taken as the beginning of modern Physics. It was one of four world changing theories published by Einstein that year, known as the a*nnus mirabilis* (miracle year). Einstein was 26.



Relativity has allowed us to examine the mechanics of the universe far beyond that of Newtonian mechanics, especially the more extreme phenomena such as black holes, dark matter and the expansion of the universe, where the usual laws of motion and gravity appear to break down.

Special relativity was the first theory of relativity Einstein proposed. It was termed as ‘special’ as it only considers the ‘special’ case — reference frames moving at constant speed. Later he developed the theory of general relativity which considers accelerating frames of reference.

Reference Frames

Relativity is all about observing events and measuring physical quantities, such as distance and time, from different reference frames. Here is an example of the same event seen by three different observers, each in their own frame of reference:

Event 1: You are reading your Kindle on the train. The train is travelling at 60 mph.

Observer	Location	Observation
1	Passenger sitting next to you	You are stationary
2	Person standing on the platform	You are travelling towards them at 60 mph
3	Passenger on train travelling at 60 mph in opposite direction	You are travelling towards them at 120 mph.

This example works well as it only involves objects travelling at relatively low speeds. The comparison between reference frames does not work that same, however, if objects are moving close the speed of light.

Event 2: You are reading your Kindle on an interstellar train. The train is travelling at 2×10^{8} m s^{-1}.

Observer	Location	Observation
1	Passenger sitting next to you	You are stationary
2	Person standing on the platform	You are travelling towards them at 2×10^{8} m s^{-1}
3	Passenger on train travelling at 2×10^{8} m s^{-1} in opposite direction	You are travelling towards them at 4×10^{8} m s^{-1}

This scenario is **impossible**[1] from the viewpoint of observer three as **you cannot observe any object travelling faster than light from any reference frame.**

[1] Observer 3 would actually see you travelling at 0.8c towards them

The Principles of Relativity

Using his imagination and performing thought experiments (or 'gedanken'), Einstein came up with two principles, or postulates, to explain the problem of fast moving reference frames. These were later proved with a vast array of data from many different experiments.

1. When two observers are moving at constant speeds relative to one another, they will observe the same laws of physics.

2. The speed of light (in a vacuum) is the same for all observers.

This means that no matter how fast you go, you can never catch up with a beam of light, since it always travels at 3×10^8 m s^{-1} relative to you.

If you (or anything made of matter) were able to travel as fast as light, light would still move away or towards you at 3×10^8 m s^{-1}, as you are stationary in your own reference frame.

The most well-known experimental proof is the Michelson-Morley interferometer experiment. Maxwell's electromagnetism equations also corroborated this postulate.

Example: If a car ship is travelling through space at 90% of the speed of light and then switches on its headlights. The passenger of the car will see the beams of the headlights travel away from them at 3×10^8 m s^{-1}.

An observer on Earth will also observe light of the beams travel at 3×10^8 m s^{-1}. Because the speed of light, c, is constant in and between all reference frames and for all observers.

These principles have strange consequences for the measurement of distance and time between reference frames.

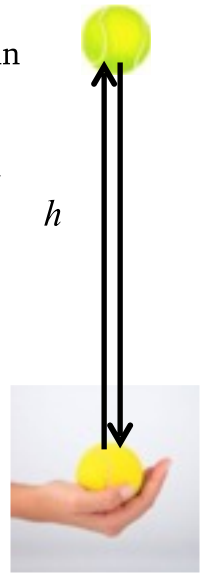
Time Dilation

We can conduct a thought experiment of our own, showing that one consequence of the speed of light being the same for all observers is that time experienced by all observers is not necessarily the same. There is no universal clock that we can all refer to — we can simply make measurements of time as we experience it.

Time is different for observers in different reference frames because the path they observe for a moving object is different.

Example 1

Event: Inside a moving train carriage, a tennis ball is thrown straight up and caught in the same hand.



Observer 1, standing in train carriage, throws tennis ball straight up and catches it in the same hand. In Observer 1's reference frame they are stationary and the ball has gone straight up and down. Observer 1 sees the ball travel a total distance of 2h.

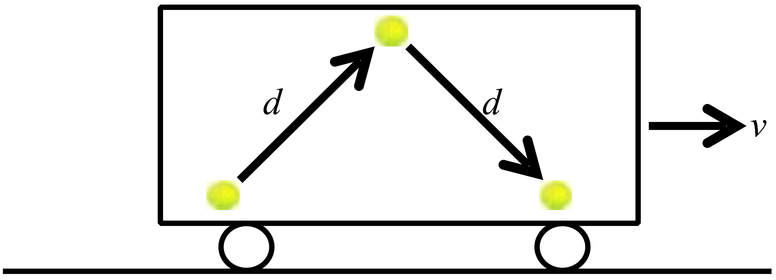
The ball is travelling at a speed s.

The period of time for the ball to return to the observers hand is:

$$t=d/v$$

$$t=s/2h$$

Observer 2, standing on the platform watches the train go past at a speed, v, and sees the passenger throw the ball. However, to them, the passenger is also travelling horizontally, at speed v. This means that, to Observer 2, the tennis ball has travelled a horizontal distance, as well as a vertical one.



Observer 2 sees the ball travel a total distance of 2d (note that d>h).

The period of time for the ball to return to the observers hand is:

$$t=s/2d$$

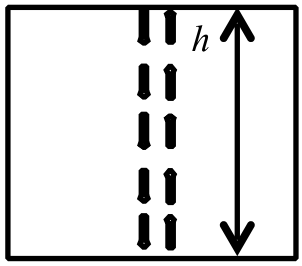
For observer 2, the ball has travelled a greater distance, in the same time.

Example 2:

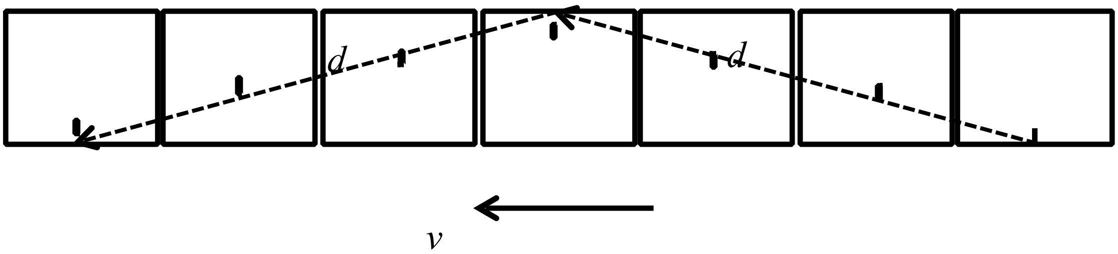
Event: You are in a spaceship travelling to the left, at speed v. Inside the spaceship cabin, a pulsed laser beam is pointed vertically up at the ceiling and is reflected back down. The laser emits another pulse when the reflected pulse is detected by a photodiode.

Reference frame 1: You, inside the cabin. The beam goes straight up, reflects of the ceiling and travels straight down.

Period of pulse = c/2h



Reference frame 2: Observer on another, stationary ship.



Period of pulse = c/2d

The time for the experiment as observed by the stationary ship ,t' (pronounced t prime), is greater than the time observed by you when moving with the photodiode, t.

t'>t

What you might observe as taking 1 second could appear to take 2 seconds to your stationary colleague. Note that you would be unaware of any difference until you were able to meet up with your colleague again and compare your data.

Time Dilation Formula and the Lorentz Transformation

$$t' = \frac{t}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

Note this is often written as:

$$t' = \gamma t$$

Where γ is known as the Lorentz Factor. It is used often in the study of special relativity and is given by:

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{v^2}{c^2}\right)}}$$

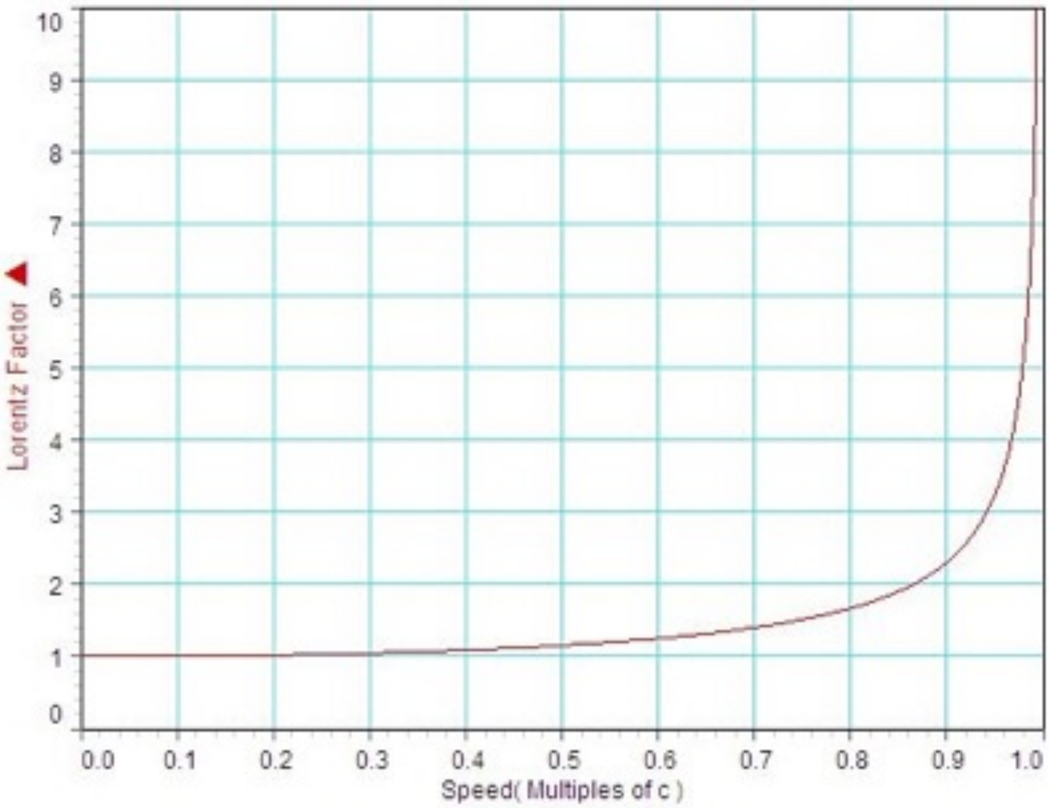
Note: $\frac{v^2}{c^2}$ is the same as $\left(\frac{v}{c}\right)^2$, which can make the calculation simpler, particularly if the speed is given in the form of a fraction of the speed of light, e.g. the speed in the above example could have been given as 0.9 times or 9/10 of the speed of light.

Example: A rocket is travelling at a constant 2.7×10^8 m s^{-1} compared to an observer on Earth. The pilot measures the journey as taking 240 minutes.

How long did the journey take when measured from Earth?

Why we do not notice relativistic time differences in everyday life?

A graph of the Lorentz factor versus speed (measured as a multiple of the speed of light) is shown below.



We can see that for small speeds (i.e. less than 0.1 times the speed of light) the Lorentz factor is approximately 1 and relativistic effects are negligibly small. Even 0.1 times the speed of light is 300,000 m s^{-1} or about 675,000mph — a tremendously fast speed compared to everyday life.

However, the speed of satellites is fast enough that even these small changes will add up over time and seriously affect the synchronisation of global positioning systems (GPS) and television satellites with users on the Earth. They have to be specially programmed to adapt for the effects of special relativity (and also general relativity, which is not covered here). Very precise measurements of these small changes in time have been performed on fast-flying aircraft and agree with predicted results within experimental error.

Further evidence in support of special relativity comes from the field of particle physics, in the form of the detection of a particle called a muon at the surface of the Earth. Muons are produced in the upper layers of the atmosphere by cosmic rays (high-energy protons from space). The speed of muons high in the atmosphere is 99·9653% of the speed of light. The half-life of muons when measured in a laboratory is about 2·2 µs.

Show, by calculation, why time dilation is necessary to explain the observation of muons at the surface of the Earth.

Solution:

$t = 2.2\mu s = 2.2\times10^{-6}s$

$v = 0.999653\times3.00\times10^{8} = 2.998956\times10^{8}\ m\ s^{-1}$

$d = ?$

$d = vt$

$d = 2.998956\times10^{8}\times2.2\times10^{-6}$

$d = 660m$

This is nowhere near far enough for a muon to get from the upper atmosphere to the Earth's surface! However we cannot use t as the muon is travelling close to the speed of light — we need to consider time dilation.

$$t' = \frac{t}{\sqrt{1-\left(v^2/c^2\right)}}$$

$$t' = \frac{2.2\times10^{6}}{\sqrt{1-0.999653^2}}$$

$t' = 84\ \mu s$

$d' = vt'$

$d' = 2.998956\times10^{8}\times84\times10^{-6}$

$d' = 2.52\times10^{4}m$

In the reference frame of an observer on Earth the half-life of the muon is recorded as $84\mu s$ and therefore from this perspective, the muon has enough time to travel the many kilometres to the Earth's surface.

Length Contraction

Another implication of Einstein's theory is the shortening of length when an object is moving. Consider the muons discussed above. Their large speed means they experience a longer half-life due to time dilation. An equivalent way of thinking about this is that the muons experience the height of the atmosphere as smaller (or contracted) by the same amount as the time has increased (or dilated). A symmetrical formula for time dilation can be derived. Note that the contraction only takes place in the direction that the object is travelling:

$$l' = l\sqrt{1-\left({}^{v}\!/\!{}_{c}\right)^{2}}$$

Where l is the length as measured by the moving observer and l' is that measured by the stationary observer.

Length Paradox

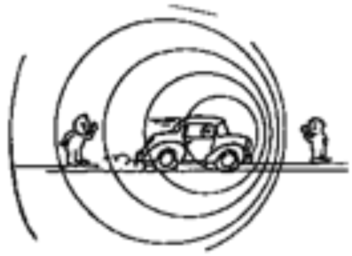
There is an apparent paradox thrown up by special relativity. Consider a train that is just longer than a tunnel. If the train travels at high speed through the tunnel does length contraction mean that, from our stationary perspective, it fits inside the tunnel? How can this be reconciled with the fact that from the train's reference frame the tunnel appears even shorter as it moves towards the train? The key to this question is simultaneity, i.e. whether different reference frames can agree on the exact time of particular events. In order for the train to fit in the tunnel the front of the train must be inside at the same time as the back of the train. Due to time dilation, the stationary observer (you) and a moving observer on the train cannot agree on when the front of the train reaches the far end of the tunnel or the rear of the train reaches the entrance of the tunnel. If you work out the equations carefully then you can show that even when the train is contracted, the front of the train and the back of the train will not both be inside the tunnel at the same time!

The Doppler Effect

The Doppler Effect is the change in the observed frequency of a wave, when the source or observer is moving.

In this course we will concentrate on a wave source moving at constant speed relative to a stationary observer.

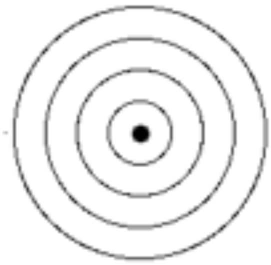
You have already experienced the Doppler Effect many times. The most noticeable is when a police car, ambulance or fire engine passes you. You hear the pitch of their siren increase as they come towards you and then decrease as they move away. Another memorable example is the sound of a very fast moving vehicle, such as a Formula 1 car passing you (or passing a microphone on the television), the sound of the engine rises and falls in frequency as it approaches, passes and moves away.



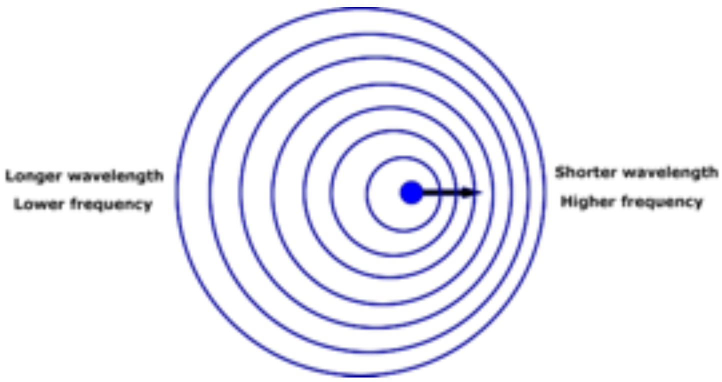
The Doppler Effect applies to all waves, including light.

What is the Doppler Effect?

A stationary sound source produces sound waves at a constant frequency f, and the wavefronts propagate symmetrically away from the source at a constant speed, which is the speed of sound in the medium. The distance between wave-fronts is the wavelength. All observers will hear the same frequency, which will be equal to the actual frequency of the source: $f = f_0$.



The sound source now moves to the right with a speed vs. The wavefronts are produced with the same frequency as before, therefore the period of each wave is the same as before. However, in the time taken for the production of each new wave the source has moved some distance to the right. This means that the wavefronts on the left are created further apart and the wavefronts on the right are created closer together. This leads to the spreading out and bunching up of waves you can see above and hence the change in frequency.



The Doppler Effect is applied in many different fields:

- Police speed guns send out a light wave (radar) and measure the Doppler shift of the reflected wave to measure the speed of an approaching car.
- An echocardiogram uses the Doppler Effect to measure the velocity of blood flow and cardiac tissue and is one of the most widely used diagnostic tests in cardiology.

More relevant to our learning in this section, the Doppler Effect is highly prominent in our observations of the universe and provides some of the strongest evidence for major theories such as the Big Bang and an expanding universe.

For a stationary observer with a moving light source, the relationship between the original frequency, f, of the source and the observed frequency, f_s is:

$$f = f_s\left(\frac{v}{v \pm v_s}\right)$$

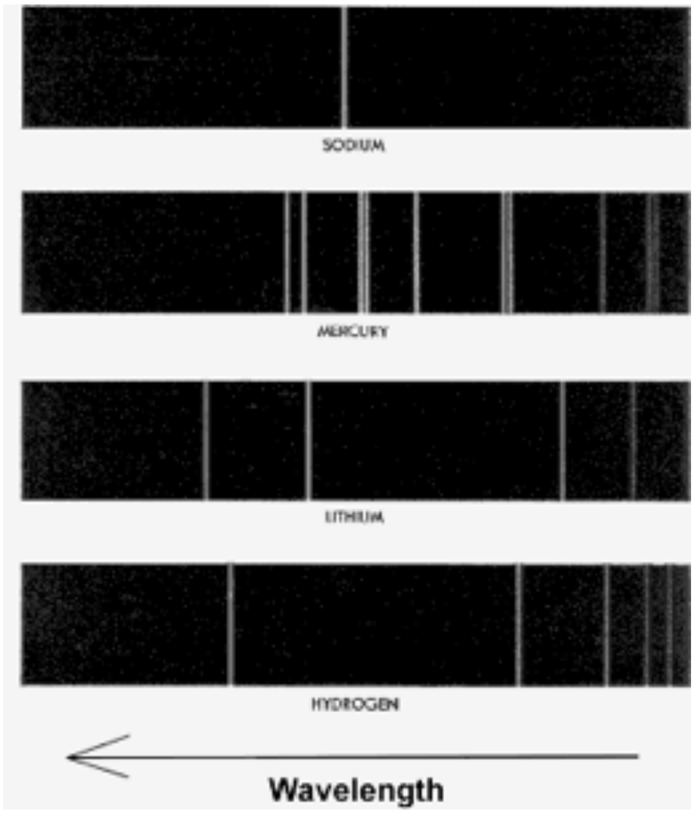
When the source is moving **away** from the observer you **add** the velocity of the source. This will cause the frequency to **decrease** or 'red shift'.

When the source is moving **towards** the observer you **subtract** the velocity of the source. This will cause the frequency to **increase** or 'blue shift'.

Red shift is exactly what is observed when we look at the light from distant stars, galaxies and supernovae. These relationships also allow us to calculate the speed at which an exoplanet is orbiting its parent star, or the velocity of stars orbiting a galactic core, which has lead us to theorise the existence of dark matter.

Redshift

Redshift is an example of the Doppler Effect. It is the term given to the change in frequency of the light emitted by stars, as observed from Earth, due to the stars moving away from us.

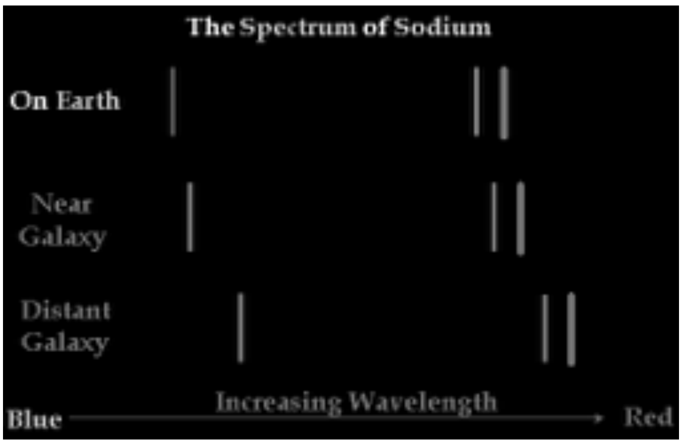


Redshift has always been present in the light reaching us from stars and galaxies but it was first noticed by astronomer Edwin Hubble, in the 1920's, when he observed that the light from distant galaxies was shifted to the red end of the spectrum.

The light emitted by a star is made up of the line spectra emitted by the different elements present in that star. Each of these line spectra is an identifying signature for an element and these spectra are constant throughout the universe. You will learn a lot more about spectra in the Particles and Waves unit of this course.

Since these line spectra are so recognisable, we can compare the spectra produced by these elements, on Earth, with the spectra emitted by a distant star or galaxy.

Hubble examined the spectral lines from various elements and found that each galaxy was shifted towards the red by a specific amount. This shift was due to the galaxy moving away from the Earth at speed, causing the Doppler Effect to be observed. The bigger the magnitude of the shift the faster the galaxy was moving.



Redshift Formula

Redshift, z, of a galaxy is given by:

$$z = \frac{\lambda_{observed} - \lambda_{rest}}{\lambda_{rest}}$$

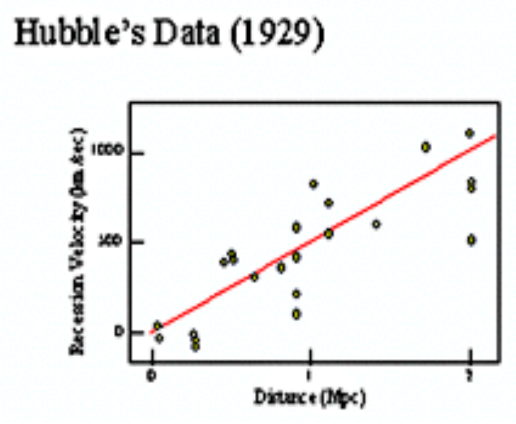
Redshift of galaxies, travelling at non-relativistic speeds, can also be shown to be the ratio of the velocity of the galaxy to the velocity of light:

$$z = \frac{v}{c}$$

Over the course of a few years Hubble examined the red shift of galaxies at varying distances from the Earth. He found that the further away a galaxy was the faster it was travelling away from us. The relationship between distance and speed of a galaxy is known as Hubble's Law.

Hubble's Law

The graph below shows the data collected by Hubble. It shows the relationship between the velocity of a galaxy, as it recedes from us, and its distance, known as Hubble's Law.



x = distance, d, in mega parsecs[1] (Mpc)

y = recession velocity, v, in km s^{-1}

The gradient is known as Hubble's constant, H_0:

$$v = H_0 d$$

The value of the Hubble constant is not known exactly, as the exact gradient of the line of best fit is subject to much debate. However, as more accurate measurements are made, especially for the distances to observable galaxies, the range of possible values has reduced. It is currently thought to lie between 50 to 80 km s^{-1} Mpc^{-1}, with the most recent[2] data putting it at 67.80 ± 0.77 km s^{-1} Mpc^{-1}.

[1] 1 Parsec = 3.262 light years = 3.086×10^{16}m

[2] Planck Mission results — published March 2013

Evidence for the Expanding Universe

Hubble's observations show that galaxies are moving away from the Earth and each other in all directions, which suggests that the universe is expanding. This means that in the past the galaxies were closer to each other than they are today. By working back in time it is possible to calculate a time when all the galaxies were at the same point in space. This allows the age of the universe to be calculated.

Example:

v = speed of galaxy receding from us

d = distance of galaxy from us

H_0 = Hubble's constant — here converted into s^{-1}

t = time taken for galaxy to reach that distance, i.e. the age of the universe

$v = H_0 d$

$$H_0 = \frac{v}{d}$$

$$\frac{1}{H_0} = \frac{d}{v}$$

$$\frac{d}{v} = t \therefore$$

$$\frac{1}{H_0} = t$$

$t = 1 \div 2\times10^{-18}$

$t = 5\times10^{17}$s

t = 15.9 billion years

Since Hubble's time, there have been other major breakthroughs in astronomy. All of these support the findings of Hubble, but allow the age of the universe to be calculated even more accurately, such as the discovery of the Cosmic Microwave Background (CMB). Observations of the CMB also support the theory that the universe is expanding out from a single point, as Hubble postulated.

The current best estimate for the age of the universe[1] is 13.798±0.037 billion years.

[1] Again from the Planck Mission results in 2013

Accelerating Universe

We have also recently discovered, that not only is the universe expanding but it is expanding at an *increasing* rate, i.e. the acceleration is increasing. This was the conclusion of astronomers in 1998 when observing distant supernovae. Their discovery was a great shock to the scientific community and was awarded the Nobel Prize in Physics in 2011.

Why such a shock?

The force of gravity acts between all matter in the universe. Matter clumps together due to gravity, such as the contraction of hydrogen gas to create new stars, the grouping of stars to create galaxies and the grouping of galaxies to create local groups and superclusters. The Hubble telescope has been able to give us a glimpse of the universe on an even larger scale and images of many of the observable galactic clusters show them gravitating towards each other to form unimaginably large structures, known as filaments.

Gravity should be an unbalanced force acting to slow the expansion down. A universe like this, which eventually collapses back in on itself is known as a closed universe. The force of gravity, determined by the mass of the universe, eventually overcomes any expansion and all matter accelerates back towards a central point.

Hubble's Law and subsequent observations, however, shows that this is not happening. The rate of expansion of the universe is increasing. This suggests that there is a force acting against the force of gravity, pushing matter apart. This force is causing a significant acceleration and so it is much greater in magnitude than gravity. As yet, astronomers and cosmologists have not been able to determine a source of energy capable of producing this force. For lack of a better term it is, for now, simply referred to as **dark energy**.

A universe which does not collapse in due to gravity but continues to expand indefinitely is called an open universe. This type of universe would undergo an end known as 'heat death'. This refers to the fact that, eventually, all energy becomes heat energy and as time goes on all matter would be so far apart that the heat energy of the universe would be spread too far apart to allow any further production of stars and galaxies.

As the magnitude of the force of gravity in the universe is dependent on the mass within that universe, it is mass which determines whether a universe is open or closed and therefore the eventual fate of that universe:

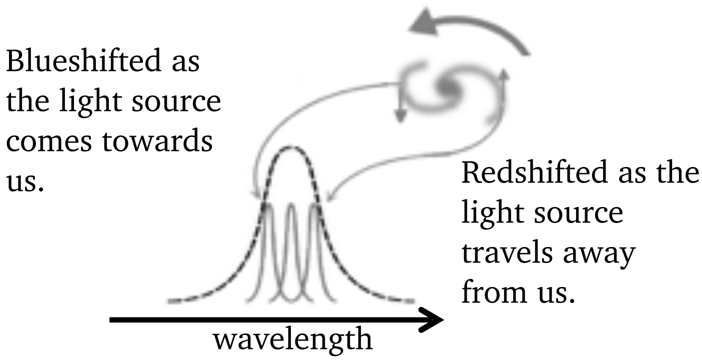
- If a universe has enough mass then the force of gravity will be greater than that produced by dark energy — expansion will decrease and the universe will be closed.

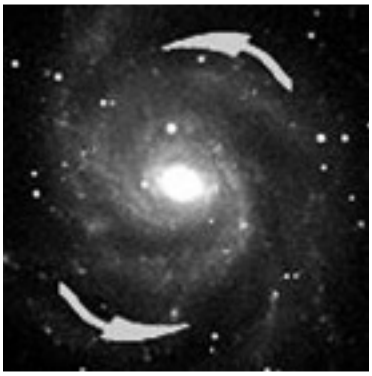
- If a universe does not have enough mass then the force produced by dark energy will be greater than gravity — expansion will continue indefinitely and the universe will be open.

Dark Matter

Observations of the Doppler Effect being evident in light observed from space has led to the development of another, equally perplexing theory — dark matter. This one comes from observing the light from individual stars within galaxies.

We know that galaxies rotate about their cores as the light observed on one side of a galaxy will be blue shifted, indicating that the source of that light is moving towards us, in comparison to the other side which will be redshifted, by the same amount.





Measuring the amount which the light is shifted by allows us to calculate the exact rotational velocity of that galaxy and thus the velocity of the stars within it.

From the section on gravitation you know that orbits are a careful balance between the gravitational field strength, created by a mass, and the velocity of a projectile.

If you give an object a great enough velocity it can escape the gravitational field it is in and escape from the orbit. An example of the same kind of action, occurring on Earth, is being on a roundabout in a park.

When on a roundabout you must hold on to stay on. If you did not provide this force your body would continue in a straight line and you would come of at a tangent to the circle. In the case of a roundabout, the force causing you to go in a circle is a force of friction. As the roundabout spins faster, so do you and you must provide a larger and larger force, by holding on more tightly. If you cannot provide a force big enough you will come off.

It is exactly the same with an orbit in space. Only here, the force causing the object to continue in a circle is gravity. As the magnitude of this force is determined by the mass of the planet, star, or galaxy, it is fixed and cannot increase. If the object travels too fast, the force of gravity will not be able to keep it on a circular path and it will escape its orbit and travel off in a straight line.

If we know:

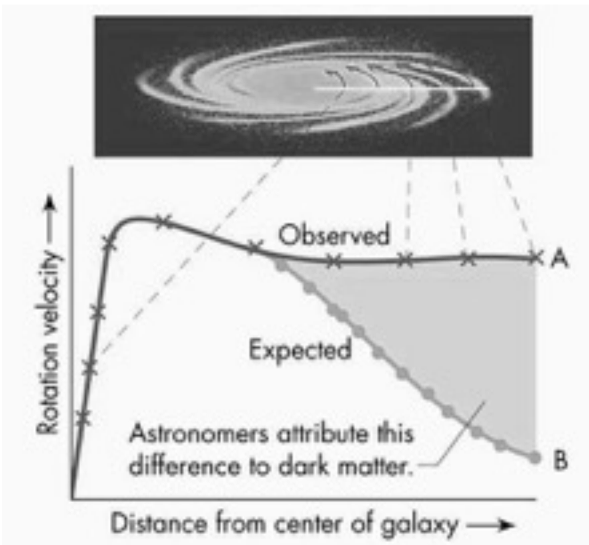
- The mass of the orbiting object, m
- The velocity of the orbiting object, v
- The radius of its orbit, r

Then we can calculate the mass of the central body, M, using the Universal Law of Gravitation.

Stars on the on the outer arms of the galaxy should travel slower than those towards the galactic core as they are further from the central mass and therefore experience a smaller gravitational force. We can directly observe the distribution of matter, and therefore mass, within a galaxy from its brightness. Matter in a galaxy is either producing light (stars) or reflecting it (nebulae and dust).

What do we observe?

The stars are travelling too fast.



As you can see from the graph above, the velocity of stars does not drop off as expected, at greater orbital radii. At these high velocities the observed mass of the galaxy should not be enough to hold on to many of its stars and we should see them fly off into intergalactic space.

The only logical conclusion that astronomers have to explain this consistent observation is that there must be a significant amount of mass that we cannot see. Hence the name — dark matter.

THE BIG BANG THEORY[1]

The Big Bang Theory postulates that the universe began with a single burst of energy. The early universe was small and incredibly hot. As the universe expanded it cooled and the energy condensed into matter which gradually formed atoms and then more and more complex structures. It took only moments for the first hydrogen and helium nuclei to form from protons and neutrons, but thousands of years for electrons to bind to them to form neutral atoms.

Discoveries in astronomy and physics have shown beyond a reasonable doubt that our universe did in fact have a beginning. Prior to that moment there was nothing. During and after that moment there was something — our universe.

Though science is still unsure how or why this happened we do know what happened next. From 10^{-43} seconds after the Big Bang (the Planck Epoch) we know that the universe had a massive density (close to infinity), was expanding rapidly and all the fundamental forces acted as one. We know relatively little about this early stage of the universe's life and virtually nothing about what the universe was like before this.

By the time that the universe was 10^{-12} seconds old the four fundamental forces(electromagnetism, gravity and the strong and weak nuclear forces) have separated. The universe is filled with an extremely hot and dense quark-gluon plasma. The universe doesn't get cool enough for protons and neutrons to form until 1 second after the Big Bang. It is thought that neutrinos came into existance around this time as well. After about 10 seconds electrons start to appear in the universe.

When the universe is about 3 minutes old it is cool enough for the protons and neutrons to form into nuclei. This nuclear fusion lasts for about 17 minutes, producing a universe consisting of about 75% Hydrogen, 25% Helium with traces of a few heavier elements such as Lithium and Beryllium. Atoms still cannot form however, thanks to the vast numbers of high energy photons.

Atoms start to appear after the universe is about 377,000 years old but it isn't until the universe is 150 million years old that the first stars start to form. 8 billion years after the Big Bang the Milky Way galaxy was formed and a billion years later (4.6 billion years ago). Our own Solar System collapsed, forming the Sun. The dust and gas around the Sun would eventually form the planets, including our own.

[1] The actual theory — not the TV show

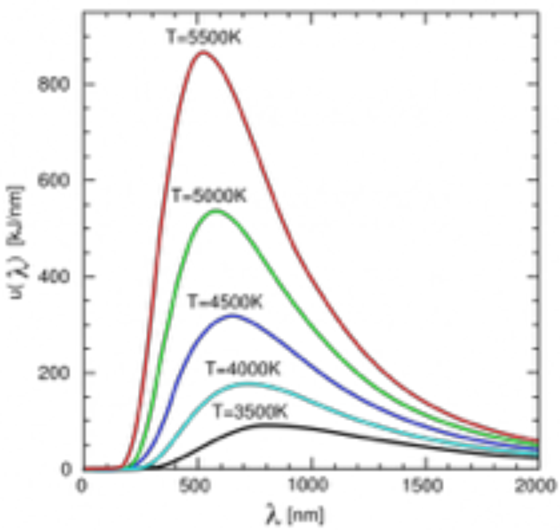
Temperature of Stellar Objects

When you look into the night sky you will see the familiar sight of white pinpoints of light. If you look a little closer, however, you will see that many of them have a colour. The colour of a star tells us the surface temperature of that star, in the same way we can tell the temperature of a flame by its colour: red is a relatively cool flame and blue is very hot.

Star	Location	Colour	Temperature / K
Betelgeuse	Orion	Red	3,500
Pollux	Gemini	Yellow (like Sun)	5,000
Rigel	Brightest star in Orion	Blue	12,130

This overall colour can, of course, be split into the spectrum of the star. Astronomers observe stars through filters to record their brightness in the different wavelengths of their spectrum.

The distribution of energy is spread over a wide range of wavelengths, however, the peak wavelength gives us the temperature.



Observations of known stars have shown that the power emitted in each wavelength is proportional to the temperature in Kelvin, to the fourth power:

$$\mathbf{P \propto T^4}$$

Remember, visible light is only a small proportion of the total radiation emitted by a star. Stars emit over a huge range, from radio to gamma rays. For example, the sun emits very powerfully in the ultraviolet from the spectral emission lines from nitrogen in its atmosphere.

Evidence of the Big Bang

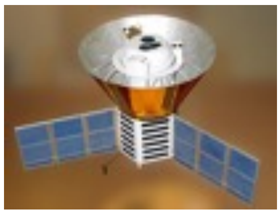
Through many generations of observing stars, as emitters of energy, we know a great deal about the relationship between the frequency of emitted energy and its temperature.

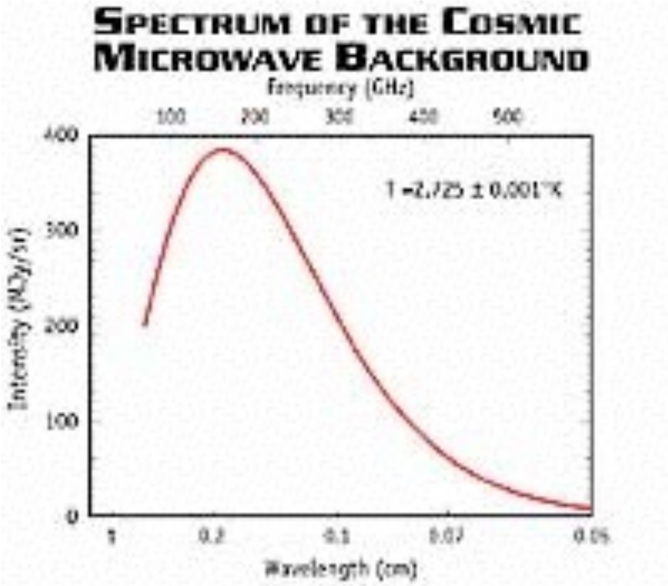
In 1948 it was suggested that if the Big Bang did happen then it would be the biggest single emission of energy in the universe — and there should be a measurable peak wavelength associated with it. The universe has cooled considerably since the Big Bang. It was predicted to be at a current temperature of 2·7K, with an associated peak wavelength in the microwave region. This radiation would be observable in every direction and spread uniformly throughout the universe — the cosmic microwave background radiation (CMB).

It was not until 1965 that the CMB was observed. Two astronomers, Arno Penzias and Robert Wilson, actually discovered the radiation left over from the Big Bang completely by accident. There was a source of excess noise in their radio telescope that they could not identify and researchers from another laboratory, who were actively looking for the CMB, recognised it. Those astronomers shared the 1978 Nobel Prize for their accidental discovery of the Big Bang's signature.

Over 20 years later, and we had the technology to measure the CMB in more detail. This was the first time we would be able to verify if the CMB followed the known profile for emitted radiation. If the spectrum was as predicted, it would have a peak wavelength proportional to 2.7 K and thus offering very strong, measurable evidence for the Big Bang.

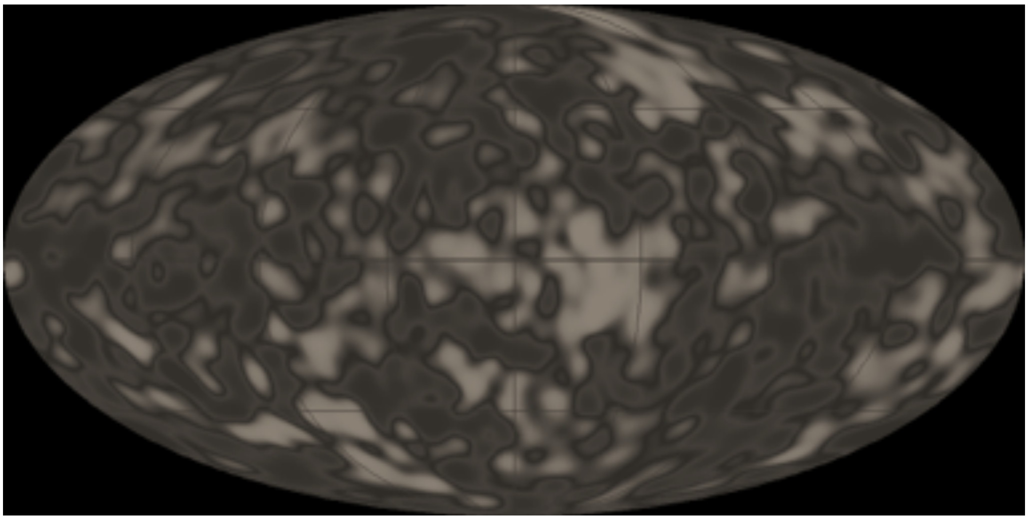
This experiment was performed by NASA, with the satellite COBE (Cosmic Background Explorer). COBE recorded the background radiation of the universe, in all directions, over three years.





There are 34 data points on the graph opposite. They and their associated uncertainty bars are so in line with the predicted values that you cannot see them beneath the predicted curve. The CMB was shown to be the **exact** radiation signature that would be left over from a single event, from which all space expanded, cooled over 13.7 billion years. The uniformity of the CMB is also a very strong indication that this event was the point of origin in the universe. If it took place after this time, somewhere inside an already expanding universe, the radiation would not be spread as evenly in all directions.

The WMAP project has imaged the CMB and shown it to be uniform across all directions to within 0.1%.



Olber's Paradox

There have been many recent advances in cosmology using complex instruments and satellites. But there is also very strong evidence for the Big Bang and the expanding universe which just requires you to look up at the sky.

On a cold, clear night, away from street lights and houses, you can look into the sky at night and see countless stars. However, you also see plenty of darkness between them. With the vast numbers of stars that are in the universe, shouldn't the night sky be bright with them, in all directions? In any line of sight there should eventually, be a star. Even though the light from greater distances is much fainter, there should be a great number of stars contributing to the irradiance from any direction, spread along the line of sight at greater and greater radii.

The Theory of the Big Bang and an expanding universe, however, explain our observation. As all stars and galaxies are expanding away from each other, from a single point, at an accelerating rate, there are many galaxies whose light will never reach us. The light will never catch up to the Earth as the space between us and the galaxy expands.

You need to know:

	✓ ? ✘
How to do suvat questions	
How to draw and interpret displacement, velocity and acceleration time graphs (motion time graphs) — especially for a bouncing ball	
How to work out displacement, velocity or acceleration from a motion time graph	
What the effects of friction are	
What terminal velocity is	
Newton's Laws	
How to do slope questions	
How to do lift questions	
How to calculate work done, potential energy, kinetic energy and power	
Law of the Conservation of Energy	
Law of Conservation of Momentum	
Collision questions, including explosions	
The difference between elastic and inelastic collisions	
How to interpret force time graphs	
How to calculate impulse from a force time graph	
That impulse = Ft = mv – mu	
How to do projectile questions	
How satellites work	
How objects in free fall behave	
Newton's Universal Law of Gravitation	
The speed of light is the same for all observers	
How to calculate length and time contraction	
What the Doppler effect is	
How to calculate Doppler shift	
How the Doppler effect can be used in Astronomy	

	✓ ? ✘
What Hubble's Law is	
How the age of the Universe can be calculated from Hubble's Law	
What evidence there is for the expansion of the universe	
How the mass of a galaxy can be estimated by the speed of the stars within it	
What evidence there is for dark matter	
What evidence there is for dark energy	
That the temperature of a star is related to the wavelengths of light it emits	
That hot objects emit light at shorter wavelengths than cool ones	
That $P \propto T^4$	
What the CMB is and how it relates to the Big Bang Theory	