# SCHOLAR Study Guide <br> CfE Higher Physics Unit 3: Electricity 

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## Contents

1 Monitoring and measuring a. c. ..... 1
1.1 The frequency of an a.c. supply ..... 3
1.2 Peak and r.m.s. values of voltage and current ..... 9
1.3 Resistors in a.c. circuits ..... 14
1.4 Summary ..... 17
1.5 Extended information ..... 17
1.6 Assessment ..... 18
2 Current, potential difference, power and resistance ..... 21
2.1 Current and charge ..... 23
2.2 Power ..... 25
2.3 Resistance ..... 26
2.4 Electrical energy and power ..... 30
2.5 Resistors in series ..... 35
2.6 Resistors in parallel ..... 37
2.7 Potential divider ..... 40
2.8 The Wheatstone bridge ..... 49
2.9 Summary ..... 55
2.10 Extended information ..... 56
2.11 Assessment ..... 57
3 Electrical sources and internal resistance ..... 59
3.1 Sources and circuits ..... 60
3.2 Load matching (an interesting application of internal resistance) ..... 65
3.3 Summary ..... 68
3.4 Extended information ..... 68
3.5 Assessment ..... 69
4 Capacitors ..... 73
4.1 What is a capacitor ..... 75
4.2 Charge and capacitance ..... 75
4.3 Capacitors in circuits ..... 81
4.4 Summary ..... 96
4.5 Extended information ..... 96
4.6 Assessment ..... 97
5 Conductors, semiconductors and insulators ..... 99
5.1 Electrical properties ..... 101
5.2 The structure of the atom ..... 101
5.3 Conductors, insulators and semiconductors ..... 103
5.4 Valence band, conduction band and band gap ..... 105
5.5 Thermistors ..... 106
5.6 The Hall effect ..... 108
5.7 Summary ..... 109
5.8 Extended information ..... 109
5.9 Assessment ..... 110
6 p-n junctions ..... 113
6.1 Doping ..... 115
6.2 p-n junctions ..... 120
6.3 Light emitting diodes and solar cells ..... 133
6.4 Summary ..... 144
6.5 Extended information ..... 144
6.6 Assessment ..... 145
7 End of unit test ..... 149
7.1 Open ended and skill based questions ..... 150
7.2 Course style questions ..... 153
7.3 End of unit assessment ..... 158
A Appendix: Units, prefixes and scientific notation ..... 165
A. 1 Physical quantities, symbols and units used in CfE Higher Physics ..... 166
A. 2 Significant figures ..... 167
A. 3 Scientific notation ..... 169
Glossary ..... 171
Hints for activities ..... 174
Answers to questions and activities ..... 186
1 Monitoring and measuring a. c. ..... 186
2 Current, potential difference, power and resistance ..... 188
3 Electrical sources and internal resistance ..... 191
4 Capacitors ..... 194
5 Conductors, semiconductors and insulators ..... 196
6 p-n junctions ..... 197
7 End of unit test ..... 199
A Appendix: Units, prefixes and scientific notation ..... 206

## Topic 1

## Monitoring and measuring a. c.

## Contents

1.1 The frequency of an a.c. supply ..... 3
1.1.1 A.c. waveforms ..... 3
1.1.2 Measuring the frequency of an alternating supply ..... 5
1.2 Peak and r.m.s. values of voltage and current ..... 9
1.2.1 Comparing the energy transformed by a.c. and d.c. supplies ..... 11
1.3 Resistors in a.c. circuits ..... 14
1.4 Summary ..... 17
1.5 Extended information ..... 17
1.6 Assessment ..... 18

## Learning objectives

By the end of this topic you should be able to:

- define a.c and d.c. current;
- describe what is meant by the frequency of an a.c. supply;
- describe how to measure the frequency of a low-voltage a.c. supply using an oscilloscope;
- relate the period of a waveform to its frequency;
- state what the abbreviation r.m.s. stands for and to explain what is meant by an r.m.s. value;
- state the relationship between peak and r.m.s. values for a sinusoidally-varying voltage and current;
- describe an experiment using an oscilloscope to measure voltage across lamps with d.c. and a.c. sources to compare peak and r.m.s. values;
- carry out calculations involving peak and r.m.s. values of voltage and current.

The electricity supply from a battery is d.c. This means that when the battery is being used, it supplies a constant voltage so the current is always in the same direction-d.c. stands for direct current.

The following graph shows that the voltage across the battery does not change value or direction with time. This means that the value and direction of the current from a battery remains constant with time.

Figure 1.1: graph of d.c. voltage against time


The electricity supply to our homes, schools and factories from the National Grid is an
a.c. supply. This means that the current from the supply constantly changes direction a.c. stands for 'alternating current'.

In Great Britain, the voltage of the supply is described as $230 \mathrm{~V}, 50 \mathrm{~Hz}$. In other countries the values may be different, but virtually all countries use a.c. for their public electricity supply.
There are two main reasons why a.c. is used.

1. Firstly that is the form of electricity generated by commercial generators.
2. Secondly transformers only work on a.c. supplies and transformers are essential to step voltages up and down for the transmission of electrical energy. Less energy is lost as heat in the cables when a high voltage and so a smaller current is used to transmit electrical energy.

In this topic we will look at what we mean by the frequency of an a.c. supply, and how we can measure it. We will also consider what is meant by the voltage of an a.c. supply, and how we can attach a number (e.g. 230) to what is a constantly changing value. Finally, we will look at resistors in a.c. circuits.

The following graph shows that the instantaneous voltage across an a.c. supply changes value and direction with time. This means that the instantaneous value and direction of the current from an a.c. supply changes with time.

Figure 1.2: graph of a.c. voltage against time


### 1.1 The frequency of an a.c. supply

This section will examine a.c. waveforms and how the frequency of an alternating supply can be measured.

### 1.1.1 A.c. waveforms

When a coil of wire is rotated at a constant rate in a magnetic field (as in an a.c. generator - an alternator) or when a magnet is rotated at a constant rate near to a coil of wire (as in Figure 1.3), then a voltage is induced in the coil.

Figure 1.3: A bicycle dynamo


The waveform of the voltage generated is shown in Figure 1.4.

Figure 1.4: voltage generated by a bicycle dynamo


It can be seen from this waveform that the voltage generated is constantly changing in amplitude - it is an a.c. waveform. This voltage is a sine wave - the most common form of a.c. supply.

For the rest of this topic, we will only consider waveforms that are sine waves, but remember the term a.c. applies to all changing waveforms, not just sine waves.
If the magnet of the bicycle dynamo is made to rotate at a faster rate, then the amplitude of the voltage generated increases. This is because the rate at which the magnetic field lines cut the coil is increased. There is another change that happens when the magnet rotates faster - the frequency of the a.c. waveform generated increases.

In Figure 1.4, there are three complete cycles shown. This corresponds to three complete revolutions of the magnet in the dynamo. If these three revolutions take one second, we say that the frequency fof the a.c. generated is 3 cycles per second, or 3 hertz ( Hz ).
If there are three waves made per second in Figure 1.4 then it follows that it takes $1 / 3$ of a second to make one complete wave. The period $T$ of the wave is $1 / 3 \mathrm{~s}$.

$$
\begin{equation*}
T=\frac{1}{f} \tag{1.1}
\end{equation*}
$$

### 1.1.2 Measuring the frequency of an alternating supply

The frequency of an alternating supply can be measured using a calibrated oscilloscope. The time-base of an oscilloscope is usually calibrated in seconds (or milli- or microseconds) per centimetre. This tells us the time it takes the spot to travel one centimetre across the screen, from left to right. To use a calibrated oscilloscope, the supply is connected to the input of the oscilloscope and the time-base adjusted to give a suitable number of waves on the screen. Using the time-base setting, along with the number of waves on the screen and the width of the screen, the time for one wave to be made is calculated. The frequency can then be calculated by using Equation 1.1.

Example A low-voltage a.c. supply is connected to an oscilloscope and four complete waves are produced when the time-base of the oscilloscope is set to $5 \mathrm{~ms} \mathrm{~cm}^{-1}$, as shown in Figure 1.5. Each square on the oscilloscope screen has a 1 cm side.

Figure 1.5: a.c. waveform seen on oscilloscope screen


$5 \mathrm{~ms} \mathrm{~cm}^{-1}$

Calculate the frequency of the supply.
Answer:
4 waves take up 8 cm across the screen so 1 wave takes up 2 cm across the screen.
The time-base is set to $5 \mathrm{~ms} \mathrm{~cm}^{-1}$ so 1 wave is made in $5 \times 2 \mathrm{~ms}=10 \mathrm{~ms}=10 \times 10^{-3} \mathrm{~s}$

$$
\begin{aligned}
T & =\frac{1}{f} \\
\text { so } f & =\frac{1}{T} \\
\therefore f & =\frac{1}{10 \times 10^{-3}} \\
\therefore f & =100 \mathrm{~Hz}
\end{aligned}
$$

The frequency of the supply is 100 Hz .

## Measuring frequency using an oscilloscope

When calculating the frequency of a signal from an oscilloscope trace, the procedure below should be followed:

1. Look at the time base setting to find the time per centimetre on the screen (change into seconds).
2. Measure the horizontal distance, in centimetres, for a wave to repeat itself on the screen.
3. Multiply this distance by the time base setting. This is the period of the signal.
4. Calculate the frequency using the formula frequency $=1 /$ period .

Q1: If the time base setting is 2.0 ms and a wave repeats on the screen every 5 squares, calculate the frequency.

Q2: If the time base setting is 1.0 ms and a wave repeats on the screen every 5 squares, calculate the frequency.

Q3: If the time base setting is $5.0 \mu \mathrm{~s}$ and a wave repeats on the screen every 2.0 squares, calculate the frequency.

## Quiz: Frequency of a.c.

Q4: Why is a.c. used for all mains electricity supplies?
i Transformers only work on a.c.
ii a.c. electricity can be stored but d.c. electricity cannot.
iii Commercial generators generate a.c.
a) (i) only
b) (ii) only
c) (iii) only
d) (i) and (ii) only
e) (i) and (iii) only

Q5: What is the correct relationship between the period ( $T$ ) and the frequency $(f)$ of a wave?
a) $T=f$
b) $T=\frac{1}{f}$
c) $T=\frac{1}{f^{2}}$
d) $T=f^{2}$
e) $T=\sqrt{f}$

Q6: A signal generator produces an alternating voltage of frequency 50 Hz .
What is the period of the wave?
a) 0.4 ms
b) 2.5 ms
c) 7.1 ms
d) 20 ms
e) 50 ms

Q7: A wave with a period of 25 ms is generated.
What is the frequency of this wave?
a) 5.0 Hz
b) 6.3 Hz
c) 25 Hz
d) 40 Hz
e) 625 Hz

Q8: An a.c. supply was connected to an oscilloscope and the following trace was obtained when the time-base of the oscilloscope was set to $2 \mathrm{~ms} \mathrm{~cm}^{-1}$.


$2 \mathrm{~ms} \mathrm{~cm}^{-1}$
Each square on the screen has a 1 cm side.
What is the frequency of the supply?
a) 125 Hz
b) 160 Hz
c) 250 Hz
d) 500 Hz
e) 1000 Hz

### 1.2 Peak and r.m.s. values of voltage and current

We have just shown that an alternating voltage is constantly changing so how can we describe a supply of electricity as, for example, a 230 V a.c. supply? Since the volt is defined as one joule per coulomb, we use this to define what we mean by the value of an a.c. voltage. We would expect a 12 V car headlamp to produce the same quantity of light (and heat) whether it is operated from a 12 V d.c. car battery or from a 12 V a.c. supply obtained from a transformer connected to the mains supply. In other words, we
would expect a 12 V supply to transform 12 joules of energy for every coulomb of charge that flows through the headlamp irrespective of whether it is a d.c. or an a.c. supply.

We compare the steady value of a d.c. voltage with the 'effective voltage' of an a.c. supply that transforms the same energy in a resistor. The average of a sine wave over any whole number of cycles is zero. We use a value called the r.m.s. voltage (r.m.s. is an abbreviation for 'root mean square') for this comparison. This is because, as you will see later, the energy transformed in a resistor depends on the square of the voltage.

$$
12 \mathrm{~V} \text { d.c }=12 \mathrm{~V} \text { r.m.s a.c. }
$$

## r.m.s. values of voltage and current

To compare a steady voltage ( $V_{\text {d.c. }}$ ) with a sinusoidally-varying alternating voltage ( $V_{\text {a.c. }}$ ) that transforms energy at the same rate in a resistor, consider the diagram.


When the a.c. voltage graph is squared, it is seen to be a sine graph of twice the frequency. Taken over a whole number of complete waves, the mean or average of the (voltage) ${ }^{2}$ graph is given by the line through the centre of the graph, with a value of $V_{\text {peak }}{ }^{2} / 2$. This is known as the mean square voltage.

If this alternating voltage is applied across a resistor, then energy is transferred at an average rate given by:
power $=\frac{\frac{1}{2} V_{\text {peak }}{ }^{2}}{R}$

A steady voltage of $V_{\text {d.c. }}$ applied across the same resistor would transfer energy at an average rate given by:
power $=\frac{V_{d, c}{ }^{2}}{R}$
so we have:

$$
\begin{aligned}
\frac{V_{\text {d.c. }}{ }^{2}}{R} & =\frac{\frac{1}{2} V_{\text {peak }}{ }^{2}}{R} \\
\therefore V_{\text {d.c. }}{ }^{2} & =\frac{1}{2} V_{\text {peak }}{ }^{2} \\
\therefore V_{\text {d.c. }} & =\frac{V_{\text {peak }}}{\sqrt{2}}
\end{aligned}
$$

The steady voltage ( $V_{\text {d.c. }}$ ) that transfers energy at the same rate is the root mean square or r.m.s. voltage ( $V_{\text {r.m.s. }}$ ).
The r.m.s. and peak values of voltage and current are given by the relationships

$$
\begin{gathered}
V_{r . m . s .}=\frac{V_{\text {peak }}}{\sqrt{2}} \approx 0.7 \times V_{\text {peak }} \\
I_{\text {r.m.s. }}=\frac{I_{\text {peak }}}{\sqrt{2}} \approx 0.7 \times I_{\text {peak }}
\end{gathered}
$$

This activity demonstrates that r.m.s. value $=0.7 \times$ peak value
This activity demonstrates that the r.m.s. and peak values of voltage and current are given by the relationships

$$
\text { r.m.s. value }=0.7 \times \text { peak value }
$$

### 1.2.1 Comparing the energy transformed by a.c. and d.c. supplies

Consider an investigation comparing the time taken to cook food using a low power microwave ( 750 W ) and a high power microwave ( 850 W ).
The time taken to cook the food using a 750 W microwave would be measured.

| Power of microwave / W | Cooking time /s |
| :---: | :---: |
| 750 | 270 |
| 850 | 240 |

## Conclusion

Both of these combinations result in the same effect: cooked food.
From these results it could be said that 270 seconds of a 750 W microwave has the same effect as 240 seconds of an 850 W microwave.

The following investigation applies a similar approach in order to compare the energy transformed in a lamp when it is connected to a d.c, supply and the energy transformed in the same lamp when it is connected to an a.c. supply. The supplies will be adjusted until the same effect is obtained: the lamp is to be equally bright using either supply.
The apparatus shown in Figure 1.6 allows us to compare the energy transformed in a lamp by a d.c. supply with the energy transformed in the same lamp by an a.c. supply.

Figure 1.6: Apparatus to compare a.c. and d.c. supplies


With the switch in position 1, the reading on the light meter (placed beside the lamp), is noted. Without changing the relative positions of the lamp and the light meter, the switch is changed to position 2. The variable resistor is adjusted until the lamp is as bright as before, using the reading on the light meter to check this. The traces seen on the oscilloscope screen when the lamp is lit equally brightly (and therefore transforming the same amount of energy) are shown in Figure 1.7.

Figure 1.7: Comparison of a.c. and d.c. supplies


We can see that the same amount of energy is transformed in the lamp by a d.c. supply that has a value approximately 0.7 times the peak value of an a.c. supply. This means that the effective or r.m.s. value of an alternating voltage ( $V_{\text {r.m.s. }}$ ) is approximately 0.7 times the peak value. (There is no commonly accepted symbol for peak voltage. You may see it written as $V_{\text {peak }}, V_{p}, V_{\text {max }}$. or $V_{m}$. Here, we have used $V_{\text {peak. }}$.) The optional activity shows the exact relationship, which is:

$$
\begin{align*}
V_{r . m . s .} & =\frac{V_{\text {peak }}}{\sqrt{2}} \\
V_{\text {peak }} & =\sqrt{2} \times V_{r . m . s .} \tag{1.2}
\end{align*}
$$

Since the current through a resistor is proportional to the potential difference across it, the relationship in Equation 1.2 also holds for current.

$$
\begin{align*}
I_{\text {r.m.s. }} & =\frac{I_{\text {peak }}}{\sqrt{2}} \\
I_{\text {peak }} & =\sqrt{2} \times I_{\text {r.m.s. }} \tag{1.3}
\end{align*}
$$

## Example

a) Calculate the peak voltage of the 230 V a.c. mains supply.
b) Remembering that the peak value of an alternating waveform is half the peak-topeak value, calculate the voltage swing of the 230 V a.c.mains supply.

Answer:
a.

$$
\begin{aligned}
& V_{\text {r.m.s. }}=\frac{V_{\text {peak }}}{\sqrt{2}} \\
& \therefore V_{\text {peak }}=\sqrt{2} \times V_{\text {r.m.s. }} \\
& \therefore V_{\text {peak }}=\sqrt{2} \times 230 \\
& \therefore V_{\text {peak }}=325 \mathrm{~V} \\
& \text { b. }
\end{aligned}
$$

$$
\begin{aligned}
V_{\text {peak-to-peak }} & =2 \times V_{m} \\
\therefore V_{\text {peak-to-peak }} & =2 \times 325 \\
\therefore V_{\text {peak-to-peak }} & =650 \mathrm{~V}
\end{aligned}
$$

So the voltage at the live wire of a 230 V a.c. mains supply swings through 650 V .

### 1.3 Resistors in a.c. circuits

In this next online activity, observe the effect on the alternating current through a resistor as the frequency of the supply is altered.

## The resistor and a.c.

At this stage there is an online activity. If however you do not have access to the internet you may try the questions which follow.
A circuit is set up that allows the frequency of the a.c. supply to be changed while keeping the potential difference constant. The supply is connected to a resistor and an ammeter. The frequency of the supply is altered and the current noted.
This is repeated for a number of frequencies.


A graph is plotted from the results.

## Current/A



Q9: By examining the graph of the results state the relationship between the frequency of a.c. supply and current.

## Quiz: Voltage and frequency

Q10: What is the correct relationship between the peak voltage ( $V_{\text {peak }}$ ) and the r.m.s. voltage ( $V_{\text {r.m.s. }}$ ) of an a.c. source obtained from the mains?
a) $\quad V_{\text {peak }}=2 V_{\text {r.m.s. }}$.
b) $V_{\text {peak }}=\sqrt{2} \times V_{\text {r.m.s. }}$.
c) $V_{\text {peak }}=V_{\text {r.m.s. }}$.
d) $V_{\text {peak }}=\frac{V_{\text {r.m.s. }}}{2}$
e) $V_{\text {peak }}=\frac{V_{r . m . s .}^{2}}{\sqrt{2}}$

Q11: The r.m.s. voltage of an a.c. supply is 10 V .
What is the peak voltage?
a) 5.0 V
b) 7.1 V
c) 10 V
d) 14 V
e) 20 V

Q12: The insulation of a certain capacitor breaks down when a voltage greater than 16 V is applied across it.
What is the greatest r.m.s. voltage that can appear across this capacitor when it is connected into an a.c. circuit?
a) 8.0 V
b) 11 V
c) 16 V
d) 23 V
e) 32 V

Q13: A resistor of resistance $27 \Omega$ is connected across a variable-frequency, fixedvoltage supply. With the frequency of the supply set at 40 Hz , the current in the resistor is recorded as 130 mA .
What will the current be when the frequency of the supply is reduced to 20 Hz ?
a) 20 mA
b) 27 mA
c) 40 mA
d) 64 mA
e) 130 mA

### 1.4 Summary

## Summary

You should now be able to:

- describe what is meant by the frequency of an a.c. supply;
- describe how to measure the frequency of a low-voltage a.c. supply using an oscilloscope;
- relate the period of a waveform to its frequency;
- state what the abbreviation r.m.s. stands for and explain what is meant by an r.m.s. value;
- state the relationship between peak and r.m.s. values for a sinusoidallyvarying voltage and current;
- describe an experiment using an oscilloscope to measure voltage across lamps with d.c. and a.c. sources to compare peak and r.m.s. values;
- carry out calculations involving peak and r.m.s. values of voltage and current;
- state the relationship between r.m.s. current and frequency in a resistive a.c. circuit.


### 1.5 Extended information

- http://www.educationscotland.gov.uk/highersciences/physics/unitthree/ electronsandenergy/monitoring.asp
Education Scotland website with student notes.
- http://www.physics-chemistry-interactive-flash-animation.com/
electricity_electromagnetism_interactive/
oscilloscope_description_tutorial_frequency_period_sine_voltage_AC.htm Interactive simulation on measuring frequency using an oscilloscope.
- http://www.allaboutcircuits.com/vol_2/chpt_1/1.html All about circuits is a useful site to explore and provides some extra information on ac current.
- http://www.school-for-champions.com/science/dc.htm

This gives a good description of dc current. The links at the side are also useful for further information on the topic of electricity.

- http://www.gcse.com/electricity/ac_dc.htm

There is plenty of information in this site to support and extend the knowledge gained in this topic.

- http://www.ndt-ed.org/EducationResources/CommunityCollege/ EddyCurrents/Physics/impedance.htm
This site can be used as good revision materials for this topic.


### 1.6 Assessment

## End of topic 1 test

The following test contains questions covering the work from this topic.
A reminder of useful data values can be found in the information sheet (opened by
$i$
clicking within a test).
The end of topic test is available online. If however you do not have access to the web, you may try the following questions.

Q14: What is the relationship between the peak and the r.m.s. values of a sinusoidallyvarying current?
a) $I_{\text {peak }}=2 I_{\text {r.m.s. }}$
b) $I_{\text {peak }}=I_{\text {r.m.s. }} / \sqrt{ } 2$
c) $I_{\text {peak }}=\sqrt{ } 2 I_{\text {r.m.s. }}$
d) $I_{\text {peak }}=I_{\text {r.m.s. }} / \sqrt{ } 2$
e) $I_{\text {peak }}=I_{\text {r.m.s. }}$

Q15: The peak of an a.c. supply is 19 V . Calculate the r.m.s. value.

Q16: A signal generator is connected to an oscilloscope and the trace shown below is obtained.


Calculate the frequency of the signal from the signal generator, if the time-base of the oscilloscope is set to $1 \mathrm{~ms} /$ division.

Q17: The diagram shows the trace on an oscilloscope screen when an a.c. signal is applied.


The oscilloscope settings are as follows:
Time base $2.0 \mathrm{~ms} \mathrm{~cm}^{-1}$
Y-gain control $2.0 \mathrm{~V} \mathrm{~cm}^{-1}$
Each square on the oscilloscope screen has a 1.0 cm side.

1. Calculate the peak voltage.
2. Calculate the r.m.s.voltage.
3. Calculate the frequency of the signal.

## Topic 2

## Current, potential difference, power and resistance

## Contents

2.1 Current and charge ..... 23
2.2 Power ..... 25
2.3 Resistance ..... 26
2.4 Electrical energy and power ..... 30
2.5 Resistors in series ..... 35
2.6 Resistors in parallel ..... 37
2.7 Potential divider ..... 40
2.8 The Wheatstone bridge ..... 49
2.8.1 The balanced Wheatstone bridge circuit ..... 49
2.9 Summary ..... 55
2.10 Extended information ..... 56
2.11 Assessment ..... 57

## Learning objectives

By the end of this topic you should be able to:

- define charge, current, voltage, electrical power;
- state the units of measurement of current, resistance;
- solve problems using the equation $Q=I t$;
- explain what is meant by electrical potential, potential difference, resistance, potential divider;
- solve problems using Ew = $Q V, P=I V=I^{2} R=V^{2} / R, V=I R$, $E w=I t V$;
- explain the kilowatt hour as a unit of energy;
- state the equations for the total resistance of a number of resistors in series or in parallel;
- solve problems involving resistors in series or in parallel;
- describe the operation of potential dividers;
- carry out calculations involving potential dividers;
- describe a Wheatstone bridge circuit and a balanced Wheatstone bridge;
- carry out calculations in a balanced Wheatstone bridge.

In previous studies of electricity you may have met current, charge, voltage, resistance, energy and power. In this topic, we will look more deeply into all of these quantities and build up a better understanding of how they are related.

### 2.1 Current and charge

Thales, an ancient Greek experimenter, noticed that when he rubbed amber with cloth, the amber attracted small pieces of straw: it exerted a force on the straw. This effect was described about 2000 years later as being due to a charge of electricity. The word electricity comes from the Greek word elektron meaning amber. The idea of charge originated because it was first thought that electricity was like a fluid that could be poured. We still sometimes say charge your glasses meaning fill them up with drink.
We now know that charge is a fundamental property of matter. The magnitude of the charge carried by one electron or one proton is known as the fundamental unit of charge $e$.

A charge of one coulomb is equal to the charge on $6.25 \times 10^{18}$ protons or electrons. It should be noted that one coulomb is an extremely large quantity of charge, and we are unlikely to encounter such a huge quantity of charge inside the laboratory.
The charge on one electron is $-1.6 \times 10^{-19} \mathrm{C}$. The charge on a proton is $+1.6 \times 10^{-19} \mathrm{C}$. The charge on the proton and the electron are the same in size but have opposite signs.

The sort of quantities of charge we are more likely to be dealing with are of the order of microcoulombs ( $1 \mu \mathrm{C}=10^{-6} \mathrm{C}$ ), nanocoulombs ( $1 \mathrm{nC}=10^{-9} \mathrm{C}$ ) or picocoulombs ( 1 pC $=10^{-12} \mathrm{C}$ ).

Experiments have shown that there are only two types of charge. More than 200 years ago these two types were called positive and negative by the American physicist Benjamin Franklin.

An object can be charged by adding negatively-charged particles such as electrons to it, in which case it becomes negatively charged, or by removing electrons from it, making it positively charged. Further experiments have shown that a negatively-charged object attracts a positively-charged object and that objects that have similar charges repel each other.

Whenever we have a flow of charged particles, we say that there is an electric current. The movement or flow of charges may be electrons carrying negative charges in a wire that forms part of an electric circuit; it may be positive ions in a solution; it could be charged particles leaving the Sun and entering the Earth's atmosphere and forming the Aurora Borealis (the Northern Lights). To be more precise in our understanding, we need to remember that electric current is the rate of flow of charge. This leads to the relationship

$$
I=\frac{\Delta Q}{\Delta t}
$$

In this relationship, $Q$ is charge, measured in coulombs (C), $t$ is time, measured in
seconds (s) and $l$ is electric current, measured in amperes (A). The symbol $\Delta$ means 'a change in' a quantity, so $\Delta Q$ means a change in charge, $Q$. $\Delta$ is meaningless on its own. The above relationship is often used in its simpler form:

$$
\begin{equation*}
Q=I t \tag{2.1}
\end{equation*}
$$

## Example : Charge transferred in an electric circuit

Calculate the total charge transferred in an electric circuit that has a steady current of 0.5 A for 5 minutes.

Answer:
We must first of all convert the time in minutes into seconds. 5 minutes $=5 \times 60=300$ s. Then using Equation 2.1, we have

$$
\begin{aligned}
Q & =I t \\
\therefore Q & =0.5 \times 300 \\
\therefore Q & =150 \mathrm{C}
\end{aligned}
$$

The charge transferred in this circuit is 150 C .

Although we have noted that charge is a fundamental property of matter, it is the ampere, the unit of current, that is nowadays taken as the basic unit, and the unit of charge is considered as a derived unit. From this relationship, it will be seen that the unit that is used for charge, the coulomb, is equivalent to the ampere second.
When there is a current in an electric circuit, the charges that move along the wires of the electric circuit are carried by electrons. So an electric current in a circuit is a flow of electrons. In the circuit, the electrons flow from the negative terminal to the positive terminal of the battery or supply.

### 2.2 Power

Starting from our definition of potential difference as work done per charge, $V={ }^{E} W / Q$, we can deduce another relationship, that may already be familiar to you. You should remember that power is the rate of transfer of energy, or the rate at which work is done, $P=E_{W / t}$. We have shown earlier that current is the rate of flow of charge, $I=Q / t$. Substituting these into our expression for potential difference, we have

$$
\begin{align*}
V & =\frac{E_{W}}{Q} \\
\therefore V & =\frac{P \times t}{I \times t} \\
\therefore V & =\frac{P}{I} \tag{2.2}
\end{align*}
$$

## Example : Car headlamp bulb

A car headlamp bulb takes a current of 4 A from the 12 V car battery.
Calculate the power delivered to the bulb.
Answer:
Using Equation 2.2, we have:

$$
\begin{aligned}
V & =\frac{P}{I} \\
\text { so } P & =V I \\
\therefore P & =12 \times 4 \\
\therefore P & =48 \mathrm{~W}
\end{aligned}
$$

The power of the lamp is 48 W .

## Quiz: Charge, current and potential difference

Q1: What is the value of the fundamental unit of charge, $e$ ?
a) $6.25 \times 10^{-19} \mathrm{C}$
b) $1.6 \times 10^{-19} \mathrm{C}$
c) $3.00 \times 10^{8} \mathrm{C}$
d) $1.6 \times 10^{18} \mathrm{C}$
e) $6.25 \times 10^{18} \mathrm{C}$

Q2: What is the current in an electric circuit if a charge of 180 C is transferred in 1 minute?
a) $5.6 \times 10^{-3} \mathrm{~A}$
b) 0.33 A
c) 3.0 A
d) 10.8 A
e) 180 A

Q3: A sandwich maker with a power rating of 690 W takes a current of 3.0 A when operating normally.
What voltage is the sandwich maker designed to operate on?
a) 3.0 V
b) 77 V
c) 230 V
d) 690 V
e) 2070 V

### 2.3 Resistance

Consider the circuit shown in Figure 2.1 which shows a resistor connected across a battery.

Figure 2.1: Resistor in a circuit


The voltage supplied by the battery is used to drive a current through the resistor - we say that the resistor has the property of resistance.

A potential difference $V$ is set up across the resistor $R$, and this potential difference can be measured by placing a voltmeter across the resistor. The resistance $R$ of a conductor is defined as the ratio of the potential difference across the conductor to the current / through it.

$$
\begin{equation*}
R=v / 1 \tag{2.3}
\end{equation*}
$$

When the voltage $V$ is measured in volts, and the current $I$ is measured in amperes, then the resistance $R$ is measured in ohms $(\Omega)$.

## Example : Car headlamp bulb

A car headlamp bulb takes a current of 4 A from the 12 V car battery.
Calculate the resistance of the bulb.
Using Equation 2.3, we have:
$R=\frac{V}{I}$
$\therefore R=\frac{12}{4}$
$\therefore R=3 \Omega$
The resistance of the car lamp is 3.0 ohms.

## Circuits

At this stage there is an online activity. If however you do not have access to the internet you may try the questions which follow.
A circuit is set up as shown.


The potential difference is altered using the slider and the current noted. The following results were obtained.

| Potential difference/ V | Current/ mA |
| :--- | :--- |
| 0 | 0 |
| 2 | 20 |
| 4 | 40 |
| 6 | 60 |
| 8 | 80 |
| 10 | 100 |

Draw a graph of the results.

Q4: Interpret the results and state the relationship between the potential difference and the current.

## Ohm's law

If the ratio $V /$, is constant for all values of $V$ (as is the case with most metallic conductors), then the conductor obeys Ohm's law and we say that the conductor is an ohmic conductor. Ohm's law states:

The current in a conductor at constant temperature is directly proportional to the potential difference across it.
When the temperature of a conductor increases its resistance also increases.

## Quiz: Resistance

Q5: Which of the following gives the correct relationship between the potential difference across, the current through, and the resistance of a conductor?
a) $R=I \times V$
b) $R=1 / v$
c) $R=I \times V^{2}$
d) $R=V / 1$
e) $R=\frac{V^{2}}{I}$

Q6: One ohm is equivalent to
a) one volt per second
b) one ampere per second
c) one coulomb per second
d) one ampere per volt
e) one volt per ampere

Q7: A mains light bulb takes a current of 0.25 A when operating. The mains voltage is 230 V .
What is the resistance of the light bulb?
a) $30.0 \Omega$
b) $57.5 \Omega$
c) $920 \Omega$
d) $3680 \Omega$
e) $211600 \Omega$

Q8: A car headlamp bulb is connected to a $0-12 \mathrm{~V}$ power supply.
Explain what happens to the resistance of the bulb as the potential difference across it is increased from 0 to 12 V , increasing the current through the bulb.
a) The resistance increases, because its temperature decreases.
b) The resistance increases, because its temperature increases.
c) The resistance remains constant because its resistance is independent of temperature.
d) The resistance decreases, because its temperature decreases.
e) The resistance decreases, because its temperature increases.

### 2.4 Electrical energy and power

Whenever there is a current in a resistor, electrical energy is transformed into heat energy. We have seen this in the filament of a light bulb - it gets hot. It is the current in a resistor that allows the resistor to transfer the energy. We are usually interested in the rate at which electrical energy is transferred by a resistor - in other words, the power.
Earlier in this Topic, we showed that electrical power is given by the relationship $P=$ $I V$. We can combine this relationship with the equation linking resistance, voltage and current, $R=v / /$, to obtain two more expressions for the power developed in a resistor.

$$
\begin{aligned}
P & =I V \\
\therefore P & =I \times(I R) \\
\therefore P & =I^{2} R
\end{aligned}
$$

also:

$$
\begin{aligned}
P & =I V \\
\therefore P & =\left(\frac{V}{R}\right) \times V \\
\therefore P & =\frac{V^{2}}{R}
\end{aligned}
$$

So we now have three relationships that can be used to calculate the power in an electrical circuit.

$$
\begin{equation*}
P=I V \tag{2.4}
\end{equation*}
$$

$$
\begin{equation*}
P=I^{2} R \tag{2.5}
\end{equation*}
$$

$$
\begin{equation*}
P=\frac{V^{2}}{R} \tag{2.6}
\end{equation*}
$$

The relationship to use for any particular situation depends on the information given.

## Examples

## 1. Power of a torch bulb

A torch bulb takes a current of 0.2 A when operating from a 6 V battery.
Calculate the power rating of the bulb.
Answer:
Using Equation 2.4, we have:

$$
\begin{aligned}
P & =I V \\
\therefore P & =0.2 \times 6 \\
\therefore P & =1.2 \mathrm{~W}
\end{aligned}
$$

The power rating of the bulb is 1.2 W .

## 2. Power lost in transmission lines

Electricity from a power station is transmitted using transmission lines that have a resistance of $0.2 \Omega \mathrm{~km}^{-1}$. The two transmission lines are each 50 km long.
If the electricity is transmitted at a current of 80 A , calculate the total power lost in the transmission lines.

Answer:
Using Equation 2.5 and remembering that there are two transmission lines, we have:

$$
\begin{aligned}
P & =I^{2} R \\
\therefore P & =80^{2} \times(0.2 \times 2 \times 50) \\
\therefore P & =128 \mathrm{~kW}
\end{aligned}
$$

The total power loss is 128 kW .

## 3. Power dissipated in a resistor

Calculate the power dissipated (in this case - transformed into heat) in a $10 \mathrm{k} \Omega$ resistor that has a voltage of 2.0 V across it.

Answer:
Using Equation 2.6, we have:

$$
P=\frac{V^{2}}{R}
$$

$\therefore P=\frac{2 \times 2}{10 \times 10^{3}}$
$\therefore P=0.4 \mathrm{~mW}$
The power dissipated in the resistor is 0.4 mW .

We are now in a position to be able to calculate how much electrical energy is transferred in an electrical circuit. We know that power is the rate at which energy is transferred, so energy = power $x$ time. We also know that power in an electrical circuit can be calculated using power $=$ current $\times$ voltage. Combining these relationships we get:

$$
\begin{aligned}
\text { energy transferred } & =\text { current } \times \text { voltage } \times \text { time } \\
E_{W} & =I V t
\end{aligned}
$$

## Example : Energy transferred by a car headlamp bulb

A car headlamp bulb takes a current of 3.0 A from the 12 V car battery.
Calculate the amount of energy transformed by the bulb every minute.
Answer:
Using Equation 2.7, and remembering to convert minutes into seconds, we have:

$$
\begin{aligned}
E_{W} & =I V t \\
\therefore E_{W} & =3 \times 12 \times 60 \\
\therefore E_{W} & =2160 \mathrm{~J}
\end{aligned}
$$

The energy transformed by the bulb every minute is 2160 J .

The SI unit of energy, the joule, is a very small unit for most practical purposes. To illustrate this, consider how many joules of energy are transferred when a 2 kW electric kettle is on for 6 minutes, about the time it would take to boil a kettleful of water. The energy transferred in this case is $2000 \times 6 \times 60=720000$ joules.

A far more practical unit of electrical energy, used by all electricity suppliers, is the kilowatt-hour (kW h). As the name of this unit suggests, it uses the kilowatt (kW) rather than the watt, and the hour (h) rather than the second. When used for electricity supply, the kilowatt-hour is sometimes simply called a 'unit' of electricity.
energy transferred in kilowatt-hours $=$ power in kilowatts $\times$ time in hours
So the energy transferred in kilowatt-hours by our 2 kW kettle in 6 minutes ( $6 / 60$ hours) is $2 \times 6 / 60=0.2 \mathrm{~kW} \mathrm{~h}$.

## Example : Joules and kilowatt-hours

How many joules are equivalent to 1 kilowatt-hour?

## Answer:

1 kilowatt is 1000 joules per second, and there are $60 \times 60$ seconds in 1 minute, so:

$$
\begin{aligned}
1 \mathrm{kWh} & =1000 \mathrm{~W} \times 60 \times 60 \mathrm{~s} \\
\therefore 1 \mathrm{kWh} & =3600000 \mathrm{~J}
\end{aligned}
$$

1 kilowatt hour is equivalent to 3600000 joules.

## Quiz: Electrical energy and power

Go online

Q9: An electric locomotive draws a current of 400 A from the 25 kV overhead supply lines.
How much electrical energy is supplied to the locomotive every second?
a) 62.5 W
b) 167 kW
c) 1.56 MW
d) 10.0 MW
e) 600 MW

Q10: How much electrical energy is transformed into other forms of energy every second in a $150 \Omega$ resistor that has a steady current of 30 mA in it?
a) 0.36 mW
b) 135 mW
c) 4.5 W
d) 8.1 W
e) 20.25 W

Q11: A ray lamp bulb is marked $12 \mathrm{~V}, 48 \mathrm{~W}$.
What is the nominal resistance of the filament of this light bulb?
a) $0.0625 \Omega$
b) $0.25 \Omega$
c) $3.0 \Omega$
d) $4.0 \Omega$
e) $192 \Omega$

Q12: A mobile phone charger supplies a current of 5.0 mA for 12 hours to charge the phone's 3.0 V battery.
How much electrical energy is transferred during the charging process?
a) 1.25 mJ
b) 180 mJ
c) 10.8 J
d) 72.0 J
e) 648 J

Q13: An energy saving light bulb uses only $20 \%$ of the energy of a conventional light bulb, for the same light output.
A 100 W conventional light bulb is operated for 8 hours per day for 350 days in a year.
Calculate how many kilowatt hours of electricity are saved every year by replacing this conventional light bulb with an energy saving light bulb, giving the same light output.
a) 56
b) 224
c) 280
d) 13440000
e) 201600000

### 2.5 Resistors in series

A series circuit is a circuit in which there is only one path for the current to flow through the components.
You will previously have met the formula to calculate the total resistance of resistors connected in series.
$R=$ the sum of all the individual resistances.
In general, for $n$ resistors connected in series.

$$
\begin{equation*}
R=R_{1}+R_{2}+\ldots R_{n} \tag{2.8}
\end{equation*}
$$

Example Four resistors with resistances $1.8 \Omega$, $2.2 \Omega, 2.7 \Omega$ and $3.3 \Omega$ are connected in series across a supply that has a potential difference of 5 V .

Figure 2.2: Series circuit


Calculate the equivalent resistance and the current in the circuit.
Answer:
The equivalent resistance of resistors in series is given by the sum of all the individual resistances, so
$R=1.9+2.2+2.7+3.3$
$R=10 \Omega$
The current in the circuit is given by $I=v / R$, so
$I=\frac{V}{R}$
$I=\frac{5}{10}$
$I=0.5 \mathrm{~A}$
The current in the circuit is 0.5 A .

## Quiz: Resistors in series

At this stage there is an online activity. If however you do not have access to the internet you may try the questions which follow.

In the questions which follow, calculate the total resistance in a circuit where the following resistors are placed in series.

Q14: Two resistors of $10 \Omega$ and $150 \Omega$

Q15: Two resistors of $220 \Omega$ and $150 \Omega$

Q16: Three resistors of $150 \Omega, 150 \Omega$ and $200 \Omega$

Q17: Two resistors of $10 \Omega$ and $50 \Omega$

### 2.6 Resistors in parallel

A parallel circuit is a circuit in which there is more than one path for the current to flow through the components

Since charge is conserved in this circuit, the rate at which charge flows into junction $P$ must equal the rate at which charge flows out of the same junction. The rate of flow of charge is current, so
$I=I_{1}+I_{2}+I_{3}$
The three resistors are connected across the supply and therefore the potential difference $V$ appears across all three resistors so, using $I=V / R$, we have

$$
\begin{aligned}
& \frac{V}{R}=\frac{V}{R_{1}}+\frac{V}{R_{2}}+\frac{V}{R_{3}} \\
& \frac{V}{R}=V\left(\frac{1}{R_{1}}+\frac{1}{R_{2}}+\frac{1}{R_{3}}\right) \\
& \frac{1}{R}=\frac{1}{R_{1}}+\frac{1}{R_{2}}+\frac{1}{R_{3}}
\end{aligned}
$$

You will previously have met the formula to calculate the total resistance of resistors connected in parallel.

Where $R$ is the equivalent resistance in the circuit.
In general, for $n$ resistors connected in parallel

$$
\begin{equation*}
\frac{1}{R}=\frac{1}{R_{1}}+\frac{1}{R_{2}}+\ldots+\frac{1}{R_{n}} \tag{2.9}
\end{equation*}
$$

## Example : Resistors in parallel

Two resistors with resistances $27 \Omega$ and $33 \Omega$ are connected in parallel across a cell that has a potential difference of 1.5 V and negligible internal resistance. Calculate the equivalent resistance and the current taken from the cell.

Figure 2.3: Parallel circuit


Answer:
The equivalent resistance of resistors in parallel is given by the relationship
$\frac{1}{R}=\frac{1}{R_{1}}+\frac{1}{R_{2}}+\frac{1}{R_{3}}$
SO
$\frac{1}{R}=\frac{1}{27}+\frac{1}{33}$
$\frac{1}{R}=0.067$
$\therefore R=15 \Omega$
The current in the circuit is given by $I=V / R$, so
$I=\frac{V}{R}$
$I=\frac{1.5}{15}$
$I=0.1 \mathrm{~A}(100 \mathrm{~mA})$
The current in the circuit is 100 mA .

In the example, you will see that the total equivalent resistance $R$ is $15 \Omega$ which is less than either individual resistance. This is always the case when resistors are connected in parallel - the total resistance is always less that each of the individual resistances.

## Quiz: Resistors in parallel

At this stage there is an online activity. If however you do not have access to the internet you may try the questions which follow.

Go online
For the following questions calculate the total resistance of the circuit when the resistors are place in parallel.

Q18: Two resistors of $10 \Omega$ and $150 \Omega$

Q19: Two resistors of $200 \Omega$ and $100 \Omega$

Q20: Three resistors of $150 \Omega, 150 \Omega$ and $50 \Omega$

Q21: Three resistors of $10 \Omega, 5 \Omega$ and $50 \Omega$

### 2.7 Potential divider

A potential divider is a circuit consisting of a number of resistors (often only two) in series, connected across a supply, that is used as a source of fixed or of variable p.d.

Figure 2.4: Potential divider circuit


In Figure 2.4, the potential difference across each of the resistors $R_{1}$ and $R_{2}$ is a fixed fraction of the potential difference of the supply $\left(V_{\mathrm{s}}\right)$. This means that the fixed potential of point P is determined by the values of the two resistors $R_{1}$ and $R_{2}$. The ratio of the potential differences across the resistors in a potential divider circuit is the same as the ratio of the resistances of the resistors. The following relationships hold for all potential divider circuits that consist of two resistors, such as the one shown in Figure 2.4.

$$
\begin{equation*}
\frac{V_{1}}{V_{2}}=\frac{R_{1}}{R_{2}} \tag{2.10}
\end{equation*}
$$

and since $V_{s}=V_{1}+V_{2}$ then

$$
\begin{equation*}
V_{1}=V_{s} \times \frac{R_{1}}{R_{1}+R_{2}} \tag{2.11}
\end{equation*}
$$

and

$$
\begin{equation*}
V_{2}=V_{s} \times \frac{R_{2}}{R_{1}+R_{2}} \tag{2.12}
\end{equation*}
$$

## Example : Fixed potential divider

A potential divider consists of $R_{1}=270 \Omega$ in series with $R_{2}=330 \Omega$, connected across a supply of $V_{\mathrm{s}}=12 \mathrm{~V}$.
Calculate the potential difference across $R_{1}$.

## Answer:

The potential divider circuit is as shown in Figure 2.4.
$V_{1}$ is the potential difference across $R_{1}$, so using Equation 2.11 we see that

$$
\begin{aligned}
V_{1} & =V_{s} \times \frac{R_{1}}{R_{1}+R_{2}} \\
\therefore V_{1} & =12 \times \frac{270}{270+330} \\
\therefore V_{1} & =12 \times \frac{270}{600} \\
\therefore V_{1} & =5.4 \mathrm{~V}
\end{aligned}
$$

The potential difference across the resistor is 5.4 V .

The usefulness of the potential divider circuit is greatly extended if the resistance of either or both of the resistors in the potential divider can be varied. You will
already have met the variable resistor - a component that has a resistance that can be changed. You should already know that the resistance of a light dependent resistor (LDR) decreases as the intensity of the light reaching it increases. You should also know that the resistance of an ntc thermistor decreases as the temperature of the thermistor increases.

These three circuit components can be used to replace the fixed resistors in a potential divider circuit, to make circuits that respond in different ways to external factors. The output potential difference from a potential divider can be varied manually or automatically, or it can increase or decrease as the light level increases, or it can increase or decrease as the temperature increases.
A variable resistor can be connected as a potential divider as shown in Figure 2.5. When connected in this way, the variable resistor is usually referred to as a potentiometer.

Figure 2.5: Potentiometer


The output of the potential divider, $V_{\text {out }}$, can be adjusted by changing the position of the slider on the potentiometer from A to B . $V_{\text {out }}$ can be varied continuously from 0 , when the slider is at end A , to $V_{\mathrm{s}}$, when the slider is at end B .

## Potential divider output

There is an online demonstration of this.


Go online

Consider Figure 2.6, which shows a potential divider that consists of a thermistor in series with a variable resistor.

Figure 2.6: Thermistor in potential divider circuit


As the temperature of the thermistor increases, its resistance decreases. Using Equation 2.10, we can see that this means that the voltage across the thermistor also decreases. Since $V_{\mathrm{s}}$ is fixed, this means that $V_{\text {out }}$ (the voltage across the variable resistor) increases. So it can be seen that in this circuit, $V_{\text {out }}$ increases as the temperature increases and decreases as the temperature decreases. The resistance of the variable resistor can be preset to give a suitable output voltage $V_{\text {out }}$ for particular ambient conditions.

## 5

## Practical potential divider circuits

Q22: Describe and explain the potential divider circuit shown in the diagram.


Q23: Describe and explain the potential divider circuit shown in the diagram.


Q24: Describe and explain the potential divider circuit shown in the diagram.

$\qquad$

You should be able to describe and explain the use of thermistors and light-dependent resistors in potential dividers to provide a potential difference that is dependent on temperature and on light intensity respectively.

## Quiz: The potential divider

The potential divider circuit shown is used in the first two questions in this quiz.


Q25: What is the relationship between $V_{1}, V_{2}, R_{1}$, and $R_{2}$ in the potential divider circuit shown?
a) $\frac{V_{1}}{V_{2}}=\frac{R_{1}}{R_{2}}$
b) $V_{1} \times V_{2}=R_{1} \times R_{2}$
c) $V_{1}=V_{2} \times \frac{R_{1}}{R_{1}+R_{2}}$
d) $V_{2}=V_{1} \times \frac{R_{2}}{R_{1}+R_{2}}$
e) $R_{1}=R_{2}=V_{1}=V_{2}$

Q26: In the potential divider circuit shown, $V_{\mathrm{s}}=15 \mathrm{~V}, R_{1}=820 \Omega$ and $R_{2}=680 \Omega$.
What is the value of $V_{1}$ ?
a) 6.8 V
b) 8.2 V
c) 15 V
d) 680 V
e) 820 V

The potential divider circuit shown below is used in the next three questions in this quiz.


Q27: Describe the action of the potential divider circuit shown.
a) The output voltage increases as the light level increases.
b) The output voltage decreases as the light level increases.
c) The output voltage increases as the resistance of the variable resistor increases.
d) The output voltage increases as the temperature increases.
e) The output voltage decreases as the temperature increases.

Q28: In the diagram, the resistance of the variable resistor is set to $1000 \Omega$. Under certain conditions, the resistance of the thermistor becomes $3500 \Omega$.
What is the output voltage, $V_{\text {out }}$, if $V_{\mathrm{s}}$ is 9.0 V ?
a) 2.0 V
b) 3.5 V
c) 4.5 V
d) 7.0 V
e) 9.0 V

Q29: In the diagram, $V_{\mathrm{s}}=9.0 \mathrm{~V}$ and the resistance of the variable resistor is set to 1000 $\Omega$.

What is the range of the output voltage, $V_{\text {out }}$, if the resistance of the thermistor varies between $500 \Omega$ and $2000 \Omega$ ?
a) 0.0 to 3.0 V
b) 0.0 to 6.0 V
c) 0.0 to 9.0 V
d) 3.0 V to 6.0 V
e) 6.0 V to 9.0 V

### 2.8 The Wheatstone bridge

The Wheatstone bridge circuit consists of a series/parallel arrangement of resistors. Although it bears his name, Sir Charles Wheatstone did not actually invent this circuit. Wheatstone was a railway engineer in the nineteenth century who was concerned with electrical signalling on the railways. He used the bridge circuit as a means of sending messages. In one application, the Wheatstone bridge circuit is used to measure resistance.

### 2.8.1 The balanced Wheatstone bridge circuit

In its usual arrangement, the Wheatstone bridge circuit consists of four resistors, connected to a source as shown in Figure 2.7.

Figure 2.7: The Wheatstone bridge circuit


Consider firstly the potential divider consisting of $R_{1}$ and $R_{2}$. Using the relationship
between voltage, current and resistance, we see that

$$
\begin{aligned}
V_{R_{1}} & =I_{1} R_{1} \\
V_{R_{2}} & =I_{1} R_{2} \\
V & =V_{R_{1}}+V_{R_{2}}
\end{aligned}
$$

In a similar way, by considering $R_{3}$ and $R_{4}$, we see that

$$
\begin{aligned}
V_{R_{3}} & =I_{2} R_{3} \\
V_{R_{4}} & =I_{2} R_{4} \\
V & =V_{R_{3}}+V_{R_{4}}
\end{aligned}
$$

By a suitable choice of resistor values, we can arrange that $V_{R_{1}}=V_{R_{3}}$ and $V_{R_{2}}=V_{R_{4}}$. For this condition, we have

$$
\begin{aligned}
I_{1} R_{1} & =I_{2} R_{3} \\
\text { so } \frac{I_{2}}{I_{1}} & =\frac{R_{1}}{R_{3}} \\
\text { and } I_{1} R_{2} & =I_{2} R_{4} \\
\text { so } \frac{I_{2}}{I_{1}} & =\frac{R_{2}}{R_{4}} \\
\therefore \frac{R_{1}}{R_{3}} & =\frac{R_{2}}{R_{4}}
\end{aligned}
$$

In this situation, points $P$ and $Q$ in Figure 2.7 are at the same potential, so $V_{P Q}$ is zero. A voltmeter connected across these points would read zero potential - it would show null deflection. In this condition, the Wheatstone bridge circuit is said to be balanced.

The balanced Wheatstone bridge is used to measure the value of an unknown resistor (say $R_{1}$ ) as follows. Two of the resistors (say $R_{3}$ and $R_{4}$ ) are fixed-value resistors that have known resistances. Usually these values are equal or close to each other. The final resistor $\left(R_{2}\right)$ is a variable resistor, often a resistance box, where the value of the resistance can be altered, but is known. When the bridge circuit is balanced, and this condition is recognised by a zero value connected across $P$ and $Q$ showing a zero value, the value of the unknown resistor $R_{1}$ is calculated from

$$
\begin{equation*}
R_{1}=R_{3}\left(\frac{R_{2}}{R_{4}}\right) \tag{2.13}
\end{equation*}
$$

The benefit of this method of measuring resistance is that the internal resistance of the meter used does not affect the reading, since the meter is read when the potential difference across it is zero, and hence there is no current in the meter branch when the bridge circuit is balanced. An accurate meter is not needed, only a sensitive one with a correctly marked zero.

## Example

The Wheatstone bridge circuit shown in Figure 2.8 is balanced when $R_{2}$ is set to 384 $\Omega$.

Figure 2.8: Wheatstone bridge


Calculate the resistance of $R_{1}$.
Answer:
Another way of representing a Wheatstone bridge circuit is shown in Figure 2.8. You should be able to see that electrically this is similar to Figure 2.7.
At balance:

$$
\begin{aligned}
R_{1} & =R_{2}\left(\frac{R_{3}}{R_{4}}\right) \\
\therefore R_{1} & =384\left(\frac{500}{400}\right) \\
\therefore R_{1} & =480 \Omega
\end{aligned}
$$

## Quiz: The Wheatstone bridge

Try the questions. If your answer is incorrect or you do not understand a question, look at the hints provided. If you read the hint and still do not understand then ask your tutor.

The Wheatstone bridge circuit shown is used in the first three questions in this quiz.


Q30: The Wheatstone bridge circuit shown is balanced.
What is the relationship between the resistances?
a) $R_{1}+R_{2}=R_{3}+R_{4}$
b) $\quad R_{1}-R_{2}=R_{3}-R_{4}$
c) $R_{1} \times R_{2}=R_{3} \times R_{4}$
d) $\frac{R_{1}}{R_{2}}=\frac{R_{3}}{R_{4}}$
e) $\quad R_{1}=\frac{R_{2}}{R_{3} \times R_{4}}$

Q31: The Wheatstone bridge circuit shown is balanced when $R_{2}=60 \Omega, R_{3}=30 \Omega$ and $R_{4}=90 \Omega$.
What is the value of $R_{1}$ ?
a) $20 \Omega$
b) $30 \Omega$
c) $60 \Omega$
d) $90 \Omega$
e) $180 \Omega$

Q32: The Wheatstone bridge circuit shown is balanced when $R_{1}=600 \Omega$ and $R_{2}=400$ $\Omega$.
The resistances of $R_{3}$ and $R_{4}$ could be
a) $R_{3} 80 \Omega ; R_{4} 120 \Omega$
b) $R_{3} 120 \Omega ; R_{4} 80 \Omega$
c) $R_{3} 400 \Omega ; R_{4} 400 \Omega$
d) $R_{3} 400 \Omega ; R_{4} 600 \Omega$
e) $R_{3} 600 \Omega ; R_{4} 600 \Omega$

In the following circuit, the supply has negligible internal resistance.


Q33: The voltage across one of the $50 \Omega$ resistors is:
a) 0 V
b) 3.0 V
c) 6.0 V
d) 8.0 V
e) 12.0 V

Q34: The voltage across the $60 \Omega$ resistors is:
a) 0 V
b) 3.0 V
c) 6.0 V
d) 8.0 V
e) 12.0 V

A voltmeter is now added to this circuit as shown below.


This circuit is now a Wheatstone bridge circuit.

Q35: The magnitude of the reading shown on the voltmeter is:
a) 0 V
b) 2.0 V
c) 4.0 V
d) 8.0 V
e) 12.0 V

### 2.9 Summary

## Summary

You should now be able to:

- state that charge is measured in coulombs;
- state that current is the rate of flow of charge;
- state that current is measured in amperes and that one ampere is one coulomb per second;
- solve problems using the equation $\mathrm{Q}=\mathrm{It}$;
- explain what is meant by electrical potential;
- define the volt as one joule per coulomb;
- explain what is meant by potential difference;
- define electrical power in terms of current and potential difference;
- solve problems using $\mathrm{E}_{\mathrm{w}}=\mathrm{QV}$ and $\mathrm{P}=\mathrm{IV}$;
- explain what is meant by resistance;
- state the unit for resistance is the ohm;
- carry out calculations using $\mathrm{V}=\mathrm{IR}$;
- define electrical power in terms of potential difference, current and resistance;
- explain the kilowatt hour as a unit of energy;
- solve problems using $P=I V=I^{2} R=V^{2} / R$;
- solve problems using $\mathrm{E}_{\mathrm{W}}=\mathrm{ItV}$;
- state the equation for the total resistance of a number of resistors in series;
- solve problems involving resistors in series;
- state the equation for the total resistance of a number of resistors in parallel;
- solve problems involving resistors in parallel;
- explain what is meant by a potential divider;
- describe the operation of potential dividers;
- describe how potential dividers are used in circuits to control electronic devices;
- carry out calculations involving potential dividers;


## Summary

- describe a Wheatstone bridge circuit;
- describe a balanced Wheatstone bridge;
- carry out calculations in a balanced Wheatstone bridge;


### 2.10 Extended information

The authors do not maintain these web links and no guarantee can be given as to their effectiveness at a particular date.
They should serve as an insight to the wealth of information available online and encourage readers to explore the subject further.

- http://www.gcse.com/volt1.htm

Use this link for support materials on voltage and current

- http://www.gcse.com/energy/kWh.htm

This link on the same site gives some practical power examples.

- http://www.allaboutcircuits.com/vol_1/chpt_8/10.html This site reinforces the theory on the Wheatstone bridge and gives some extended
- http://www.electronics-tutorials.ws/resistor/res_3.htm This page can be used to revise resistors in series.
- http://www.electronics-tutorials.ws/resistor/res_4.html This page can be used to revise resistors in parallel.
- http://www.citycollegiate.com/wheatstone_bridge.htm This is a very good summary of the wheatstone bridge.
- http://physics.csustan.edu/ian/java/tutorials/EM/ElectronGun/ElectronGun.htm This simulation allows the user to adjust the accelerating voltage in an electron gun and the user has to calculate the final velocity of the electron after it has been accelerated.
- http://www.walter-fendt.de/ph14e/wheatstone_e.htm An interactive simulation that allows a Wheatstone bridge circuit to be balanced.
- http://www.educationscotland.gov.uk/highersciences/ physics/unitthree/electronsandenergy/current.asp
An Education Scotland link to student notes on series and parallel circuits along with Wheatstone bridge circuits.


### 2.11 Assessment

## End of topic 2 test

The following test contains questions covering the work from this topic.
A reminder of useful data values can be found in the information sheet (opened by clicking
$\square$ within a test).

The end of topic test is available online. If however you do not have access to the web, you may try the following questions.

Q36: Resistors of resistance $3.3 \Omega$ and $1.2 \Omega$ are available

1. Calculate the total resistance in $\Omega$ when they are connected in series. (Give your answer to this question to one decimal place.)
2. Calculate the total resistance in $\Omega$ when they are connected in parallel. (Give your answer to this question to one decimal place.)

Q37: Forty lamps, each of resistance $1.25 \Omega$ are connected in series and operated from the 230 V mains supply.
Calculate the current, in A, taken from the supply

Q38: Two resistors $R_{1}$ and $R_{2}$ are connected in parallel.
The resistance of $R_{1}$ is less that the resistance of $R_{2}$. The total resistance $R_{T}$ is
a) less than the resistance of $R_{1}$
b) equal to the resistance of $R_{1}$
c) between the resistance of $R_{1}$ and $R_{2}$
d) equal to the resistance of $R_{2}$
e) greater than the resistance of $R_{2}$

Q39: A Wheatstone bridge circuit is shown.


Calculate the resistance of $R_{2}$, in $\Omega$, needed to balance the bridge.

Q40: What value of resistor, in $\Omega$, must be connected in parallel with a $54 \Omega$ resistor to reduce the total resistance to $18 \Omega$ ?

## Topic 3

## Electrical sources and internal resistance

## Contents

3.1 Sources and circuits ..... 60
3.2 Load matching (an interesting application of internal resistance) ..... 65
3.3 Summary ..... 68
3.4 Extended information ..... 68
3.5 Assessment ..... 69

## Learning objectives

By the end of this topic you should be able to:

- explain what is meant by the e.m.f. internal resistance and the terminal p.d of a source;
- carry out calculations using the relationships involving the e.m.f., the terminal p.d. and the internal resistance of a source;
- explain that the maximum power output from a source occurs when the external resistance (the load) matches the internal resistance;
- determine internal resistance and e.m.f. using graphical analysis;
- analyse circuits containing ideal supplies, short circuits or open circuits.

In previous topics on electricity, you will have met charge, voltage, current and resistance. The electric circuit will also be familiar to you. An electric circuit in its simplest form consists of a source of energy (perhaps a battery), a component to transform the electrical energy into some other form of energy, and a closed path of conductors. In this topic, we will look again at the electric circuit, and in particular at the source of energy used in a circuit.

### 3.1 Sources and circuits

We have already seen that an electrical source, such as a cell, a battery or even the mains supply or a thermocouple, supplies the energy to the charges in an electrical circuit. The term used for this quantity is electromotive force, or e.m.f. for short. Electromotive force ( $E$ ), like potential difference, is measured in volts.

Figure 3.1: Measuring the e.m.f. of an electrical source


A voltmeter placed across the terminals of an electrical source, as in Figure 3.1, measures the e.m.f. of the source. This is because under these conditions the voltmeter is measuring the open circuit potential difference across the terminals of the source (the terminal potential difference or t.p.d. of the source). The open circuit t.p.d. of a source is equal to the e.m.f. of the source, as we will see shortly.
Consider a battery sending a current round an electrical circuit. This current is a flow of electrical charges around the circuit, both externally to the battery (in the external circuit, often called the load resistor, or more simply, the 'load') but also internally in the battery itself. If an 'ideal' source were to exist then it would supply a constant e.m.f. between its terminals no matter what current is taken from it. Such an ideal source does not exist (just like a frictionless surface does not exist). All practical sources of electrical energy present an opposition to the movement of the charges through the source itself - they have an internal resistance, given the symbol $r$.

We now have to think of a source of electrical energy as a supplier of e.m.f. $E$ in series with an internal resistance $r$. Although the internal resistance appears between the plates of the source as in Figure 3.2(a), it is often more convenient to separate the source of e.m.f. from the internal resistance, and picture the source as in Figure 3.2(b).

Figure 3.2: Source of electrical energy showing the internal resistance


The internal resistance of a source obeys Ohm's law and so $r$ is constant and is independent of the current I through it. The e.m.f. of a source is defined as the energy given by the source to each one coulomb of charge as it passes through the source. For example, a source of e.m.f 9 V gives 9 J of energy to each 1 C of charge as the charge passes through the source.

The e.m.f. of a source is also expressed as being equal to the sum of all the potential differences across all of the resistors in the circuit, including the potential difference across the internal resistance, $r$.

Consider the circuit shown in Figure 3.3.

Figure 3.3: Source connected to an external resistor


In this circuit, the e.m.f. of the source $E$ is equal to the potential difference $V$ across the external resistor of resistance $R$, plus the potential difference across the internal resistance $r$.

$$
\begin{align*}
E & =I R+I r \\
E & =V+I r \\
E-V & =I r \\
r & =\frac{E-V}{I} \tag{3.1}
\end{align*}
$$

In Equation 3.1, the term $V$ is the potential difference that appears at the terminals of the source. For this reason it is called the terminal potential difference (t.p.d.). The term Ir represents the potential difference that is 'lost' across the internal resistance of the source, and never appears in the external circuit. This term is often called the lost volts. It is worth noting that both $E$ and $r$ are properties of the source and are constant (at least in the short term, if the source is not abused). On the other hand both the terminal potential difference and the lost volts depend on the current taken from the source, and so are not constant.

Example A cell has an e.m.f. of 1.5 V . Its terminal potential difference falls to 1.2 V when driving a current through an external resistor of resistance $5.0 \Omega$.

Calculate the current in the circuit, the lost volts and the internal resistance of the cell.
Answer:
Applying Ohm's law to the external resistor:

$$
\begin{aligned}
V & =I R \\
\text { so } I & =\frac{V}{R} \\
I & =\frac{1.2}{5.0} \\
I & =0.24 \mathrm{~A}
\end{aligned}
$$

Re-arranging Equation 3.1:
Ir $=E-V$
Ir $=1.5-1.2$
lost volts $=0.3 \mathrm{~V}$
Using Equation 3.1:
Lost volts $=I r$
$0.3=0.24 \times r$

$$
r=1.25 \Omega
$$

The current in the circuit is 0.24 A , the lost volts is 0.3 V and the internal resistance of the cell is 1.25 ohms.

## Measuring the e.m.f. and internal resistance of a source

The circuit shown was used to find the e.m.f. $E$ and the internal resistance $r$ of a cell.


The following results were obtained.

| Voltage $V\left(\mathrm{~V}_{\text {tpd }}\right)$ | Current $I(\mathrm{~A})$ |
| :---: | :---: |
| 1.110 | 1.110 |
| 1.246 | 0.831 |
| 1.368 | 0.507 |
| 1.446 | 0.371 |
| 1.485 | 0.265 |
| 1.524 | 0.186 |

Plot a graph of voltage ( $\mathrm{V}_{\mathrm{tpd}}$ ) against current, and from it calculate:

1. the e.m.f. $E$ of the cell, (Remember the e.m.f. of the cell is the potential difference when no current is drawn from the cell).
2. the internal resistance $r$ of the cell, (You can use equation 3.1).
3. the maximum current that the cell is capable of delivering (the short-circuit current).

The e.m.f. and internal resistance of a source can both be calculated from a graph of voltage against current. Quiz: e.m.f. and internal resistance

First try the questions. If you get a question wrong or do not understand a question, use the Hints. If you read the hint and still do not understand then ask your teacher or tutor.

Q1: Which of the following terms is equivalent to internal resistance?
a) $\frac{V}{E}$
c) $\frac{E-V}{I}$
d) $I R$
e) $I r$

Q2: Which of the following terms is known as the 'lost volts'?
a) $E$
b) $V$
c) $I R$
d) $I r$
e) $\frac{E-V}{I}$

Q3: A cell has an e.m.f. of 1.54 V . When it is in series with a $1.00 \Omega$ resistor, the reading on a voltmeter across the cell is 1.40 V .
What is the internal resistance of the cell?
a) $0.10 \Omega$
b) $0.14 \Omega$
c) $0.71 \Omega$
d) $1.40 \Omega$
e) $1.54 \Omega$

Q4: A cell has an e.m.f. of 1.52 V .
What is its internal resistance if its short-circuit current is 2.50 A ?
a) $4.02 \Omega$
b) $3.80 \Omega$
c) $1.64 \Omega$
d) $0.980 \Omega$
e) $0.608 \Omega$

Q5: A battery of e.m.f. 9.0 V and internal resistance $3.0 \Omega$ is connected to a resistance of $15 \Omega$.
What is the potential difference across the terminals of the battery?
a) 0.5 V
b) 1.5 V
c) 7.5 V
d) 9.0 V
e) 15 V

### 3.2 Load matching (an interesting application of internal resistance)

When choosing a power supply for an application it is important to match the external resistance of the appliance to the internal resistance of the power supply.

## Load matching

Power = 1.44 watts


In this circuit the resistance of the external resistor (the load) is altered and the current produced by the supply is measured.

Q6: Calculate the power delivered by the supply at each resistance value using the formula $P=I^{2} R$ and complete the table.
The first is done for you as per the diagram.

| Resistance $\Omega$ | Current A | Power W |
| :--- | :--- | :--- |
| 1 | 1.2 | 1.44 |
| 2 | 1.0 |  |
| 3 | 0.857 |  |
| 4 | 0.75 |  |
| 5 | 0.667 |  |
| 6 | 0.6 |  |
| 7 | 0.545 |  |
| 8 | 0.5 |  |
| 9 | 0.461 |  |
| 10 | 0.428 |  |
| 11 | 0.4 |  |
| 12 | 0.375 |  |

Draw a graph of the resistance values against the power values.
When the power output is plotted against the load resistance it is clear that the maximum power output from a source takes place when the load is equal to the internal resistance of a source.

## Examples

1. A circuit containing a loudspeaker, an ammeter and a signal generator of e.m.f 12 V and internal resistance $r$ is constructed as shown.


When the loudspeaker is producing maximum power, the current in the circuit is measured as 0.75 A .
Step 1 Calculate total resistance

$$
\begin{aligned}
& V=I \times R_{t} \\
& 12=0.75 \times R_{t} \\
& R_{t}=16 \Omega
\end{aligned}
$$

Step 2 Calculate value of internal resistance $r$.

$$
R_{t}=R_{r}+R_{\text {loudspeaker }}
$$

At maximum power out, the value of the internal resistance $r$ and the resistance of the load, the loudspeaker, must be equal. $\Rightarrow r=8 \Omega$.
2. Two 8.0 ohm speakers have to be connected in parallel to a signal generator.

The signal generator has three outputs marked 4.0 ohms, 16.0 ohms, 32.0 ohms. To which output should the speakers be connected?
Calculate the internal resistance $r$ of the signal generator.
Answer:
Calculate the total load resistance. As the two speakers are connected in parallel we must calculate their total effective resistance.

$$
\begin{aligned}
\frac{1}{\mathrm{R}_{\mathrm{T}}} & =\frac{1}{\mathrm{R}_{1}}+\frac{1}{\mathrm{R}_{2}} \\
\frac{1}{\mathrm{R}_{\mathrm{T}}} & =\frac{1}{8}+\frac{1}{8} \\
\frac{1}{\mathrm{R}_{\mathrm{T}}} & =\frac{2}{8} \\
\mathrm{R}_{\mathrm{T}} & =\frac{8}{2} \\
\mathrm{R}_{\mathrm{T}} & =4 \text { ohms }
\end{aligned}
$$

The speakers should therefore be connected to the 4.0 ohm output to ensure the maximum transfer of power from the signal generator.

### 3.3 Summary

## Summary

You should now be able to:

- explain what is meant by the e.m.f. of a source;
- describe an electrical source in terms of e.m.f. and internal resistance;
- carry out calculations using the relationships involving the e.m.f., the terminal p.d. and the internal resistance of a source;
- determine internal resistance and e.m.f. from a graph of $\mathrm{V}_{\text {tpd }}$ against current;
- analyse circuits containing ideal supplies, short circuits or open circuits.


### 3.4 Extended information

The authors do not maintain these web links and no guarantee can be given as to their effectiveness at a particular date.
They should serve as an insight to the wealth of information available online and encourage readers to explore the subject further.

- http://www.absorblearning.com/advancedphysics/demo/units/020301.htmI\#EMFbasics There are many extra questions available in this site which complement the work already covered. There will be a fuller description on LEDs in a later topic of the SCHOLAR course.
- http://www.antonine-education.co.uk/Pages/Physics_1/Electricity/EL_08/electricity_8.htm This tutorial includes an explanation of internal resistance and tutorial questions.
- http://www.science-campus.com/engineering/electrical/ dc_theory/chapter6/dctheory_6_7.html
This site shows the charges being energised as they pass through the cell and de-energies as they pass through resistors.
- http://www.educationscotland.gov.uk/highersciences/physics/ unitthree/electronsandenergy/electricalsources.asp
Notes and questions specifically written for the Higher physics course.


### 3.5 Assessment

## End of topic 3 test

The following test contains questions covering the work from this topic.

A reminder of useful data values can be found in the information sheet (opened by

Q7: A cell has an e.m.f. of 1.51 V , and an internal resistance of $0.059 \Omega$.
Calculate the short-circuit current of the cell, in A.

Q8: A battery with an e.m.f. of 9.0 V and an internal resistance of $0.3 \Omega$, is connected to an external resistor of resistance $4.2 \Omega$.


1. Calculate the current in the circuit, in A .
2. Calculate the lost volts, in V .
3. Calculate the terminal potential difference of the battery, in V .
4. What value of total external resistance should be connected to the battery to give the maximum power output from the battery?

Q9: A d.c. generator produces an e.m.f. of 230 V on open circuit. When it is connected to a load that takes a current of 22 A , the terminal potential difference of the generator falls to 220 V .


Calculate the internal resistance of the generator, in $\Omega$.

Q10: A car battery has an e.m.f. of 14.0 V , and an internal resistance of $0.10 \Omega$.


While in normal use, a voltmeter connected across the battery indicates that the terminal potential difference of the battery is 12.2 V .

1. Calculate the current taken from the battery, in A.
2. Calculate the terminal potential difference, in V , of the battery when the starter motor, which takes a current of 95A, is operated.

## Topic 4

## Capacitors

## Contents

4.1 What is a capacitor ..... 75
4.2 Charge and capacitance ..... 75
4.2.1 Energy stored in a capacitor ..... 77
4.3 Capacitors in circuits ..... 81
4.3.1 Charging a capacitor ..... 81
4.3.2 Discharging a capacitor ..... 85
4.3.3 Capacitors in a.c. circuits ( for interest only) ..... 89
4.3.4 Applications of capacitors ..... 91
4.4 Summary ..... 96
4.5 Extended information ..... 96
4.6 Assessment ..... 97

## Learning objectives

By the end of this topic you should be able to:

- state that the charge stored on two parallel conducting plates is proportional to the potential difference across the plates, and describe the principles of a method to demonstrate this;
- define capacitance;
- state that the unit of capacitance is the farad, and that one farad is equal to one coulomb per volt;
- carry out calculations using the relationship $C=Q_{V}$;
- explain why work must be done to charge a capacitor;
- state that the work done in charging a capacitor is given by the area under the Q-V graph;
- state the expressions $E_{c}=\frac{1}{2} Q V=\frac{1}{2} C V^{2}=\frac{1}{2} \frac{Q^{2}}{C}$ for the energy $E_{c}$ stored on a capacitor, and carry out calculations using these expressions;
- sketch graphs of voltage and current against time for charging and discharging capacitors in series CR circuits;
- carry out calculations on voltage and current in series CR circuits;
- describe and explain some applications of capacitors.


### 4.1 What is a capacitor

A capacitor is made of two pieces of metal separated by an insulator. In the 1700s one of the first capacitors used for research was built using a glass jar. One piece of metal was placed on the inside of the jar and the other on the outside. The glass acted as the insulator.

Capacitors are now made in many shapes and sizes but all have this same basic design of two pieces of metal separated by an insulator.

If the outside piece of metal is charged negatively then the inside piece of metal will become equally positively charged. Similarly if the outside piece of metal was charged positively then the inside piece would become negatively charged.

When the capacitor becomes charged there will be a potential difference across the two pieces of metal.

The circuit symbol for a capacitor is shown in Figure 4.3.
The capacitance of a capacitor is measured in farads ( $F$ ) or more commonly microfarads ( $\mu \mathrm{F}, \times 10^{-6}$ ) or nanofarads ( $\mathrm{nF}, \times 10^{-9}$ ).
The capacitance of a capacitor depends on its construction not the charge on it or the potential difference across it.

### 4.2 Charge and capacitance

We are going to consider the case of two parallel conducting plates, which have a fixed separation between them. When there is an electric field between the plates, then there is a potential difference V across them. The following activity allows you to investigate the relationship between V and the amount of charge Q stored on the plates.

## Investigating $\mathbf{V}_{\mathbf{c}}$ and $\mathbf{Q}_{\mathbf{c}}$

A circuit is set up to investigate the relationship between the voltage and the charge stored on the capacitor.

Go online


The voltage of the supply is increased (between 0.1 V and 1.0 V ) and the charge on the capacitor is noted.

The results obtained are used to produce the following graph.


Since there is a linear relationship (shown by a straight line graph through the origin) between charge $(Q)$ and p.d. (V), we can state that

$$
\begin{equation*}
\mathrm{Q} \propto \mathrm{~V} \tag{4.1}
\end{equation*}
$$

Rather than leave the relationship Equation 4.1 with a proportional to sign, we can put in a constant of proportionality $C$ (equal to the gradient of the $Q-V$ plot), so that Equation 4.1 becomes

$$
\begin{align*}
\mathrm{Q} & =C V \\
\text { or } \mathrm{C} & =\frac{Q}{V} \tag{4.2}
\end{align*}
$$

$C$ is called the capacitance. The unit of capacitance is the farad $F$, where $1 F=1 C^{-1}$. The farad is named after the English physicist Michael Faraday (1791-1867). Equation 4.2 shows that capacitance is the ratio of charge to p.d.

A system of two parallel conductors is called a capacitor. In fact, any two conductors that are insulated from each other form a capacitor, but in this course we will only be studying the parallel-plate capacitor. For any parallel-plate capacitor, its capacitance
$C$ depends on the surface area of the plates, the distance between the plates and the insulating material (air or another insulator) that separates the plates.

It is worth pointing out that 1 F is a very large capacitance, and we will never encounter such a huge capacitance in practice. Practical capacitors have capacitances in the microfarad ( $1 \mu \mathrm{~F}=1 \times 10^{-6} \mathrm{~F}$ ), nanofarad ( $1 \mathrm{nF}=1 \times 10^{-9} \mathrm{~F}$ ) or picofarad ( $1 \mathrm{pF}=1 \times 10^{-12}$ F) regions.

## Example

A $50 \mu \mathrm{~F}$ capacitor is charged to 0.40 mC . Calculate the potential difference between the plates.

Answer:
To answer this question, we will use Equation 4.2, remembering to convert the capacitance into farads, and the charge into coulombs.

$$
\begin{aligned}
C & =\frac{Q}{V} \\
\therefore 50 \times 10_{-6} & =\frac{0.40 \times 10^{-3}}{V} \\
\therefore V & =8.0 \mathrm{~V}
\end{aligned}
$$

The potential difference between the plates is 8.0 V .

### 4.2.1 Energy stored in a capacitor

Let us consider the charged parallel-plate capacitor shown in Figure 4.1.
Figure 4.1: Electric field between two charged plates


Suppose we take an electron from the left-hand plate and transfer it to the right-hand
plate. We have to do work in moving the electron since the electrical force acting on it opposes this motion. The more charge that is stored on the plates, the more difficult it will be to move the electron since the electric field between the plates will be larger.

The work that is done in placing charge on the plates of a capacitor is stored as potential energy in the charged capacitor. The more charge that is stored on the capacitor, the greater the stored potential energy.

Figure 4.2 shows the plot of charge against potential difference that we saw in the previous section.

Figure 4.2: Plot of charge against p.d. for a capacitor


It can be shown that the area under the graph (between the plotted line and the p.d.axis) is equal to the work done in charging the capacitor. Since the graph is a straight line, the area between the line and the p.d.-axis forms a right-angled triangle. The area of a right-angled triangle $=1 / 2$ base $\times$ height, so for a capacitor with charge $Q$ and p.d. $V$, the work done is

Work done $=1 / 2 \times Q \times V$

As we have seen, this work done is equal to the energy stored on the capacitor. Using Equation 4.2, we can state three equivalent expressions for the energy $E_{c}$ stored on a capacitor:

$$
\begin{align*}
& E_{c}=\frac{1}{2} Q V \\
& E_{c}=\frac{1}{2} C V^{2} \\
& E_{c}=\frac{1}{2} \frac{Q^{2}}{C} \tag{4.3}
\end{align*}
$$

## Energy stored on a capacitor

$$
\begin{aligned}
\mathrm{E}_{c} & =\frac{1}{2} Q V \\
E_{c} & =\frac{1}{2} C V^{2} \\
E_{c} & =\frac{1}{2} \frac{Q^{2}}{C}
\end{aligned}
$$

These are three expressions for the energy stored on a charged capacitor. Starting from the equation $E_{c}=1 / 2 Q V$, and using $Q=C V$ or $C=Q / v$, can you show that these three expressions are equivalent?

Example Let us return to the $50 \mu \mathrm{~F}$ capacitor charged to 0.40 mC . How much energy is stored on the capacitor?
Answer:

$$
\begin{aligned}
& \mathrm{E}_{c}=\frac{1}{2} \frac{Q^{2}}{C} \\
& \therefore E_{c}=\frac{1}{2} \times \frac{\left(0.40 \times 10^{-3}\right)^{2}}{50 \times 10^{-6}} \\
& \therefore E_{c}=\frac{1}{2} \times \frac{1.6 \times 10^{-7}}{50 \times 10^{-6}} \\
& \therefore E_{c}=1.6 \times 10^{-3} \mathrm{~J}
\end{aligned}
$$

The energy stored in the capacitor is $1.6 \times 10^{-3} \mathrm{~J}$.

## Extra Help: Using the energy relationships $E=Q V$ and $E=1 / 2 \mathbf{Q V}$

At this stage there is an online activity which revises energy relationships.

Go online

## Quiz: Capacitors

Q1: 1 F is equivalent to
a) $1 \mathrm{C} \mathrm{V}^{-1}$
b) $1 \mathrm{JC}^{-1}$
c) 1 CV
d) $1 \mathrm{VC} \mathrm{C}^{-1}$
e) $1 \mathrm{CJ}^{-1}$

Q2: The potential difference across a 500 nF capacitor is 12 V . How much charge is stored on the capacitor?
a) 3.0 pC
b) 24 pC
c) 42 nC
d) $6.0 \mu \mathrm{C}$
e) $72 \mu \mathrm{C}$

Q3: When the charge stored on a certain capacitor is $8.0 \times 10^{-8} \mathrm{C}$, the p.d. across it is 3.6 V . What is the capacitance of this capacitor?
a) 4.5 nF
b) 6.2 nF
c) 22 nF
d) 290 nF
e) $1.0 \mu \mathrm{~F}$

Q4: How much work is done in charging a $40 \mu \mathrm{~F}$ capacitor to $6.4 \times 10^{-4} \mathrm{C}$ ?
a) $1.25 \mu \mathrm{~J}$
b) 5.1 mJ
c) 31 mJ
d) 8.0 J
e) 130 J

Q5: When the charge on a capacitor is $1.4 \mu \mathrm{C}$, the potential difference across the capacitor is 0.45 V . What is the p.d. across the capacitor when the charge on it is 5.6 $\mu \mathrm{C}$ ?
a) 0.11 V
b) 1.8 V
c) 1.9 V
d) 3.5 V
e) 16 V

### 4.3 Capacitors in circuits

In this section of the topic, we will be looking at how capacitors behave when they are connected as components in d.c. circuits.
The circuit symbol for a capacitor is shown in Figure 4.3.

Figure 4.3: Capacitor circuit symbol


A capacitor is effectively a break in the circuit, and charge cannot flow across it. We will see now how this influences the current in capacitive circuits.

### 4.3.1 Charging a capacitor

## Charging a capacitor

There is an activity online at this stage. The activity provides a circuit with a capacitor and resistor which can be altered. The shape of the output graphs is also given.

Figure 4.4 shows a simple d.c. circuit in which a capacitor is connected in series to a battery and resistor. This is often called a series CR circuit.

Figure 4.4: Simple d.c. capacitive circuit


When the switch S is closed, charge can flow on to (but not across) the capacitor C . At the instant the switch is closed the capacitor is uncharged, and it requires little work to add charges to the capacitor. As we have already discussed, though, once the capacitor has some charge stored on it, it takes more work to add further charges. Figure 4.5 shows graphs of current / through the capacitor (measured on the ammeter) and charge $Q$ on the capacitor, against time.

Increasing the R or C value increases the rise time however the final p.d. across the capacitor will remain the same. The final p.d. across the capacitor will equal the e.m.f., E , of the supply.

Figure 4.5: Plots of current and charge against time for a charging capacitor


Since the potential difference across a capacitor is proportional to the charge on it, then a plot of p.d. against time will have the same shape as the plot of charge against time shown in Figure 4.5.

Suppose the battery in Figure 4.4 has e.m.f $E$ and negligible internal resistance. The sum of the p.d.s across $C$ and $R$ must be equal to $E$ at all times. That is to say,
$V_{C}+V_{R}=E$
where $V_{C}$ is the p.d. across the capacitor and $V_{R}$ is the p.d. across the resistor. At the instant switch $S$ is closed there is no charge stored on the capacitor, so $V_{C}$ is zero, hence $V_{R}=E$. The current in the circuit at the instant the switch is closed is given by

$$
\begin{equation*}
I=\frac{E}{R} \tag{4.4}
\end{equation*}
$$

As charge builds up on the capacitor, so $V_{C}$ increases and $V_{R}$ decreases. This is shown in Figure 4.6.

Figure 4.6: Plots of p.d. against time for a capacitive circuit


The charge and potential difference across the capacitor follow an exponential rise. (The current follows an exponential decay). The rise time (the time taken for the capacitor to become fully charged) depends on the values of the capacitance $C$ and the resistance $R$. The relationship between rise time, $C$ and $R$ is quite complex, but it is enough for us to be able to state that the rise time increases if either $C$ or $R$ increases. So, for example, replacing the resistor R in the circuit in Figure 4.4 by a resistor with a greater resistance will result in the p.d. across the capacitor C rising more slowly, and the current in the circuit dropping more slowly.

## Example

Consider the circuit in Figure 4.7, in which a $40 \mathrm{k} \Omega$ resistor and an uncharged $220 \mu \mathrm{~F}$ capacitor are connected in series to a 12 V battery of negligible internal resistance.

Figure 4.7: Capacitor and resistor in series


1. What is the potential difference across the capacitor at the instant the switch is closed?
2. After a certain time, the charge on the capacitor is $600 \mu \mathrm{C}$. Calculate the potential differences across the capacitor and the resistor at this time.

Answer:

1. At the instant the switch is closed, the charge on the capacitor is zero, so the p.d. across it is also zero.
2. We can calculate the p.d. across the capacitor using Equation 4.2:

$$
\begin{aligned}
V_{c} & =\frac{Q}{C} \\
\therefore V_{c} & =\frac{600 \times 10^{-6}}{220 \times 10^{-6}} \\
\therefore V_{c} & =2.7 \mathrm{~V}
\end{aligned}
$$

Since the p.d. across the capacitor is $2.7 \mathrm{~V}_{\mathrm{c}}$, the p.d. across the resistor is 12 $2.7=9.3 \mathrm{~V}_{\mathrm{c}}$.

### 4.3.2 Discharging a capacitor

## Discharging a capacitor

There is an activity online at this stage showing how the capacitor charges and discharges.

The circuit shown in Figure 4.8 can be used to investigate the charging and discharging of a capacitor.

Figure 4.8: Circuit used for charging and discharging a capacitor


When the switch $S$ is connected to $x$, the capacitor $C$ is connected to the battery and resistor R, and will charge in the manner shown in Figure 4.5. When $S$ is connected to $y$, the capacitor is disconnected from the battery, and forms a circuit with the resistor R. Charge will flow from $C$ through $R$ until $C$ is uncharged. A plot of the current against time is given in Figure 4.9.

Figure 4.9: Current as the capacitor is charged, and then discharged


Remember that the capacitor acts as a break in the circuit. Charge is not flowing across the gap between the plates, it is flowing from one plate through the resistor to the other plate. Note that the direction of the current reverses when we change from charging to discharging the capacitor. The energy which has been stored on the capacitor is dissipated in the resistor.
Charging current:The initial charging current is very large. Its value can be calculated by

$$
I=\frac{V_{\text {supply }}}{R}
$$

The current is only at this value for an instant of time. As the capacitor charges, the p.d. across the capacitor increases so the p.d. across the resistor decreases causing the current to decrease.

Discharging current: The initial discharging current is very large. Its value can be calculated by

$$
I=\frac{V_{\text {capacitator }}}{R}
$$

During discharge the circuit is not connect to the supply so it is the p.d. across the capacitor, not the p.d. across the supply, which drives the current. If however the capacitor had been fully charged, the initial p.d. across the capacitor would equal the p.d. across the supply.

The current is only at this value for an instant of time. As the capacitor discharges, the p.d. across the capacitor decreases so the p.d. across the resistor also decreases causing the current to decrease. When the capacitor is fully discharged, the p.d. across it will be zero hence the current will also be zero.

Figure 4.9 shows us that at the instant when the capacitor is allowed to discharge, the size of the current is extremely large, but dies away very quickly. This leads us to one of the applications of capacitors, which is to provide a large current for a short amount of time. One example is the use of a capacitor in a camera flash unit. The capacitor is charged by the camera's batteries. At the instant the shutter is pressed, the capacitor is allowed to discharge through the flashbulb, producing a short, bright burst of light.

## Using the energy stored on a capacitor

At this stage there is an online activity. If however you do not have access to the internet you should ensure that you understand the following explanation.

Go online

When a lamp is lit from a d.c. supply directly it gives a steady dim energy output.


It is now connected to a capacitor and charged as shown


The capacitor is then discharged through the lamp by changing the switch position as shown.


- When the capacitor powers the lamp, a large current flows for a very short period
of time. This produces a bright flash of light.
- The current flows for only a short time while the capacitor discharges.
- Before the flash can be used again the capacitor must be recharged from the supply.


### 4.3.3 Capacitors in a.c. circuits ( for interest only)

What happens when we replace the d.c. source in a $C R$ circuit with an a.c. supply?

Figure 4.10: a.c. capacitive circuit


Let us consider the situation shown in Figure 4.10, in which the current is measured using an a.c. ammeter. Remember that we saw earlier when a d.c. supply is used, the current rapidly drops to zero once the switch is closed.

In an a.c. circuit there is a steady current through the capacitor. This is because the capacitor is charging and discharging every time a.c changes direction. if the frequency of the a.c. is high enough this will appear to give a steady reading on the a.c. ammeter.

Use the following online activity to investigate the relationship between r.m.s. current and frequency in a capacitive circuit.

## The capacitor and a.c. ( for interest only)

This activity will demonstrate that the impedance of a capacitive circuit is inversely proportional to the frequency of the supply.

At this stage there is an online activity. If however you do not have access to the internet you should ensure that you understand the explanation which follows.

A circuit is set up to investigate the relationship between frequency of an a.c. supply and current in a capacitive circuit.

The r.m.s. voltage of the supply is kept constant.


The frequency of the supply is increased and the r.m.s. current is measured.
The results obtained are used to produce the following graph.


We can see that the r.m.s current increases as the frequency increases - a CR circuit passes high frequency a.c. much better than it does low frequency a.c. or d.c. Why is this?

You should remember that charge does not flow across the plates of a capacitor. It accumulates on the plates, and the more charge that has accumulated, the more work is required to add extra charges. At all times, the total charge on the plates of the capacitor is zero. Charges are merely transferred from one plate to the other via the external circuit when the capacitor is charged. At low frequency, as the applied voltage oscillates, there is plenty of time for lots of charge to accumulate on the plates, which means the current drops more at low frequency (see Figure 4.5). At high frequency, there is only a short time for charge to accumulate on the plates before the direction of the current is reversed, and the capacitor discharges.

### 4.3.4 Applications of capacitors

We have already seen how capacitors can be used to store electrical energy and deliver a short, high energy burst of electricity. In this section we will look at some other applications of capacitors in electrical circuits.

Flashing indicators:The control circuits for flashing lights often contain a capacitor. Examples of these circuits include home security systems, advertising signs and road signs warning drivers of a school.
The control circuit for a flashing neon lamp is shown below.
The neon lamp is in parallel with the capacitor so they p.d. across the neon lamp will equal the p.d. across the capacitor.
The neon lamp will light when the p.d. across it reaches 100 V . It will continue to until the p.d. across it falls below 80 V .

The trace produced on the oscilloscope is shown below.
On a rising part of the graph: The p.d. across the capacitor and lamp is less than 100 V so the lamp is off. The capacitor is charging.


At 100 V :The p.d. across the capacitor and the lamp has reached 100 V so the lamp will light.


On a falling part of the graph:The neon lamp is lit and the capacitor is discharging through the neon lamp. The p.d. across the capacitor and lamp is falling.


At 80 V : The p.d. across the capacitor and lamp has fallen to 80 V so the lamp goes off and the capacitor will begin to charge again.


80 V


Reducing the value of the resistor will allow the capacitor to charge in less time so the lamp will flash more frequently. The time between each flash will be less.

Reducing the value of the capacitor will allow the capacitor to charge in less time so the lamp will flash more frequently. The time between each flash will be less.
$\Rightarrow$ As the capacitance is less, the capacitor will discharge in less time so the lamp will be lit for less time each flash.
$\Rightarrow$ Also as the capacitance is less, the capacitor will store less energy so the flash will be less bright.
In recent years touch screens have become a feature of devices such as mobile phones and computers.


Source: Kenneth M Pennington
When a person touches the screen the device registers where the contact has been made and relays this information to the device's processor. These screens work by 'capacitive sensing'. The glass screen is coated with a transparent conductor such as indium tin oxide and a small voltage is applied to this layer. Human skin is a (poor) conductor and when it comes into contact with the screen a capacitor is formed at that point. The device then senses the formation of the capacitor, a process call capacitive sensing and sends the information to a processor. You may have noticed that these devices do not work when you are wearing gloves - this is because the gloves are an insulator and the capacitor can only form when a conductor, like skin, comes into contact with it.

## Quiz: Capacitors in circuits

In the following quiz, the first three questions refer to the circuit shown. The circuit contains an uncharged $60 \mu \mathrm{~F}$ capacitor and a $36 \Omega$ resistor connected to a 9.0 V battery of negligible internal resistance.


Q6: What is the current in the circuit at the instant the switch is closed?
a) 150 mA
b) 250 mA
c) 600 mA
d) 4.0 A
e) 6.7 A

Q7: What is the p.d. across the resistor at the instant when the current is 50 mA ?
a) 1.4 mV
b) 450 mV
c) 1.8 V
d) 3.0 V
e) 9.0 V

Q8: What is the p.d. across the capacitor at the instant when the current is 50 mA ?
a) 450 mV
b) 3.0 V
c) 7.2 V
d) 8.3 V
e) 9.0 V

Q9: A capacitor is connected in series to an a.c. power supply and an a.c. ammeter. As the frequency of the a.c. is slowly increased from 20 Hz to 2500 Hz , whilst its r.m.s. voltage remains unchanged, the current measured by the meter
a) increases.
b) decreases.
c) is constant and non-zero.
d) is zero at all times.
e) increases, then decreases.

### 4.4 Summary

## Summary

You should now be able to

- state that the charge stored on two parallel conducting plates is proportional to the potential difference across the plates, and describe the principles of a method to demonstrate this;
- state that capacitance is the ratio of charge to potential difference;
- state that the unit of capacitance is the farad, and that one farad is equal to one coulomb per volt;
- perform calculations using the relationship $C=Q / V$;
- explain why work must be done to charge a capacitor;
- state that the work done in charging a capacitor is given by the area under the $Q-V$ graph;
- state the expressions $E_{c}=\frac{1}{2} Q V=\frac{1}{2} C V^{2}=\frac{1}{2} \frac{Q^{2}}{C}$ for the energy $E_{c}$ stored on a capacitor, and carry out calculations using these expressions;
- sketch graphs of voltage and current against time for charging and discharging capacitors in series $C R$ circuits;
- carry out calculations on voltage and current in series CR circuits;
- describe and explain some applications of capacitors.


### 4.5 Extended information

The authors do not maintain these web links and no guarantee can be given as to their effectiveness at a particular date.
They should serve as an insight to the wealth of information available online and encourage readers to explore the subject further.

- http://www.technologystudent.com/elec1/capac1.htm

This site gives similar information to the content but leads to some other uses for capacitors.

- http://phet.colorado.edu/en/simulation/circuit-construction-kit-dc

An interactive simulation that enable students to build circuits and investigate the effects of changing $R$ and $C$ values.

- http://www.educationscotland.gov.uk/highersciences/physics/ unitthree/electronsandenergy/capacitors.asp
An Education Scotland resource specifically for Higher Physics.


### 4.6 Assessment

## End of topic 4 test

The following test contains questions covering the work from this topic.

The following data should be used when required:
Magnitude of the charge on an electron $\mathrm{e}=1.6 \times 10^{-19} \mathrm{C}$

The end of topic test is available online. If however you do not have access to the web, you may try the following questions.

Q10: The potential difference across a 22 nF capacitor is 6.9 V .

1. Calculate how much charge in $C$ is stored on the capacitor.
2. Calculate how much energy in J is stored on the capacitor.

Q11: This question refers to the circuit below, in which a battery of e.m.f. E is connected in series to a capacitor (capacitance C) and a resistor (resistance).


The battery has e.m.f. $\mathrm{E}=20 \mathrm{~V}$ and negligible internal resistance. $\mathrm{C}=340 \mu \mathrm{~F}$ and $\mathrm{R}=$ $6.3 \mathrm{k} \Omega$. The d.c. ammeter records the current in the circuit in mA .

1. State the p.d.(in V ) across the resistor at the instant the switch is closed.
2. Calculate the current in $(\mathrm{mA})$ recorded on the ammeter at the instant the switch is closed.
3. After several seconds, the ammeter reads zero current. State the value of the p.d. (in V ) across the resistor now.
4. State the p.d. (in V ) across the capacitor when the current in the circuit has dropped to zero.

Q12: A $38 \mu \mathrm{~F}$ capacitor is charged by transferring electrons from one plate to the other, until the potential difference between the plates is 1.9 V .

1. Calculate the potential energy stored on the capacitor, in J.
2. Calculate the number of electrons that have been transferred from one plate to the other.

Q13: A 620 nF capacitor and a $350 \mathrm{k} \Omega$ resistor are connected in series to a switch and a 12 V battery.
Shortly after the switch is closed, the potential difference across the resistor is 5.3 V . Calculate the p.d. (in V ) across the capacitor at this time.

Q14: When the capacitor in the circuit shown is fully charged by a 9.0 V supply, it is found that the charge stored on it is $1.9 \times 10^{-5} \mathrm{C}$.


Calculate the charge (in C) stored on the capacitor if it is charged by a 15 V supply.

## Topic 5

## Conductors, semiconductors and insulators

## Contents

5.1 Electrical properties ..... 101
5.2 The structure of the atom ..... 101
5.3 Conductors, insulators and semiconductors ..... 103
5.4 Valence band, conduction band and band gap ..... 105
5.5 Thermistors ..... 106
5.6 The Hall effect ..... 108
5.7 Summary ..... 109
5.8 Extended information ..... 109
5.9 Assessment ..... 110

## Learning objectives

By the end of this topic you should be able to:

- state solids can be categorised into conductors, semiconductors or insulators by their ability to conduct electricity;
- state that the electrons in atoms are contained in energy levels;
- state that when atoms come together to form solids, the electrons then become contained in energy bands separated by gaps;
- explain that in metals, which are good conductors, the highest occupied band is not completely full and this allows the electrons to move and therefore conduct. This band is known as the conduction band;
- explain that in an insulator the highest occupied band (called the valence band) is full and that the first unfilled band above the valence band is the conduction band;
- explain that for an insulator the gap between the valence band and the conduction band is large and at room temperature there is not enough energy available to move electrons from the valence band into the conduction band where the electrons would be able to contribute to conduction and therefore there is no electrical conduction in an insulator;
- explain that in a semiconductor the gap between the valence band and conduction band is smaller and at room temperature there is sufficient energy available to move some electrons from the valence band into the conduction band allowing some conduction to take place;
- state that the space between the valence band and the conduction band is known as the band gap;
- explain that the bigger the band gap, the more energy that is required to move an electron from the valence band to the conduction band;
- explain that an increase in temperature increases the conductivity of a semiconductor.

In this topic we will study the electrical properties of materials. We start the topic with a look at the electrical properties of different materials and see how they can be classified into three groups; conductors, insulators and semiconductors. We will then go on to look more closely at semiconductors, and in particular how their characteristics are affected by temperature.

### 5.1 Electrical properties

Early on in your study of science you learned that materials are either conductors or insulators. In reality the difference between them is more a matter of degree than of type.

Every substance resists the flow of charge to some extent but some have a higher resistance than others. If you put a big enough voltage across any material, charge will flow through it; think about what happens when lightning hits a building or a tree.

As you know, conductors have a low resistance and insulators have a high resistance but there is no clear distinction between the two; there is no cutoff resistance that defines whether a material should be classed as one thing or the other. However this is not a problem in choosing materials for a specific purpose since there are plenty of good conductors and even more good insulators to choose from.
Some materials have a resistance somewhere between that of good conductors and good insulators. For instance the human body is a poor conductor but the voltage of the mains supply is quite enough to deliver a fatal electric shock.

Table 5.1 shows a selection of conductors, insulators and semiconductors. In general, metals and alloys such as steel are conductors while most non-metals and compounds are insulators. The semiconductors are the substances with a resistance somewhere between that of conductors and insulators, that also have electrical properties that can be manipulated by the addition of impurities as shown in a later topic.

Table 5.1: Electrical properties

| Conductors | Insulators | Semiconductors |
| :---: | :---: | :---: |
| Iron | Carbon (Diamond) | Silicon |
| Tungsten | Wood | Germanium |
| Carbon (Graphite) | Plastic | Selenium |

### 5.2 The structure of the atom

You should know from your previous studies that the atom consists of a central positively charged nucleus with electrons orbiting the nucleus in fixed orbits. These orbits are sometimes called shells or bands. When we are considering these orbits in this section
we will often regard them as energy levels. We say that the electrons occupy energy levels inside the atom.


To help us visualize this situation we can produce an energy level diagram as shown in Figure 5.1.

The lowest line in the diagram represents the lowest energy that an electron can have and is called the ground state. Electrons on higher levels are said to be in an excited state and the top line represents the ionisation level of the atom. Notice that the energy levels get closer together the further they are from the ground state.

The values refer to the energy an electron must gain to reach the ionisation level. Every element has its own unique energy level diagram and the energies shown in Figure 5.1 refer to an atom of hydrogen.

Figure 5.1: Energy level diagram
$\left.\left.\begin{array}{l}W_{4}=\overline{W_{3}=-1.30 \times 10^{-19} \mathrm{~J}} \\ W_{2}=-2.43 \times 10^{-19} \mathrm{~J} \\ W_{1}=-5.44 \times 10^{-19} \mathrm{~J} \\ W_{0}=-21.8 \times 10^{-19} \mathrm{~J}\end{array}\right\} \begin{array}{c}\begin{array}{c}\text { Ionisation } \\ \text { level }\end{array} \\ \text { Excited } \\ \text { states }\end{array}\right\}$ Ground state

### 5.3 Conductors, insulators and semiconductors

You should know from the previous section that electrons occupy different energy levels within an atom. These levels are sometimes called shells, orbits or bands with the highest energy levels corresponding to the outermost shells and lower energy levels to inner shells.

The electrical properties of an element depend on the number of electrons in its outermost, or valence shell, as the inner electrons are more tightly bound to the nucleus of the atom and don't take part in conduction. The maximum number of electrons that an element can have in its valence shell is eight. A full valency shell is very stable - it is difficult to remove electrons from a full shell. Elements that have a full valency shell are very good insulators.

Metals have a low number of valence electrons and these are easily removed from the atom; this means that metals are good conductors. Non-metals have four or more valence electrons but they are more tightly bound to the nucleus and are much harder to remove; this makes them good insulators.

This means that in metals the highest occupied band is not completely full. This is known as the conduction band. This is the first unfilled band above the valence band.

For insulators the energy gap between the valence band and the conduction band is large. This means that at room temperature the electrons are unable to gain enough energy to move from the valence band to the conduction band and so insulators do not conduct electricity.

It is the elements with four electrons in their outer shell that are used as semiconductors. If we look at a diagram of pure silicon with its four outer electrons (Figure 5.2), we see that the atoms form a regular pattern and that the electrons pair up with those from neighbouring atoms. By sharing, each atom 'sees' the stable 8 electron closed outer shell and so the material has a very high resistance. Pure semiconductors are also referred to as intrinsic semiconductors.

Figure 5.2: Silicon crystal structure


However for these materials the energy gap between the valence band and the conduction band is much smaller than in insulators. This means that at room temperature it is possible for some electrons to gain enough energy to move from the valence band to the conduction band.

When this happens some conduction will take place.
When the temperature of a semiconductor is increased the electrons will gain more energy and so more will move from the valence band to the conduction band. This means that as the temperature of a semiconductor increases its conductivity will increase. The opposite is true for metallic conductors: when temperature increases their conductivity falls.

Figure 5.3 allows a comparison to be made.

Figure 5.3: Conductivity


### 5.4 Valence band, conduction band and band gap

The diagram below shows the relative energies of the valence band and conduction band of an insulator, semiconductor and conductor. The gap between the valence and conduction bands is known as the band gap. No electrons are allowed in this gap.

In order for a material to conduct there must be electrons free to move in the conduction band. This occurs when the conduction band is only partially filled.

The larger the band gap the greater the energy that is required to move an electron from the valence band to the conduction band.


In an insulator the valence band is filled by electrons and the band gap is large. At room temperature electrons do not normally gain enough energy to make the transition from the valence band to the conduction band. This means that there will be no electrons in the conduction band so conduction is not possible.
In a semiconductor the valence band is filled by electrons but the band gap is smaller than that in an insulator. At room temperature it is possible that some electrons will gain enough energy to make the transition from the valence band to the conduction band. This means that there may be some electrons in the conduction band so some conduction is possible.

In a metal conductor there is only one band of interest, the conduction band, and it is partially filled by electrons so can always conduct. As the temperature of the metal conductor increases the number of the electrons in the conduction band increases to such an extent that their freedom to move and conduct decreases, see Figure 5.3. This means that the resistance of a metal conductor will increase with temperature.

### 5.5 Thermistors

Semiconductors can be used to manufacture components called thermistors. Thermistors are electrical components that alter their resistance at different temperatures. The symbol for a thermistor is shown.


## Temperature effect on a negative coefficient thermistor

The effect of temperature on a negative coefficient thermistor can be investigated using a circuit.

At this stage there is an online activity. If however you do not have access to the internet you may try the question which follows.


When the temperature of the water is altered the following data is obtained.

| Temperature (degrees Celsius) | Resistance (ohms) |
| :--- | :--- |
| 5 | 26000 |
| 10 | 20000 |
| 15 | 16000 |
| 20 | 13000 |
| 25 | 10000 |
| 30 | 8000 |
| 35 | 6500 |
| 40 | 5300 |
| 45 | 4300 |
| 50 | 3600 |
| 55 | 3000 |
| 60 | 2500 |
| 65 | 2100 |
| 70 | 1700 |
| 75 | 1500 |
| 80 | 1250 |

Q1: Use the data and draw a graph of resistance against temperature ( $x$-axis). What is your conclusion of the effect of altering the temperature of the water?

### 5.6 The Hall effect

When a current carrying conductor is placed in a magnetic field the electrons are deflected in their path according to the direction of the magnetic field and the direction of the current. This is known as the Hall effect.


This causes a voltage to develop between the top and the bottom of the conductor shown in the diagram above. It should be emphasised that this voltage is very small and has no noticeable effect in most situations.

However this can be put to practical uses. Hall effect probes can be used to detect and measure magnetic fields. One of the most practical uses of the Hall effect is to find electrical cables in walls. As current flows through the cables it produces a magnetic field this can be detected using a Hall probe. Hall probes can also be used to measure current without inserting a meter into the circuit and thus interfering with the flow of current.

Hall sensors are now found in many modern applications such as measuring the speed of a rotating motor and in pressure sensors.

### 5.7 Summary

## Summary

You should now be able to:

- state solids can be categorised into conductors, semiconductors or insulators by their ability to conduct electricity;
- state that the electrons in atoms are contained in energy levels;
- state that when atoms come together to form solids, the electrons then become contained in energy bands separated by gaps;
- explain that in metals, which are good conductors, the highest occupied band is not completely full and this allows the electrons to move and therefore conduct. This band is known as the conduction band;
- explain that in an insulator the highest occupied band (called the valence band) is full and that the first unfilled band above the valence band is the conduction band;
- explain that for an insulator the gap between the valence band and the conduction band is large and at room temperature there is not enough energy available to move electrons from the valence band into the conduction band where the electrons would be able to contribute to conduction and therefore there is no electrical conduction in an insulator;
- explain that in a semiconductor the gap between the valence band and conduction band is smaller and at room temperature there is sufficient energy available to move some electrons from the valence band into the conduction band allowing some conduction to take place;
- state that the space between the valence band and the conduction band is known as the band gap;
- explain that the bigger the band gap, the more energy that is required to move an electron from the valence band to the conduction band;
- explain that an increase in temperature increases the conductivity of a semiconductor.


### 5.8 Extended information

The authors do not maintain these web links and no guarantee can be given as to their effectiveness at a particular date.
They should serve as an insight to the wealth of information available online and encourage readers to explore the subject further.

- http://www.physicsclassroom.com/class/estatics/u811d.cfm

This site provides a detailed explanation on conductors and insulators

- http://nobelprize.org/educational/physics/semiconductors/index.html

This is an excellent site with good explanations. It will be useful for all students.

- http://www.st-andrews.ac.uk/~www_pa/Scots_Guide/info/comp/conduct/conduct.htm The St Andrews site provides good materials which can be used to reinforce student understanding of this topic.
- http://www.electronics-tutorials.ws/diode/diode_1.html

Students should find this site useful as revision for this topic and for others in this unit.

- http://www.educationscotland.gov.uk/highersciences/physics/unitthree/ electronsatwork/conductors.asp
This Education Scotland website has been written specifically to support students following the Higher Physics course.
- http://www.chemistry.wustl.edu/~edudev/LabTutorials/PeriodicProperties/ MetalBonding/MetalBonding.html
This site contains background information on band structure and applications of semiconductor devices which will be covered in the next topic.


### 5.9 Assessment

## End of topic 5 test

The following test contains questions covering the work from this topic.
Go online
The end of topic test is available online. If however you do not have access to the web, you may try the following questions.

Q2: Sort these materials into conductors, insulators and semiconductors.
Iron, Carbon (Diamond), Wood, Germanium, Tungsten, Silicon

| Conductors | Insulators | Semiconductors |
| :--- | :--- | :--- |
|  |  |  |
|  |  |  |
|  |  |  |

Q3: Which of the following statements is/are true?

1. Electrons in atoms are contained in energy levels.
2. In insulators some of the electrons are in the conduction band.
3. In conductors all of the electrons are in the valence band.
4. The first band above the valence band is known as the conduction band.
5. The greater the band gap, the greater the energy required by an electron to move from the valance band to the conduction band.

Q4: Which of these elements would show a decrease in conductivity if the temperature was increased?

1. Germanium
2. Carbon
3. Silicon
4. Aluminium

## Topic 6

## p-n junctions

## Contents

6.1 Doping . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 115
6.1.1 Band structure diagrams for intrinsic and doped semiconductors . . . . 118
6.2 p-njunctions . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 120
6.2.1 Forward bias . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 123
6.2.2 Reverse bias . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 125
6.2.3 Voltage against current graph . . . . . . . . . . . . . . . . . . . . . . . . 126
6.2.4 Band structure diagrams . . . . . . . . . . . . . . . . . . . . . . . . . . . 127
6.3 Light emitting diodes and solar cells . . . . . . . . . . . . . . . . . . . . . . . . 133
6.3.1 Light emitting diodes . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 133
6.3.2 Band structure of an LED . . . . . . . . . . . . . . . . . . . . . . . . . . 136
6.3.3 Colour of LED . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 137
6.3.4 Photodiodes, solar cells and the photovoltaic effect . . . . . . . . . . . . 138
6.4 Summary . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 144
6.5 Extended information . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 144
6.6 Assessment . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 145

## Learning objectives

By the end of this topic you should be able to

- explain that during manufacture; the conductivity of semiconductors can be controlled, resulting in two types: p-type and n-type;
- state than an electron is a negative charged carrier;
- state that a hole acts as a positive charged carrier;
- explain that a hole is actually a position where an electron is missing;
- explain how the doping process results in free electrons in n-type semiconductors and holes in p-type semiconductors;
- explain that when p-type and n-type materials are joined, a depletion layer is formed at the junction. The electrical properties of this depletion layer are used in a number of devices;
- describe the movement of charge through a forward or reverse biased p-n junction diode;
- explain that LEDs are p-n junctions which emit photons when a current is passed through the junction;
- explain that solar cells are p-n junctions designed so that a potential difference is produced when photons enter the depletion layer. This is the photovoltaic effect;
- explain the operation of the above devices in terms of band theory.

In this topic we will study how semiconductors are produced and some of their uses. In particular we will examine diodes, solar cells and LEDs (light emitting diodes).

### 6.1 Doping

You studied the basic properties of semiconductor materials in the last topic. You should know that some materials (e.g. silicon, germanium) lie somewhere between conductors and insulators. These materials are called semiconductors. These materials have four electrons in the valence band but the energy gap between the valence band and the conduction band is quite small. This means that a few electrons are available for conduction.

We also noted that when temperature increased more electrons moved from the valence band to the conduction band and so conductivity increased with temperature in semiconductor materials.

Another way of improving the conductivity of semiconductors is by doping.
Doping is the process of adding tiny amounts of impurity atoms into a crystal of pure semiconductor material. By tiny amounts we mean less than one impurity atom per billion. The impurity atoms chosen will either have three or five valence electrons and we can see the effect of this on the crystal structure of silicon in Figure 6.1

Figure 6.1: Doping


If we add atoms with five valence electrons one of the electrons will be loosely bound and able to move freely within the crystal structure, effectively lowering the resistance of the material. This is known as n-type semiconductor as it has extra negative charge carriers (electrons) within it.
Adding atoms with three outer electrons results in p-type semiconductor with 'holes' in its electron structure. These holes can be thought of as positive charge carriers since electrons from neighbouring atoms can move into them and a new hole will be formed. In this way it looks as though the holes are moving in the opposite direction to the electrons
(Figure 6.2).
Figure 6.2: Hole movement


It is important to realise that both $n$-type and $p$-type semiconductors are electrically neutral since they still have equal numbers of protons and electrons. It is only the electron arrangement in the valence bands that is different.

### 6.1.1 Band structure diagrams for intrinsic and doped semiconductors

The band structure for intrinsic, n-type and p-type semiconductors are shown below.


## Intrinsic semiconductor

At zero Kelvin the valence band will be filled by electrons. At room temperature some electrons will gain enough energy to move to the conduction band. Each electron which moves to the conduction band will leave behind a hole in the valence band.

Both the holes in the valence band and the electrons in the conduction band are free to move.

## n-type semiconductor

When a crystal is doped with a group 5 atom there will be a free electron. This loosely bound electron will be in the conduction band. The result is that the n-type semiconductor will now be able to conduct by the movement of electrons. The doping of the crystal reduces the resistance of the crystal.

## p-type semiconductor

When a crystal is doped with a group 3 atom there will be a free hole. This hole will be in the valence band. The result is that the p -type semiconductor will now be able to conduct by the movement of holes. The doping of the crystal reduces the resistance of the crystal.

## Quiz: Semiconductors

Q1: Which one of the following statements about pure silicon is true?
a) It has a low resistance
b) It is a metal
c) It has five electrons in its outer shell
d) It is electrically neutral
e) It has positive holes in its outer electron shell

Q2: Which one of the following statements about semiconductor doping is false?
a) The semiconductor material has equal numbers of protons and electrons
b) The impurity atoms make up about $10 \%$ of the new material
c) n-type material is made by doping with atoms having 5 electrons in their outer shell
d) The majority charge carriers in p-type material are positive holes
e) Doping has the effect of lowering the resistance of the semiconductor

Q3: Which of the following statements about n-type semiconductors is/are true?
(i) Electrons are the majority charge carriers.
(ii) They contain more electrons than a pure semiconductor of the same size.
(iii) They have the same number of electrons as a pure semiconductor of the same size.
a) (i) only
b) (ii) only
c) (iii) only
d) (i) and (ii) only
e) (i) and (iii) only

Q4: Which of the following statements about p-type semiconductors is/are true?
(i) Electrons are the majority charge carriers.
(ii) They contain more electrons than a pure semiconductor of the same size.
(iii) They have fewer electrons than a pure semiconductor of the same size.
a) (i) only
b) (ii) only
c) (iii) only
d) (i) and (ii) only
e) (i) and (iii) only

Q5: Which of the following statements about semiconductors is/are true?
(i) Holes are protons.
(ii) Hole movement is really the movement of electrons filling holes and leaving new holes in the atom they came from.
(iii) Hole movement is caused by protons moving from atom to atom.
a) (i) only
b) (ii) only
c) (iii) only
d) (i) and (ii) only
e) (i) and (iii) only

Q6: A crystal of semiconductor material is doped. The band structure of the doped crystal is shown below.

## conduction band



Which of the following statements is/are true?

- (i) This is the band structure diagram for n-type semiconductor.
- (ii) The free charge carriers are holes.
- (iii) The doping of this crystal reduces its resistance.
a) (i) only
b) (ii) only
c) (iii) only
d) (i) and (ii) only
e) (i) and (iii) only


## $6.2 \mathrm{p}-\mathrm{n}$ junctions

Semiconductors are crystalline and are manufactured, or grown, in very clean conditions to ensure that they are not contaminated by any impurities apart from the doping elements. During the manufacturing process the type of doping can be changed and so a semiconductor can be made that is half p-type and half n-type. In this form it is known as a p -n junction diode. Although there are other types of diodes, they all perform the same basic function and so we will refer to the $p$-n junction diode simply as a diode. Figure 6.3 shows various diodes along with a close up and the circuit symbol for a diode.

Figure 6.3: p-n junction diodes


You may already know that a diode will allow current to pass in one direction only but we will look at why this is the case. There are two ways that a diode can be connected to a battery, known as forward and reverse bias, Figure 6.4. If the p-type end is connected to the positive side of the battery and the $n$-type is connected to the negative side then the diode is said to be forward-biased. If the diode is connected the other way round then it is reverse-biased. The resistor in the circuit is there to protect the diode from high currents.

Figure 6.4: Forward and reverse biased diodes


Before we look at bias in more detail we will consider what happens at a p-n junction with no voltage applied.

When a diode is first made there are free electrons in the n-type material and free holes in the p-type, although both sides are electrically neutral. Free electrons are more concentrated in the n-type material and they diffuse across the junction and combine with some of the holes. This has the effect of creating a middle section with no excess charges, known as the depletion layer. Remember that the atoms in the diode are electrically neutral and so when an electron and hole combine, two ions are produced one on each side of the junction. A potential difference, of the order of a few hundred millivolts, is set up between the ends of the depletion layer due to these ions, with the effect that any more charges trying to cross the junction are unable to overcome the potential barrier, or junction voltage. This barrier must be overcome before the diode can conduct.

Figure 6.5: Depletion layer


Depletion layer

The formation and effect of the depletion layer, or junction region, can be described using band theory.

At temperatures above zero kelvin electrons in the n-type material and holes in the ptype material will diffuse. Those near the junction will be able to diffuse across it.

When this happens they will recombine ie an electron will "fill" in a hole. This causes there to be a shortage of electrons in the n-type and the conduction band is lowered in the $n$-type material. The lack of holes in the p -type material raises the valence band.
The band structure diagram for an unbiased $p-n$ junction diode is show below.


As the conduction band in the $n$-type material lowers and the valence band in the p-type material rises, the slope of the conduction band in the depletion layer increases and the depletion layer acts as a potential barrier (about 0.6/0.7 V).
This barrier must be overcome before the diode can conduct.

### 6.2.1 Forward bias

If we look closely at a forward-biased diode (Figure 6.6), we see that the junction voltage opposes the applied voltage from the supply battery. In the case of a silicon based diode the junction voltage is about 0.6 volts and as long as the supply voltage is less than this value it cannot overcome the barrier. As the supply voltage is increased beyond the junction voltage, majority charge carriers are able to cross the junction; electrons from the $n$-type to the $p$-type and holes in the opposite direction. This has the effect of reducing the width of the depletion layer and so the diode conducts.

Figure 6.6: Forward bias


As the bias voltage is increased the current through the diode will also increase although the current is not directly proportional to the voltage. In other words diodes do not generally obey Ohm's law and are sometimes referred to as 'non-ohmic conductors'.
When the p-type material of the p-n junction diode is connected to the positive of the supply the electrons at that side have less potential energy than under no bias. This has the effect of lowering the bands on the $p$-type side from where they were originally. This reduces the slope in the depletion layer and makes it easier for electrons to flow across
the barrier.
The holes are similarly able to flow in the opposite direction across the junction towards the negative side of the battery.

When the supply voltage is greater than the depletion layer voltage the free electrons in the conduction band of the n-type semiconductor can be "pushed" across the depletion layer and up into the conduction band of the p-type semiconductor allowing conduction to take place.

An alternative name for the depletion layer is the junction region. It is named the depletion layer because it is depleted (reduced number) of free charge carriers. It is named the junction region because this is where the p-type and n-type semiconductors meet.

The supply provides the increase in energy required to move the free electrons from the conduction band of the $n$-type into the conduction band of the p -type. This is shown in Figure 6.8.

### 6.2.2 Reverse bias

Figure 6.7 shows the situation inside a reverse-biased diode. The free electrons in the $n$-type material will be attracted by the positive terminal of the supply battery and electrons from the battery will enter the p-type end of the diode and combine with some of the holes. This has the effect of removing some of the charge carriers from the diode and increasing the width of the depletion layer. The resistance of the junction becomes very large and so there is no current in the circuit.

Figure 6.7: Reverse bias


[^0]Although it is useful to think of a reverse-biased diode as an open switch cutting off the current, it is not entirely true. In reality the diode acts more like a very high value resistor than an open switch and so it is possible for there to be a tiny current through the diode. This is known as the leakage current and can be ignored in most cases.

When the p-type material of the p-n junction diode is connected to the negative of the supply the electrons at that side have more potential energy than under no bias. This has the effect of raising the bands on the p-type side from where they were originally. This increases the slope in the depletion layer and makes it harder for electrons to flow across the barrier.

The band structure diagram for a p-n junction diode connected in reverse bias shows the depletion layer has become a greater barrier to the movement of electrons from the conduction band of the n-type to the conduction band of the p-type. This results in the diode not conducting and there being no current through the diode when it is connected in reverse bias. This is shown in Figure 6.8.

### 6.2.3 Voltage against current graph

The following circuits show a p-n junction diode connected in reverse and forward bias.


The following graph shows the current through a p-n junction diode against the voltage across a p-n junction diode in both reverse and forward bias.

Figure 6.8:


1. No current flows when the p-n junction diode is reverse bias.
2. A minimum forward bias voltage is required before a current will flow.
3. This minimum forward bias voltage is required to overcome the junction voltage, about 0.6 volts, which is across the depletion layer.
4. As the forward bias voltage increases above the junction voltage the current through the $p$-n junction diode increases.
5. The current is not proportional to the voltage across the diode.

### 6.2.4 Band structure diagrams

## Forward bias diode

The circuit diagram and the band structure for a forward bias p - n junction diode are shown below.


Key:

- (black dot) free electron, free negative charge carrier.
- (white dot) free hole, free positive charge giver.


When the supply voltage is greater than the depletion layer voltage the free electrons in the conduction band of the n-type semiconductor can be "pushed" across the depletion layer and up into the conduction band of the p-type semiconductor allowing conduction to take place.

An alternative name for the depletion layer is the junction region. It is named the
depletion layer because it is depleted (reduced number) of free charge carriers. It is named the junction region because this is where the p -type and n -type semiconductors meet.

The supply provides the increase in energy required to move the free electrons from the conduction band of the n-type into the conduction band of the p-type.

## Reverse bias diode




The band structure diagram for a p-n junction diode connected in reverse bias shows the depletion layer has become a greater barrier to the movement of electrons from the conduction band of the $n$-type to the conduction band of the p -type.

Looking at the circuit diagram it can be seen that the supply voltage is not pushing or pulling electrons or holes across the diode.

This results in the diode not conducting and there being no current through the diode when it is connected in reverse bias. This is shown in Figure 6.8.

Quiz: p-n junction diodes
Q7: Which one of the following statements about diodes is true?
a) p-n junction diodes are positively or negatively charged.
b) direct current (d.c.) cannot pass through a diode.
c) diodes can be n-type or p-type.
d) they allow current to pass in one direction only.
e) reversing a diode in a d.c. circuit reverses the direction of the current.

Q8: Which one of the following statements about $\mathrm{p}-\mathrm{n}$ junction diodes is false?
a) the depletion layer contains no free charge carriers inside it.
b) a potential difference exists between the ends of the depletion layer.
c) the depletion layer is a region of high resistance.
d) they perform the same function as other types of diode.
e) they are made from doped semiconductor material.

Q9: Which one of the following statements about a forward-biased diode is false?
a) the p-type terminal is connected to the positive supply.
b) forward bias reduces the width of the depletion layer.
c) the bias voltage must be greater than the junction voltage for the diode to conduct.
d) there is a limit to the current that the diode can handle.
e) the diode obeys Ohm's law.

Q10: Which one of the following statements about a reverse-biased diode is false?
a) the depletion layer is wider than in an unbiased diode.
b) there is a limit to the size of bias voltage that a diode can withstand.
c) free electrons in the n-type material move to the positive side of the supply.
d) free holes in the $p$-type material move to the negative side of the supply.
e) the p-type terminal is connected to the positive supply.

Q11: Which bulb(s) would light in the circuit shown?

a) bulb 1 only
b) bulb 2 only
c) bulb 3 only
d) bulbs 1 and 2 only
e) bulbs 1 and 3 only

Q12: The band structure of a p-n junction diode is shown in the following diagram.


Which of the following statements about the diode is true?
a) electrons can flow freely though the depletion layer
b) energy must be supplied to move an electron through the depletion layer
c) holes can flow freely though the depletion layer
d) there is no minimum voltage required to make the diode conduct
e) when a diode is connected in reverse bias there is a smaller energy difference across the depletion layer than when connected in forward bias

### 6.3 Light emitting diodes and solar cells

This section is in two parts:

- light emitting diodes
- Photodiodes, solar cells and the photovoltaic effect


### 6.3.1 Light emitting diodes

What happens when electrons and holes combine inside a forward-biased diode?
You should remember that electrons occupy energy levels inside an atom. When electrons in an atom absorb energy they can move to a higher energy level or even
escape from the atom. The electrons can also give up their energy and drop into lower energy levels if there is room.

As you have already seen, when a diode is forward biased, electrons move through the diode by jumping from one atom to another and combining with the positive holes in the region of the junction. When this happens the electrons are actually dropping into lower energy levels and giving out energy in the form of photons of light.

By careful choice of semiconductor material, the released photons can be in the visible spectrum. This is the basis of the light emitting diode (LED). Knowing the energy level structure of different elements allows materials to be chosen so that the LED will give out particular colours. The first visible LEDs gave out red light but advances in the technology have resulted in many more colours. As the light is produced by electrons jumping between energy levels, LEDs normally have a very narrow range of frequencies. However it is now possible to produce LEDs that give out white light. Some LEDs are shown in Figure 6.9 along with the circuit symbol and close-up of an LED. The arrows in the symbol represent the light given out by the LED.

Figure 6.9: Light emitting diodes

Symbol

Illustration


LEDs are very efficient producers of light due to the fact that very little heat is produced. LEDs are very low power devices and so are useful as 'power-on' and 'standby' indicators for electronic systems. Due to their low power consumption LEDs work on small voltages and are usually protected by a resistor connected in series.

It is not only 'visible' LEDs that are useful. LEDs that produce photons in the infrared range are used extensively in the remote control of electronic devices such as televisions and video recorders etc.

## LED

At this stage there is an online activity. If however you do not have access to the internet Go online you should ensure that you understand the process shown in the diagrams.


Electrons and hole recombining at the junction
In a forward bias LED electrons and holes move towards the junction. When they meet at the junction they recombine.


Photons of light are produced

When they recombine at the junction energy is released in the form of a photon. Every time an electron recombines with a hole a photon of light can be released.

### 6.3.2 Band structure of an LED

The band structure of an LED and the direction of movement of charges when it is emitting light are shown in the following diagram.
Key:

- (black dot) free electron, free negative charge carrier.
- (white dot) free hole, free positive charge giver.


1. The supply voltage forces electrons into the depletion layer.
2. The supply voltage forces holes into the depletion layer.
3. When an electron moves into the depletion layer of the p - n junction diode it drops down to the valence band and into a hole.
4. This transition causes the release of energy in the form of a photon which may be visible light. Since an LED is being described, light will be emitted but other types of diodes can be manufactured which will emit photons of infrared or ultraviolet radiation.

The energy of the photon released equals the energy of the band gap through which the electron falls.

### 6.3.3 Colour of LED

Different colours of light can be produced from LEDs.
Each colour of light has a different frequency. The higher the frequency of light the greater the amount of energy required to produce that colour.

When an electron recombines with a hole in a forward biased p-n junction energy is released. The amount of energy released determines the colour of light emitted. Different materials and levels of doping can be used to construct the $p$ and $n$ materials that make up the diode so that when an electron recombines with a hole at the junction different amounts of energy are released. This allows different LEDs to produce different colours. Some LEDs seem to produce more than one colour but these are in fact two or more LEDs connected inside one enclosure.

The band structure of an LED connected in forward bias in shown in the following diagram.

Key:

- (black dot) free electron, free negative charge carrier.
- (white dot) free hole, free positive charge giver.


The band gap energy in this LED is shown as $3.3 \times 10^{-19} \mathrm{~J}$. This means that when an electron drops from the conduction band to the valence band a photon of energy $3.3 \times 10^{-19} \mathrm{~J}$ is released.
The frequency of the light produced can be calculated.

$$
\begin{aligned}
& E=h f \\
& 3.3 \times 10^{-19}=6.63 \times 10^{-34} x f \\
& f=5.0 \times 10^{14} \mathrm{~Hz}
\end{aligned}
$$

The colour of this photon of light is therefore red/orange.
Since most of the electrons will fall from the bottom of the conduction band to the top of the valence band most of the photons will be of the same energy so the light is almost monochromatic.

It is not perfectly monochromatic since some electrons will fall from bands above the bottom of the conduction band to bands lower than the top of the valence band. Since these electrons undergo a greater energy transition the resulting photon will have a greater frequency and will therefore be nearer the blue end of the spectrum.

### 6.3.4 Photodiodes, solar cells and the photovoltaic effect

In the last section we saw how knowledge of atomic energy levels led to the production of semiconductors that could transform electrical energy into light - the light emitting diode or LED.

The reverse of this process is possible. So what happens if we shine light onto a p-n junction diode?

Photodiodes are basically the same as light emitting diodes in that they consist of a slice of semiconductor doped as p-type at one end and n-type at the other end. There are some differences in the actual structure to enable the photodiode to absorb light as efficiently as possible but it is still the p-n junction that is responsible for the energy transformation. The main differences in design are that the p-type section at the top of the photodiode is much thinner than the n-type and it is covered with a material that transmits light. Both of these design differences maximize the light reaching the junction region.

When light is incident upon a photodiode, electron-hole pairs are created in the junction region. This is due to electrons absorbing the energy of the photons and escaping from the atom, thus leaving behind holes. The number of electron-hole pairs that are created depends on the intensity of light reaching the junction of the photodiode.

Figure 6.10 Shows an example of a photodiode along with a close up view and its circuit symbol. You will notice that the symbol is very similar to that of an LED, the only difference being in the direction of the arrows; these represent the light shining onto the photodiode.

Figure 6.10: Photodiode


There are two basic ways, or modes, in which a photodiode can be used. Although in both of these modes light falling on the photodiode results in the production of free charges, the effect of this depends on the way that the photodiode is connected in the circuit.

When used in photovoltaic mode, photodiodes can provide the energy for solar powered equipment such as calculators or telecommunication satellites. As the photodiode effectively becomes the power supply when used in this way it does not require a bias voltage. Figure 6.11 shows a simple circuit diagram to show how a photodiode is connected in photovoltaic mode.

Figure 6.11: Photovoltaic mode


The amount of energy available from a photodiode will depend on the area exposed to light and on the intensity (and frequency) of the light reaching the photodiode. The efficiency of a typical photodiode is less than $20 \%$ but sunlight provides a totally free
energy source. Photodiodes are also very reliable and can give many years of service.
An array of photodiode connected in this way is what makes up a solar cell.
The other mode in which photodiodes can be used is the photoconductive mode. This mode is beyond the scope of this course.

## Solar cells and voltage

The effect of light level on the output voltage from a solar cell can be measured in a simple experiment.

The brightness of the light source is increased and the voltage from the solar cell is noted.


The results are:

| light level (units) | Voltage (V) |
| :--- | :--- |
| 0 | 0 |
| 1 | 0.15 |
| 2 | 0.28 |
| 3 | 0.4 |
| 4 | 0.45 |
| 5 | 0.46 |
| 6 | 0.46 |
| 7 | 0.46 |
| 8 | 0.46 |
| 9 | 0.46 |
| 10 | 0.46 |
| 11 | 0.46 |
| 12 | 0.46 |
| 13 | 0.46 |
| 14 | 0.46 |
| 15 | 0.46 |
| 16 | 0.46 |
| 17 | 0.46 |
| 18 | 0.46 |
| 19 | 0.46 |
| 20 | 0.46 |

Q13: Use these results to draw a graph of voltage against light level ( x -axis). How would you describe the effect?

### 6.3.4.1 Band structure of photodiodes

The band structure a photodiode is shown in following diagram.
Key:

- (black dot) free electron, free negative charge carrier.
- (white dot) free hole, free positive charge giver.


1. Photon of light/energy enters p-type semiconductor.
2. Photon of light/energy enters n-type semiconductor.
3. If the energy of a photon is greater than the band gap an electron is moved from the valence band of the p-type to the conduction band, resulting in excess electrons in the conduction band of the $p$-type.
4. If the energy of a photon is greater than the band gap an electron is moved from the valence band of the $n$-type to the conduction band, resulting in excess holes in the valence band of the n-type.
5. Excess electrons drift from the conduction band of the p-type to the conduction band of the $n$-type.
6. Excess holes drift from the valence band of the $n$-type to the valence band of the p-type.

It is the upward transition of the electrons which enables the photodiode to absorb energy which produces a voltage.

Whenever an electron is moved from the valence band to the conduction band a hole will be created in the valence band. Therefore there will always be equal numbers of electrons and holes able to move through the depletion layer. These are often referred to as "electron/hole pairs".

The production of a voltage by this method is known as the photovoltaic effect and is used in solar cells to drive a current to operate many devices including calculators and satellites.

It is important to notice that as with the photoelectric effect it is a "one hit process". One photon can only move one electron from the valence band to the conduction band.

If the irradiance increases then more photons hit the photodiode per second so more electrons move per second and a greater voltage will be produced.

## Quiz: Photodiodes

Q14: Which one of the following statements about a photodiode operating in photovoltaic mode is true?
a) It is forward biased.
b) It is reverse biased.
c) It acts as a source of emf.
d) It works best in dark conditions.
e) It transforms electrical energy to light energy.

Q15: Which of the following statements about photodiodes is/are true?
(i) Light energy produces electron-hole pairs at the junction region.
(ii) They are covered with a reflective coating.
(iii) The top layer is very thin.
a) (i) only
b) (ii) only
c) (i) and (ii) only
d) (ii) and (iii) only
e) (i) and (iii) only

Q16: Which of the following devices would not use a photodiode?
a) Barcode scanner
b) Remote controlled TV
c) Solar powered calculator
d) fibre optic telephone system
e) Car water temperature sensor

### 6.4 Summary

## Summary

You should now be able to use band theory to explain:

- the properties of $n$-type semiconductor;
- the properties of $p$-type semiconductor,
- the operation of a p-n junction diode in both forward and reverse bias;
- the operation of an LED;
- the operation of a photodiode and solar cell.


### 6.5 Extended information

The authors do not maintain these web links and no guarantee can be given as to their effectiveness at a particular date.
They should serve as an insight to the wealth of information available online and encourage readers to explore the subject further.

- https://phet.colorado.edu/en/simulation/semiconductor

Simulation of a p-n junction in a circuit. Make it forward and reverse bias and notice how the electron distribution in the valence and conduction bands changes. Make the "battery force" (battery voltage) overcome the "internal force" (depletion layer voltage) and watch the electrons move through the depletion layer to allow a current to flow.

- http://www.educationscotland.gov.uk/highersciences/physics/unitthree/ electronsatwork/pnjunctions.asp
Education Scotland materials covering this work.
- http://www.st-andrews.ac.uk/~www_pa/Scots_Guide/info/comp/conduct/ movechrg/movechrg.htm
A nice simulation confirming valance and conduction bands, the formation of electron/hole pairs and the motion of the electron/hole pair when a voltage is applied.
- http://photonicswiki.org/index.php?title=What_is_a_Light_Emitting_Diode\%3F A simple simulation showing the formation of the depletion layer in a p-n junction.
- http://www.allaboutcircuits.com/vol_3/chpt_2/6.html

This site provides similar explanations on p-n junctions

- http://hyperphysics.phy-astr.gsu.edu/hbase/electronic/led.html\#c2

This site gives an in depth study of leds and will be useful for students who want to explore this area further.

- http://hyperphysics.phy-astr.gsu.edu/hbase/solids/pnjun.html

This site is worth a visit. Students who have time may wish to explore other links on this site.

- http://www.allaboutcircuits.com/vol_3/chpt_2/6.html

The diagrams are very clear on this site. There are also very good links providing further study materials relevant to the course.

### 6.6 Assessment

## End of topic 6 test

The following test contains questions covering the work from this topic.
The end of topic test is available online. If however you do not have access to the web, you may try the following questions.

Q17: Which one of the following elements is a semiconductor?.
a) iron
b) glass
c) aluminium
d) copper
e) silicon

Q18: What are the majority charge carriers in $n$-type semiconductor material?

Q19: To which terminal of the power supply should the p-type end of a reverse-biased diode be connected?

Q20: The number of valence shell electrons in an n-type doping atom, a pure semiconductor atom and a p-type doping atom are, in order:
a) $3,4,5$
b) $2,4,6$
c) $5,4,3$
d) $3,3,3$
e) $4,4,4$

Q21: What is the overall charge on an unbiased piece of $p$-type semiconductor material?
a) positive
b) negative
c) neutral

Q22: What bias conditions are necessary in order for an LED to emit light?
a) reverse bias
b) no bias
c) forward bias
d) either forward or reverse bias

Q23: What name is given to the area on each side of the junction of a $p-n$ junction diode in which there are no free charge carriers?

Q24: Which of the following statements about photodiodes is/are true?
(i) The junction region is very close to the surface.
(ii) The energy change that takes place within the junction is electricity to light.
(iii) Photodiode operation depends on the recombination of holes and electrons.
a) (i) only
b) (ii) only
c) (iii) only
d) (i) and (iii) only
e) (ii) and (iii) only

Q25: What name is given to the mode of operation of a photodiode in which it can supply power to a load circuit?

Q26: The band structure of an LED is shown below.
Key:

- (black dot) free electron, free negative charge carrier.
- (white dot) free hole, free positive charge giver.


## Conduction band



Valence band

$$
0000
$$

In order for a photon of light to be released which of the following statements is/are true?
(i) LED must be forward bias.
(ii) Electrons move from conduction band to valence band.
(iii) Holes move from valence band to conduction band.
a) (i) only
b) (ii) only
c) (iii) only
d) (i) and (iii) only
e) (ii) and (iii) only

## Topic 7

## End of unit test

## Contents

7.1 Open ended and skill based questions ..... 150
7.2 Course style questions ..... 153
7.3 End of unit assessment ..... 158

### 7.1 Open ended and skill based questions

## Open ended and skill based questions

Q1:
Go online A group of students were evaluating an experiment to investigate the relationship between the output from a solar cell and the distance from the solar cell.

The diagram below shows some of the apparatus used and its positioning in the laboratory:


- Student A stated: "I think we should use only long distances between the lamp and the solar cell. This would reduce the uncertainty in our distance measurements."
- Student B stated: "I think we should repeat our experiment tomorrow. Repeating and averaging our measurements will make our results more reliable."
- Student C stated: "I think we should use one solar cell of greater area. This would increase the voltage that will be measured."
- Student D stated: "As long as no clouds pass in front of the sun we should still be able to find the expected relationship between the distance between the lamp and the solar cell and the output from the solar cell."

Using your knowledge of experimental analysis, comment on these statements.

## Q2:

A student reads the following newspaper headline: "The real reasons your phone battery is rubbish."

The article is about the issue of mobile phone batteries not storing enough energy.
The student decides to investigate how much energy is supplied to a mobile phone during one charging of the mobile phone battery.
The student uses the following circuit:


- The student's mobile phone indicates that it needs to be recharged.
- The student presses the reset button on the energy meter so that it reads zero.
- The student then switches on the mains and waits until the phone indicates that it is fully charged.
- When the phone is fully charged, the energy meter shows a reading of zero units of energy having been supplied to the phone.

The energy meter is working and the mobile phone operates normally after this charging. Using only the same equipment, describe how the student could change her/his procedure in order to obtain a more accurate estimate of the energy supplied during one charging of the mobile phone.
Your answer must be structured so that it is logical when read.

Q3:
A student investigates the relationship between the capacitance,

$$
C
$$

, of a parallel plate capacitor and the area of overlap,
A
, of the plates.
The data the student obtains is presented in the following graph.

## Capacitance

C / x $10^{-13} \mathrm{~F}$


Using data from the graph, determine the relationship between the capacitance,

$$
C
$$

, of a parallel plate capacitor and the area,
, of overlap of the plates. Include all constants in the relationship.

### 7.2 Course style questions

## Course style questions

Q4:
(a) A signal generator is connected to an oscilloscope.

Go online
The following diagram shows the trace obtained, the Y-gain and the time base controls on the oscilloscope.


1. Calculate the peak voltage of the signal generator.
2. Calculate the frequency of the signal.
3. Without altering the settings, this signal generator is now connected across a $64 \Omega$ resistor.
Calculate the r.m.s. value of the current in the resistor.
(4)
(b) A circuit is set up as shown below.


The power supply has an emf of 12 V and negligible internal resistance.

1. Initially the switch $S$ is open.

Calculate the p.d. across the $22 \mathrm{k} \Omega$ resistor.
2. Switch $S$ is now closed.

Explain how this changes the p.d. across the $22 \mathrm{k} \Omega$ resistor.

## Q5:

In an investigation to measure the internal resistance of a battery a student sets up the following circuit.


The student varies the setting of the variable resistor Rv and records readings of voltage and current.
The readings obtained are presented in the following graph.
potential difference / V

(a) Use information from the graph to determine:

1. the e.m.f of the battery;
2. the internal resistance of the battery.
(b) From the graph it can be predicted that the maximum possible current from the battery is 900 mA .
3. Describe an experimental procedure which the student could use to verify this prediction.
4. Explain why carrying out this procedure could damage the cell.
(2)
(c) The student now collects two more identical batteries and connects the following circuit.


Calculate the reading shown on the ammeter.

Q6: A student sets up the circuit shown to investigate the charging and discharging of a capacitor.


The battery has an e.m.f. of 9.0 V and negligible internal resistance. Initially the capacitor is uncharged.
(a) Switch S is moved to the charging position so that the capacitor charges.

1. Sketch a graph of the potential difference across the capacitor against time from the moment the switch is moved to the charging position for until the capacitor is fully charged.
Numerical values are only required on the voltage axis.
2. Sketch a graph of the current in the capacitor against time from the moment the switch is moved to the charging position for until the capacitor is fully charged. Numerical values are only required on the current axis.
(b) Calculate the energy stored in the capacitor when it is fully charged.
(c) Switch S is now moved to the discharging position so that the fully charged capacitor can discharge through the lamp.
3. At one instant during the discharge, the potential difference across the capacitor is 3.0 V . Calculate the current in the lamp.
4. Explain how the time taken for the capacitor to discharge through the lamp compares to the time taken to charge the capacitor.

Marks (13)

### 7.3 End of unit assessment

## 5

## End of unit test

An online assessment is available, with questions covering all the topics in this unit.
Go online
Q7: A 50 Hz a.c. supply is connected to an oscilloscope. The screen of the oscilloscope is 8 cm wide. Two full waves are seen on the screen. The time-base setting of the oscilloscope is
a) $10 \mathrm{~ms} \mathrm{~cm}^{-1}$
b) $2.5 \mathrm{~ms} \mathrm{~cm}^{-1}$
c) $12.5 \mathrm{~ms} \mathrm{~cm}^{-1}$
d) $5 \mathrm{~ms} \mathrm{~cm}^{-1}$
e) $80 \mathrm{~ms} \mathrm{~cm}^{-1}$

Q8: The r.m.s current from a mains supply is I. What is the peak current?
a) $I \sqrt{ } 2$
b) $1 / \sqrt{ } 2$
c) $1 / 2 \sqrt{ } 1$
d) $\mathrm{I}^{2} \sqrt{ } 2$
e) $2 \sqrt{ } 1$

Q9: The potential difference across the terminals of an electric motor is 20 V . This means that
a) 1 J of work is done by 20 A passing through the motor
b) 20 J of work are done by 1 C passing through the motor
c) 20 J of work are done by 1 A passing through the motor
d) 1 J of work are done by 20 C passing through the motor
e) the motor exerts a force of 20 N when 1 A flows

Q10: A sinusoidally-varying voltage that has a frequency of 4420 Hz , is applied across a $0.5 \mathrm{~W}, 15 \Omega$ resistor.
Calculate the peak value of the voltage, in V , such that the resistor is not overloaded.

Q11: During a lightning discharge, 45 C of charge move through a potential difference of $4.7 \times 10^{7} \mathrm{~V}$, in a time of $2.7 \times 10^{-2} \mathrm{~s}$.
Calculate the energy released by this lightning bolt, in J .

Q12: In the circuit below, the reading on the ammeter is 3.0 A .


What is the total current between $P$ and $Q$ ?
a) 1.5 A
b) 3.0 A
c) 4.5 A
d) 5.0 A
e) 6.0 A

Q13: A student sets up the following circuit to find the resistance of a resistor


The internal resistance of the d.c supply is negligible.
The voltmeter may be connected across XY or XZ.
The student obtains the following results.

| Voltmeter connected <br> across | Voltmeter reading (V) | Ammeter reading (mA) |
| :---: | :---: | :---: |
| $X Y$ | 1.20 | 40.0 |
| $X Z$ | 1.40 | 40.0 |

1. Calculate the resistance of the resistor
2. Calculate the resistance of the ammeter
3. The voltmeter is removed and a resistor of resistance $200 \Omega$ is connected across XY.
What is the reading of the ammeter?
4. Calculate the voltage across XY.

Q14: The Wheatstone bridge shown below is used to measure the resistance of a thermistor at various temperatures.


At a certain temperature, a zero reading is obtained on the ammeter when
$\mathrm{P}=2.00 \mathrm{k} \Omega, \mathrm{Q}=200 \Omega$ and $\mathrm{R}=220 \Omega$.
Calculate the resistance of the thermistor at this temperature.

Q15: A battery of e.m.f. 9.0 V is connected to a $15 \Omega$ resistor. The terminal potential difference is 7.5 V . The current in the circuit is
a) 0.5 A
b) 0.1 A
c) 2.0 A
d) 1.1 A

Q16: A battery delivering a current of 2.5 A has a terminal voltage of 3.5 V . What is the internal resistance of the battery, in $\Omega$, if its open circuit voltage (e.m.f.) is 3.75 V ?

Q17: Two resistors, with resistances of $6 \Omega$ and $11 \Omega$ are joined in series and connected to a battery of e.m.f. 20 V that has an internal resistance of $3 \Omega$.


1. Calculate the current in the circuit, in A.
2. Calculate the potential difference across the $6 \Omega$ resistor, in V .
3. Calculate the potential difference across the $11 \Omega$ resistor, in V .
4. Calculate the potential difference across the battery terminals in V , when it is delivering current.
5. What value of resistor should be connected to this battery in order for the maximum power to be delivered to the load?

Q18: The farad is equivalent to the
a) volt per ohm
b) coulomb per volt
c) joule per coulomb
d) coulomb per second
e) volt per ampere

Q19: The circuit below is used to study the charging of a 100 nF capacitor.


The capacitor is initially uncharged.

1. Describe what happens to the reading on the ammeter after the switch is closed.
2. How would you know when the potential difference across the capacitor has reached its maximum value?
3. Calculate the energy stored by the capacitor when the p.d. across the resistor is 6.0 V .
4. How does this compare with the maximum energy stored by the capacitor?
5. $10 \mathrm{k} \Omega$ resistor is replaced with a resistor of lower resistance. Explain the effect that this has on the maximum energy stored on the capacitor.

Q20: An initially uncharged capacitor is charged using a constant current of $70 \mu \mathrm{~A}$. After 60 s , the voltage across the capacitor is 5 V .

1. Calculate how much charge, in mC , is stored on the capacitor after 60 s .
2. Calculate how much energy, in mJ , is stored in the capacitor after 60 s .
3. A second capacitor, with a smaller capacitance, is charged for the same time using the same current. How does the charge stored on the second capacitor compare with the first?
4. A second capacitor, with a smaller capacitance, is charged for the same time using the same current. How does the voltage across the second capacitor compare with the first?
5. A second capacitor, with a smaller capacitance, is charged for the same time using the same current. How does the energy stored in the second capacitor compare with the first?

Q21: Which of the following statements about semiconductors is/are true?
i The addition of impurity atoms to a pure semiconductor decreases its resistance.
ii In an n-type semiconductor, the majority of the charge carriers are negative.
iii In a p-type semiconductor, the majority of the charge carriers are positive.
a) (i) only
b) (ii) only
c) (iii) only
d) (ii) and (iii) only
e) (i), (ii) and (iii)

## Appendix A

## Appendix: Units, prefixes and scientific notation

## Contents

A. 1 Physical quantities, symbols and units used in CfE Higher Physics ..... 166
A.1.1 Unit 3: Electricity ..... 166
A. 2 Significant figures ..... 167
A. 3 Scientific notation ..... 169
A.3.1 Prefixes ..... 170

## A. 1 Physical quantities, symbols and units used in CfE Higher Physics

## A.1.1 Unit 3: Electricity

| Physics Quantity | Symbol | Unit | Unit Abbreviation |
| :--- | :---: | :--- | :---: |
| electric current | $I$ | ampere | A |
| peak voltage | $V_{p}$ | volt | V |
| root mean square <br> voltage | $V_{r m s}$ | volt | V |
| peak current | $I_{p}$ | ampere | A |
| root mean square <br> current | $I_{r m s}$ | ampere | A |
| resistance | $R$ | ohm | $\Omega$ |
| total resistance | $R_{T}$ | ohm | $\Omega$ |
| electromotive force | or | $\varepsilon$ | volt |
| internal resistance | $r$ | ohm | $\Omega$ |
| capacitance | $C$ | farad | F |

## A. 2 Significant figures

It is important when calculating numerical values that the final answer is quoted to an appropriate number of significant figures.
As a general rule, the final numerical answer that you quote should be to the same number of significant figures as the data given in the question.

The above rule is the key point but you might like to note the following points:

1. The answer to a calculation cannot increase the number of significant figures that you can quote.
2. If the data is not all given to the same number of significant figures, identify the least number of significant figures quoted in the data. This least number is the number of significant figures that your answer should be quoted to.
3. When carrying out sequential calculations carry many significant figures as you work through the calculations. At the end of the calculation, round the answer to an appropriate number of significant figures.
4. In the Higher Physics course quoting an answer to three significant figures will usually be acceptable.

## Examples

1. The current in a circuit is 6.7 A and the voltage across the circuit is 21 V . Calculate the resistance of the circuit.

Note: Both of these pieces of data are given to two sig. figs. so your answer must also be given to two sig figs.
$\mathrm{I}=6.7 \mathrm{~A}$
$\mathrm{V}=21 \mathrm{~V}$
$\mathrm{R}=$ ?

$$
\begin{aligned}
& V=I R \\
& 21=6.7 \times R \\
& R=3.1343 \\
& R=3.1 \Omega
\end{aligned}
$$

round to 2 sig figs

## 2. A 5.7 kg mass accelerates at $4.36 \mathrm{~m} \mathrm{~s}^{-2}$.

Calculate the unbalanced force acting on the mass.
Note: The mass is quoted to two sig. figs and the acceleration is quoted to three sig. figs. so the answer should be quoted to two sig figs.

```
\(\mathrm{m}=5.7 \mathrm{~kg}\)
\(\mathrm{a}=4.36 \mathrm{~m} \mathrm{~s}^{-2}\)
\(\mathrm{F}=\) ?
```

$$
\begin{aligned}
& F=m a \\
& F=5.7 \times 4.36 \\
& F=24.852 \\
& F=25 \mathrm{~N}
\end{aligned}
$$

round to 2 sig figs
3. A car accelerates from $0.5037 \mathrm{~m} \mathrm{~s}^{-1}$ to $1.274 \mathrm{~m} \mathrm{~s}^{-1}$ in a time of 4.25 s .

The mass of the car is 0.2607 kg .
Calculate the unbalanced force acting on the car.
Note: The time has the least number of sig figs, three, so the answer should be quoted to three sig figs.

$$
\begin{aligned}
& \mathrm{u}=0.5037 \mathrm{~m} \mathrm{~s}^{-1} \\
& \mathrm{v}=1.274 \mathrm{~m} \mathrm{~s}^{-1} \\
& \mathrm{t}=4.25 \mathrm{~s} \\
& \mathrm{~m}=0.2607 \mathrm{~kg}
\end{aligned}
$$

Step 1: calculate a

$$
\begin{aligned}
& a=\frac{v-u}{t} \\
& a=\frac{1.274-0.5037}{4.25} \\
& a=0.181247 \mathrm{~ms}^{-2}
\end{aligned}
$$

## Step 2: calculate F

$$
\begin{aligned}
F & =m a \\
F & =0.2607 \times 0.18147 \\
F & =0.0472511 \\
F & =0.0473 \mathrm{~N} \\
& \text { round to } 3 \text { sig figs }
\end{aligned}
$$

## Quiz questions

Q1: A car travels a distance of 12 m in a time of 9.0 s .
a) 1.3333
b) 1.33
c) 1.3
d) 1.4
e) 1

Q2: A mass of 2.26 kg is lifted a height of 1.75 m . The acceleration due to gravity is $9.8 \mathrm{~m} \mathrm{~s}^{-2}$.
The potential energy gained by the mass is:
a) 38.759 J
b) 38.76 J
c) 38.8 J
d) 39 J
e) 40 J

Q3: A trolley of 5.034 kg is moving at a velocity of $4.03 \mathrm{~m} \mathrm{~s}^{-1}$. The kinetic energy of the trolley is:
a) 40.878 J
b) 40.88 J
c) 40.9 J
d) 41 J
e) 40 J

## A. 3 Scientific notation

When carrying out calculations, you should be able to use scientific notation. This type of notation has been used throughout the topics where necessary, so you will already be familiar with it

Remember scientific notation is used when writing very large or very small numbers. When a number is written in scientific notation there is usually one, nonzero number, before the decimal point.

## Examples

1. The speed of light is often written as $3 \times 108 \mathrm{~m} \mathrm{~s}^{-1}$.

This can be converted into a number in ordinary form by moving the decimal point 8 places to the right, giving $300000000 \mathrm{~m} \mathrm{~s}^{-1}$.
2. The capacitance of a capacitor may be 0.000022 F .

This very small number would often be written as $2.2 \times 10^{-5} \mathrm{~F}$. The $\times 10^{-5}$ means move the decimal point 5 places to the left.

Make sure you know how to enter numbers written in scientific notation into your calculator.

## A.3.1 Prefixes

There are some prefixes that you must know. These are listed in the following table:

| Prefix | Symbol | Symbol |
| :--- | :---: | :--- |
| pico | $p$ | $\times 10^{-12}$ |
| nano | $n$ | $\times 10^{-9}$ |
| micro | $\mu$ | $\times 10^{-6}$ |
| milli | $m$ | $\times 10^{-3}$ |
| kilo | $k$ | $\times 10^{3}$ |
| mega | $M$ | $\times 10^{6}$ |
| giga | $G$ | $\times 10^{9}$ |

In Higher Physics you are expected to know and remember the meaning of all of these prefixes.

## Glossary

a.c.
alternating current. The current from an a.c. supply constantly changes direction.

## Alternator

an a.c. generator.

## Capacitance

the ratio of electric charge to potential difference between any two conductors separated by an insulating material. The capacitance of a system of conductors describes the ability of the system to store electric charge.

## Capacitor

two (or more) conductors separated by an insulator that can be used to store charge
d.c.
direct current. The current from a d.c. supply always moves in the same direction around an electric circuit.

## Depletion layer

the area surrounding the p-n junction of a diode where the electrons have combined with the holes leaving no free charges

## Electric current

a net flow of charged particles

## Electromotive force

the electromotive force of a source is the electrical potential energy that is given to each unit of charge that passes through the source

## Excited state

any atomic energy level higher than the ground state

## Forward-biased

a diode connected in a circuit such that the p-type terminal is more positive than the $n$-type terminal

## Frequency

the number of complete cycles of a wave passing a given point in a given time, usually per second. Frequency is measured in hertz (Hz) where $1 \mathrm{~Hz}=1$ wave per second.

## Fundamental unit of charge

$e$; the magnitude of charge carried by one electron or one proton. Equal to 1.60 x $10^{-19}$ coulombs.

## Ground state

the lowest energy level of an atom

## Hall effect

the deflection of charge carriers in a conductor caused by a magnetic field

## Instantaneous

at one point in time or at one particular instant in time

## Internal resistance

the opposition to current in a source of electrical energy

## Intrinsic semiconductors

semiconductor material with no impurities

## Ionisation level

the energy level at which an electron can break free from an atom

## Junction voltage

the potential difference between the ends of the depletion layer inside a $p$-n junction diode

## Leakage current

the tiny current in a reverse-biased diode

## Load resistor

the resistor, or combination of resistors, that forms the external part of an electrical circuit

## Lost volts

the potential difference that is used to drive a current through the internal resistance of a source. Lost volts is given by the expression Ir where $r$ is the internal resistance of the source.

## Monochromatic

one energy, one frequency, one wavelength, one colour

## N -type semiconductor

semiconductor material that has an excess of free electrons

## Ohm's law

the current in a conductor at constant temperature is directly proportional to the potential difference across it

## Open circuit

a circuit in which the current is zero. In the circuit there is a gap or an infinite resistance.

## Period

the time to make one complete wave. Period is measured in seconds.

## Photodiode

a type of p-n junction diode that responds to light intensity

## Photovoltaic mode

the mode of operation of a photodiode where it can supply power to a load. This is the basis of a solar cell.

## Potential divider

a circuit consisting of a number of resistors (often only two) in series, connected across a supply, that is used as a source of fixed or of variable p.d.

## P-type semiconductor

semiconductor material that has an excess of free holes

## Resistance

the opposition that a conductor offers to a current through it. Defined as the ratio of potential difference across the conductor to the current through it.

## Reverse-biased

a diode connected in a circuit such that the p-type terminal is more negative than the n-type terminal

## short-circuit

a circuit in which the current is at its maximum. In this type of circuit the resistance connected across the terminals of the source is $0 \Omega$. The only resistance in the circuit will be the internal resistance, $r$.

## Short-circuit current

the maximum current that a source can supply. The current drawn from the supply when there is zero resistance in the external circuit (when the terminals of the source are joined together or 'short-circuited').

## Terminal potential difference (t.p.d.)

the terminal potential difference is the potential difference that appears across the terminals of a source when the source is supplying a current to a circuit. It is the potential difference that appears across the external resistance, or load resistor, in the circuit.

## Thermistor

a resistor in which the resistance depends on its temperature

## Valence shell

the atomic energy level that contains the outermost electrons of the atom. It is the electrons in this shell that determine the chemical reactions between elements.

## Wheatstone bridge circuit

a resistor network, consisting of a series/parallel combination that can be used to measure resistance when balanced. In the out-of-balance condition, a small p.d. that is proportional to the change in resistance is produced.

## Hints for activities

## Topic 1: Monitoring and measuring a. c.

## Quiz: Frequency of a.c.

## Hint 1:

See the introduction at the beginning of this topic.

## Hint 2:

$$
T=\frac{1}{f}
$$

## Hint 3:

This is a straight application of the relationship

$$
T=\frac{1}{f}
$$

## Hint 4:

This is a straight application of the relationship

$$
T=\frac{1}{f}
$$

## Hint 5:

First count the number of squares for one complete cycle - e.g between adjacent troughs. Multiply this by the time-base setting to get the period.

## Quiz: Voltage and frequency

## Hint 1:

$$
\begin{aligned}
V_{\text {r.m.s. }} & =\frac{V_{\text {peak }}}{\sqrt{2}} \\
V_{\text {peak }} & =\sqrt{2} \times V_{\text {r.m.s. }}
\end{aligned}
$$

## Hint 2:

This is a straight application of

$$
\begin{aligned}
V_{\text {r.m.s. }} & =\frac{V_{\text {peak }}}{\sqrt{2}} \\
V_{\text {peak }} & =\sqrt{2} \times V_{\text {r.m.s. }}
\end{aligned}
$$

Comparing the energy transformed by a.c. and d.c. supplies.

## Hint 3:

The peak voltage must not be greater than the voltage at which the insulation in the capacitor breaks down.

## Hint 4:

Power varies with the square of the r.m.s. current.
Hint 5:
Changing frequency has no effect on the current in circuit that contains only resistors.

## Topic 2: Current, potential difference, power and resistance

## Quiz: The Wheatstone bridge

## Hint 1:

See the section titled The balanced Wheatstone bridge circuit.

## Hint 2:

This is a straight application of the relationship for a balanced Wheatstone bridge refer to the section with this title.

## Hint 3:

"The ratio $R_{1}: R_{2}$ is equal to $3: 2$."

## Hint 4:

The two resistors are of equal value so how will the supply voltage split?

## Hint 5:

Use the voltage divider equation to calculate the share of the supply voltage across the $60 \Omega$ resistor.

Hint 6:
This is an out of balance Wheatstone bridge circuit.
The voltmeter measures the difference in potential across the two left hand ( $50 \Omega$ and $60 \Omega$ ) resistors or the two right hand resistors ( $50 \Omega$ and $30 \Omega$ ).

## Topic 3: Electrical sources and internal resistance

Measuring the e.m.f. and internal resistance of a source
Hint 1: e.m.f. of the cell is the potential difference when no current is drawn from the cell.

## Quiz: e.m.f. and internal resistance

Hint 1:

$$
\begin{aligned}
E & =I R+I r \\
E & =V+I r \\
E-V & =I r \\
r & =\frac{E-V}{I}
\end{aligned}
$$

## Hint 2:

$$
\begin{aligned}
E & =I R+I r \\
E & =V+I r \\
E-V & =I r \\
r & =\frac{E-V}{I}
\end{aligned}
$$

In the equation above, the term V is the potential difference that appears at the terminals of the source. For this reason it is called the terminal potential difference (t.p.d.). The term Ir represents the potential difference that is 'lost' across the internal resistance of the source, and never appears in the external circuit. This term is often called the 'lost volts'. It is worth noting that both $E$ and $r$ are properties of the source and are constant (at least in the short term, if the source is not abused). On the other hand both the terminal potential difference and the lost volts depend on the current taken from the source, and so are not constant.

## Hint 3:

This is a straight application of

$$
r=\frac{E-V}{I}
$$

## Hint 4:

When a battery is short-circuited, the internal resistance of the battery is the only resistance in the circuit.

## Hint 5:

First find the total resistance in the circuit and use this to find the current.

## Topic 4: Capacitors

## Quiz: Capacitors

## Hint 1:

In all Physics relationships, units are equivalent on both sides - apply this to the following relationship.

$$
\begin{aligned}
Q & =C V \\
\text { or } C & =\frac{Q}{V}
\end{aligned}
$$

## Hint 2:

This is a straight application of the following relationship from the section titled Capacitance.

$$
\begin{aligned}
Q & =C V \\
\text { or } C & =\frac{Q}{V}
\end{aligned}
$$

## Hint 3:

This is a straight application of the following relationship from the section titled Energy stored in a capacitor.

$$
\begin{aligned}
\mathrm{E}_{c} & =\frac{1}{2} Q V \\
E_{c}=\frac{1}{2} C V^{2} & \\
E_{c} & =\frac{1}{2} \frac{Q^{2}}{C}
\end{aligned}
$$

## Hint 4:

Use the data given in the first sentence to find the capacitance - then use this with the data given in the second sentence.

## Quiz: Capacitors in circuits

## Hint 1:

This is a straight application of the following relationship from the section titled Charging a capacitor.
$I=E / R$

## Hint 2:

This is a straight application of Ohm's Law .
$R=v / 1$

## Hint 3:

The sum of the p.d. across the capacitor plus the p.d across the resistor is equal to the p.d across the battery.

## Hint 4:

Run through the activity online and try again.

## Topic 6: p-n junctions

## Quiz: Semiconductors

## Hint 1:

The structure of pure silicon is described in the section titled Electrical properties.

## Hint 2:

Eliminate the statements you know are true - read the section titled Doping if you are not sure about some statements - when you have eliminated four statements the only statement left is false.

## Hint 3:



Hint 4:


Hint 5:


Hint 6:


## Quiz: p-n junction diodes

## Hint 1:

Read the section titled $p-n$ junctions then try again.

## Hint 2:

See the following to find out about the depletion layer.


## Depletion layer

## Hint 3:

See the following figure to see what happens when the diode is forward biased.


## Forward bias

## Hint 4:

See the following figure to see what happens when the diode is reverse biased.


## Hint 5:

The diode conducts only when it is forward-biased.

## Hint 6:

The depletion layer acts as a barrier to the movement of electrons.

## Quiz: Photodiodes

## Hint 1:

In photovoltaic mode the photodiode converts the energy of incident radiation to electrical energy.

## Hint 2:

The design of a photodiode is described in the section titled Photodiode.

## Hint 3:

Which device does not use light or infrared radiation.

## Appendix A: Appendix: Units, prefixes and scientific notation <br> Quiz questions

Hint 1: Data is quoted to 2 sig figs so answer must be quoted to 2 sig figs.
Hint 2: The acceleration due to gravity is quoted to only 2 sig figs so the answer must be given to 2 sig figs.

Hint 3: The mass of the trolley is given to 4 sig figs and the velocity is given to 3 sig figs.

## Answers to questions and activities

1 Monitoring and measuring a. c.
Measuring frequency using an oscilloscope (page 7)
Q1: 100 Hz
Q2: 200 Hz
Q3: $100,000 \mathrm{~Hz}$

Quiz: Frequency of a.c. (page 8)
Q4: e) (i) and (iii) only
Q5: b) $T=\frac{1}{f}$
Q6: d) 20 ms
Q7: d) 40 Hz
Q8: a) 125 Hz

The resistor and a.c. (page 14)
Q9: The frequency has no effect on the current.

## Current/A



The alternating current in a resistor is independent of frequency.

## Quiz: Voltage and frequency (page 15)

Q10: b) $V_{\text {peak }}=\sqrt{2} \times V_{\text {r.m.s. }}$
Q11: d) 14 V
Q12: b) 11 V
Q13: e) 130 mA

## End of topic 1 test (page 18)

Q14: c) $I_{\text {peak }}=\sqrt{ } 2 I_{\text {r.m.s. }}$
Q15: 13.4 V
Q16: 500 Hz
Q17:

1. $V_{\text {peak }}=6.0 \mathrm{~V}$
2. $\mathrm{V}_{\mathrm{r} . \mathrm{m} . \mathrm{s}}=4.24 \mathrm{~V}$
3. 125 Hz

## 2 Current, potential difference, power and resistance

Quiz: Charge, current and potential difference (page 25)
Q1: b) $1.6 \times 10^{-19} \mathrm{C}$
Q2: c) 3.0 A
Q3: c) 230 V

## Circuits (page 28)

Q4: The potential difference is directly proportional to current. The graph is a straight line through the origin.

$V \propto I$ so $V=1 k$
The constant of proportionality is the resistance of the component measured in ohms. $V=I R$

## Quiz: Resistance (page 29)

Q5: d) $R=v / /$
Q6: e) one volt per ampere
Q7: c) $920 \Omega$
Q8: b) The resistance increases, because its temperature increases.

## Quiz: Electrical energy and power (page 33)

Q9: d) 10.0 MW
Q10: b) 135 mW
Q11: c) $3.0 \Omega$
Q12: e) 648 J
Q13: b) 224

## Quiz: Resistors in series (page 36)

## Expected answer

When resistors are connected in series with each other and the total resistance measured with an ohmmeter we find that the total resistance is equal to the sum of the individual resistances.
$R=R_{1}+R_{2}+\ldots R_{n}$

Q14: $160 \Omega$
Q15: $370 \Omega$
Q16: $500 \Omega$
Q17: $60 \Omega$

## Quiz: Resistors in parallel (page 39)

## Expected answer

When resistors are connected in parallel with each other and the total resistance measured with an ohmmeter we find that the total resistance is always lower than the resistance of the lowest of the individual resistances.
$1 / R=1 / R_{1}+1 / R_{2} \cdots+1 / R_{n}$

Q18: $9.375 \Omega$
Q19: $66.667 \Omega$
Q20: $30 \Omega$
Q21: $3.125 \Omega$

## Practical potential divider circuits (page 44)

Q22: This potential divider circuit consists of a variable resistor in series with a thermistor. As the temperature of the thermistor falls, its resistance increases. This means that the voltage across the thermistor ( $V_{\text {out }}$ ) also increases. This potential divider circuit therefore gives an increasing output voltage as the temperature decreases. The resistance of the variable resistor can be preset to give a suitable output voltage $V_{\text {out }}$ for particular ambient conditions.

Q23: This potential divider circuit consists of a light dependent resistor (LDR) in series with a variable resistor. As the intensity of light falling on the LDR rises, its resistance decreases. This means that the voltage across the LDR also decreases. Since $V_{\mathrm{s}}$ is fixed, this means that $V_{\text {out }}$ (the voltage across the variable resistor) increases. So it can be seen that in this circuit, $V_{\text {out }}$ increases as the light intensity increases and decreases as the light intensity decreases. The resistance of the variable resistor can be preset to give a suitable output voltage $V_{\text {out }}$ for particular ambient conditions.

Q24: This potential divider circuit consists of a variable resistor in series with a light dependent resistor (LDR). As the intensity of light reaching the LDR decreases, its resistance increases. This means that the voltage across the LDR ( $V_{\text {out }}$ ) also increases. This potential divider circuit therefore gives an increasing output voltage as the light intensity decreases. The resistance of the variable resistor can be preset to give a suitable output voltage $V_{\text {out }}$ for particular ambient conditions.

## Quiz: The potential divider (page 47)

Q25: a) $\frac{V_{1}}{V_{2}}=\frac{R_{1}}{R_{2}}$
Q26: b) 8.2 V

Q27: e) The output voltage decreases as the temperature increases.
Q28: d) 7.0 V
Q29: d) 3.0 V to 6.0 V

## Quiz: The Wheatstone bridge (page 52)

Q30: d) $\frac{R_{1}}{R_{2}}=\frac{R_{3}}{R_{4}}$
Q31: a) $20 \Omega$
Q32: b) $R_{3} 120 \Omega ; R_{4} 80 \Omega$

Q33: c) 6.0 V
Q34: d) 8.0 V

Q35: b) 2.0 V

## End of topic 2 test (page 57)

Q36:

1. $4.5 \Omega$
2. $0.88 \Omega$

Q37: 4.6amps
Q38: a) less than the resistance of $R_{1}$
Q39: $240 \Omega$
Q40: $27 \Omega$

## 3 Electrical sources and internal resistance

## Measuring the e.m.f. and internal resistance of a source (page 63)

## Expected answer



1. The e.m.f. $E$ is the terminal potential difference when the cell is not driving a current, so this is the intercept of the graph on the voltage axis.
$E=1.6 \mathrm{~V}$
2. The internal resistance $r$ is given by $r=\frac{E-V_{t p d}}{I}$ and this is the negative of the gradient of the graph.
$r=-\frac{1.6-0.5}{0-2.5}$
$r=-\frac{1.1}{-2.5}$
$r=0.44 \Omega$
3. The short-circuit current is the current when all of the e.m.f. appears across the internal resistance. From the graph, this is the intercept on the current axis when the terminal potential difference is zero.

$$
\begin{aligned}
I_{s c} & =\frac{E}{r} \\
I_{s c} & =\frac{1.6}{0.44} \\
I_{s c} & =3.6 \mathrm{~A}
\end{aligned}
$$

## Quiz: e.m.f. and internal resistance (page 64)

Q1: c) $\frac{E-V}{I}$
Q2: d) $I r$
Q3: a) $0.10 \Omega$
Q4: e) $0.608 \Omega$
Q5: c) 7.5 V

## Load matching (page 65)

## Expected answer

Here is a sample graph drawn from the results obtainable in the activity.


Q6:

| Resistance $\Omega$ | Current A | Power W |
| :--- | :--- | :--- |
| 1 | 1.2 | 1.44 |
| 2 | 1.0 | 2.0 |
| 3 | 0.857 | 2.204 |
| 4 | 0.75 | 2.25 |
| 5 | 0.667 | 2.222 |
| 6 | 0.6 | 2.16 |
| 7 | 0.545 | 2.082 |
| 8 | 0.5 | 2 |
| 9 | 0.461 | 1.917 |
| 10 | 0.428 | 1.836 |
| 11 | 0.4 | 1.76 |
| 12 | 0.375 | 1.687 |

## End of topic 3 test (page 69)

Q7: 25.6 A
Q8:

1. 2.0 A
2. 0.6 V
3. 8.4 V
4. $0.3 \Omega$

Q9: $0.45 \Omega$
Q10:

1. 18 A
2. 4.5 V

## 4 Capacitors

## Energy stored on a capacitor (page 79)

## Expected answer

We will begin with the equation $E_{c}=\frac{1}{2} Q V$. We can use $Q=C V$ and hence substitute for $Q$ :

$$
\begin{aligned}
E_{c}= & \frac{1}{2} Q V \\
& \therefore E_{c}=\frac{1}{2} \times(C V) \times V \\
& \therefore E_{c}=\frac{1}{2} C V^{2}
\end{aligned}
$$

We can also rearrange

$$
\begin{aligned}
Q & =C V \\
\therefore V & =\frac{Q}{C}
\end{aligned}
$$

We can now substitute for $V$ in the energy equation:

$$
\begin{aligned}
& E_{c}=\frac{1}{2} Q V \\
& \therefore E_{c}=\frac{1}{2} Q \times \frac{Q}{C} \\
& \therefore E_{c}=\frac{1}{2} \frac{Q^{2}}{C}
\end{aligned}
$$

So we have shown that the three equations for the energy stored on a capacitor are equivalent.

## Quiz: Capacitors (page 80)

Q1: a) $1 \mathrm{CV}^{-1}$
Q2: d) $6.0 \mu \mathrm{C}$
Q3: c) 22 nF
Q4: b) 5.1 mJ
Q5: b) 1.8 V

## Quiz: Capacitors in circuits (page 94)

Q6: b) 250 mA
Q7: c) 1.8 V
Q8: c) 7.2 V
Q9: a) increases.

## End of topic 4 test (page 97)

Q10:

1. 152 nC
2. 524 nJ

Q11:

1. 20 V
2. 3.17 mA
3. 0 V
4. 20 V

Q12:

1. $68.6 \mu \mathrm{~J}$
2. 125000000000000

Q13: 6.7 V
Q14: $3.2 \times 10^{-5} \mathrm{C}$

## 5 Conductors, semiconductors and insulators

## Temperature effect on a negative coefficient thermistor (page 106)

Q1:


We can see from the graph that as the temperature increases the resistance decreases. This is in keeping with the fact that semiconductor conductivity increases with temperature.

This type of thermistor has many practical uses from digital thermometers to controlling the speed of electric cooling fans.

## End of topic 5 test (page 110)

Q2:

| Conductors | Insulators | Semiconductors |
| :--- | :--- | :--- |
| Iron | Carbon(diamond) | Germanium |
| Tungsten | Wood | Silicon |

Q3:

1. true
2. false
3. false
4. true
5. true

Q4: Aluminium

## 6 p-n junctions

## Quiz: Semiconductors (page 119)

Q1: d) It is electrically neutral
Q2: b) The impurity atoms make up about $10 \%$ of the new material
Q3: d) (i) and (ii) only
Q4: c) (iii) only
Q5: b) (ii) only
Q6: e) (i) and (iii) only

## Quiz: p-n junction diodes (page 130)

Q7: d) they allow current to pass in one direction only.
Q8: a) the depletion layer contains no free charge carriers inside it.
Q9: e) the diode obeys Ohm's law.
Q10: e) the p-type terminal is connected to the positive supply.
Q11: d) bulbs 1 and 2 only
Q12: b) energy must be supplied to move an electron through the depletion layer

## Solar cells and voltage (page 140)

## Q13:



We can see that the output voltage from the solar cell quickly rises and then remains constant. This is a feature of modern solar cells. It should be noted that while the voltage
quickly reaches a maximum with increasing light level the same is not true of current delivered from the solar cell. When the light level is increased the current delivered by the cell continues to rise.

## Quiz: Photodiodes (page 143)

Q14: c) It acts as a source of emf.
Q15: e) (i) and (iii) only
Q16: e) Car water temperature sensor

## End of topic 6 test (page 145)

Q17: d) copper
Q18: electrons
Q19: positive
Q20: c) $5,4,3$
Q21: a) positive
Q22: c) forward bias
Q23: depletion layer
Q24: a) (i) only
Q25: photovoltaic
Q26: d) (i) and (iii) only

## 7 End of unit test

## Open ended and skill based questions (page 150)

## Q1:

You would not be expected to comment on each of the above statements.
There is no one set of answers that you must include but the following points could be included in your answer. You may wish to expand on some of these points.

- Student A:

The reading uncertainty in the distance measurements will be constant for both short and long distances.
Longer distances would give smaller percentage uncertainties in distance measurements.
Longer distances result in lower voltages from solar cell so there will be greater percentage uncertainties in the voltage readings.

- Student B:

Repeating and averaging results to improve reliability is only appropriate if the experimental conditions remain constant.
Repeating next day might not be appropriate since the background light level may have changed.

- Student C:

Increasing area would increase voltage produced.
Since the reading uncertainty in the voltage data would remain the same, the percentage uncertainty in these reading would reduce.

- Student D:

So long as the background light remains the same then Student D is correct.
Plotting a graph of voltage across solar cell against distance between lamp and solar cell should produce a graph of the correct shape with all of the voltage levels too high by the same value due to the voltage produced by light from the sun.
This would be a systematic uncertainty.
Your answer must be structured so that it is logical when read.

## Q2:

Issue: It is likely that the energy supplied in one charging is less than 0.1 unit of energy.
Possible response: Measure the energy supplied during a greater number of chargings.

- Reset meter at start of first charging.
- Charge phone.
- Use phone until it needs recharging.
- Without resetting meter recharge phone.
- Repeat this several times.
- After, say, 10 chargings, read the meter.
- Divide the units supplied by number of chargings to get the average energy supplied during each charging.

Q3:

$$
C \alpha A
$$

since the graph is a straight line through origin.
Find gradient:

$$
\begin{aligned}
& \text { gradient }=\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& \text { gradient }=\frac{44 \times 10^{-13}-0}{0.025-0} \\
& \text { gradient }=1.8 \times 10^{-10} \\
& \rightarrow C=1.8 \times 10^{-10} \times A
\end{aligned}
$$

## Course style questions (page 153)

Q4: (a)
1.

$$
\begin{aligned}
V_{p} & =\text { amplitude } \times y-\text { gain } \\
V_{p} & =2 \times 6 \\
V_{p} & =12 V
\end{aligned}
$$

2. 

Step 1

$$
\begin{aligned}
& T=\lambda \times \text { timebase } \\
& T=2 \times 5 \times 10^{-3} \\
& T=0.01(s)
\end{aligned}
$$

Step 2

$$
\begin{aligned}
& f=\frac{1}{T} \\
& f=\frac{1}{0.01} \\
& f=100 \mathrm{~Hz}
\end{aligned}
$$

3. 

Step 1

$$
\begin{aligned}
& V_{p}=\sqrt{2} \times V_{r m s} \\
& 12=\sqrt{2} \times V_{r m s} \\
& V_{r m s}=8.49(V)
\end{aligned}
$$

## Step 2

$$
\begin{align*}
& V_{r m s}=I_{r m s} \times R \\
& 8.49=I_{r m s} \times 64 \\
& I_{r m s}=0.13 \mathrm{~A} \tag{4}
\end{align*}
$$

(b)
1.

$$
\begin{aligned}
V_{1} & =\left(\frac{R_{1}}{R_{1}+R_{2}}\right) V_{s} \\
V_{1} & =\left(\frac{22}{22+69}\right) \times 12 \\
V_{1} & =2.9 \mathrm{~V}
\end{aligned}
$$

2. Closing switch $S$
$\Rightarrow$ resistance of parallel branch halves, now only $48 \mathrm{k} \Omega$
$\Rightarrow$ share of supply voltage across this lower resistance is less than when switch S was open
$\Rightarrow 22 \mathrm{k} \Omega$ will get a greater share of the 12 V supply voltage across it.

Marks (14)
Q5: (a)

1. The e.m.f., E , is the terminal potential difference when the current is zero. Extend line back until it meets the y axis.

$$
\begin{equation*}
E=3.6 \mathrm{~V} \tag{1}
\end{equation*}
$$

2. The internal resistance, $r$, can be found from finding the gradient of this graph. Take care, the current is in mA so best to convert to A.

$$
\begin{aligned}
& \text { gradient }=\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& \text { gradient }=\frac{0.4-2.8}{0.8-0.2} \\
& \text { gradient }=-4.0
\end{aligned}
$$

Internal resistance, $r=4.0 \Omega$
(3)
(b)
1.

Short circuit current when load resistance, $R_{v}$, is zero.
Procedure:
Reduce $R_{v}$ to zero or replace $R_{v}$ with a piece of thick wire.
2. When the battery is short circuited, a high current flows.

This high current is passing through the internal resistor of the battery and makes the battery heat up.
This heating can damage the battery.
(c)

Step 1 notice that one of the batteries has been reversed, so its e.m.f. of 3.8 V must be subtracted from the total 7.6 V e.m.f. of the other two batteries.

$$
V_{\text {circuit }}=3.6 \mathrm{~V}
$$

Step 2 find the total resistance:

$$
\begin{gathered}
R_{t}=R_{1}+R_{2}+R_{3}+R_{v} \\
R_{t}=4+4+4+8 \\
R t=20 \Omega
\end{gathered}
$$

Step 3 calculate the current in the circuit.

$$
\begin{gathered}
V=I \times R \\
3.6=I \times 20 \\
I=0.18 \mathrm{~A}
\end{gathered}
$$

Q6: (a)
. Remember labels and origin.

2.

Step 1 calculate initial maximum current.
Must use $18 \mathrm{k} \Omega$ resistance as this is the resistance through which capacitor is charged.

$$
\begin{gathered}
V=I \times R \\
9.0=I \times 18000 \\
I=5.0 \times 10^{-4} \mathrm{~A}
\end{gathered}
$$

Step 2 draw graph, remember labels and origin:

(b)

$$
\begin{align*}
& E=\frac{1}{2} C V^{2} \\
& E=\frac{1}{2} \times 470 \times 10^{-6} \times 9.0^{2} \\
& E=0.019 J \tag{3}
\end{align*}
$$

(c)

1. Since the potential difference across the capacitor is 3 V then the pd across the lamp will also be 3 V .

$$
\begin{gathered}
V=I \times R \\
3.0=I \times 45 \\
I=0.067 \mathrm{~A}
\end{gathered}
$$

2. Resistance of lamp is much smaller than resistance in charging circuit $\Rightarrow$ time taken to discharge is much less.

Marks (13)

## End of unit test (page 158)

Q7: d) $5 \mathrm{~ms} \mathrm{~cm}^{-1}$
Q8: a) $I \sqrt{ } 2$
Q9: b) 20 J of work are done by 1 C passing through the motor
Q10: 2.74 V
Q11: 2115000000 J
Q12: e) 6.0 A
Q13:

1. $30 \Omega$
2. $5.0 \Omega$
3. 45 mA
4. 1.17 V

Q14: $2200 \Omega$
Q15: a) 0.5 A
Q16: $0.1 \Omega$
Q17:

1. 2.0 A
2. 12 V
3. 38 V
4. 50 V
5. $3.0 \Omega$

Q18: b) coulomb per volt
Q19:

1. decreases
2. the ammeter reading is zero
3. 450 nJ
4. it is less
5. it has no effect

## Q20:

1. 4.2 mC
2. 10.5 mJ
3. it is the same
4. it is greater
5. it is greater

Q21: e) (i), (ii) and (iii)

## A Appendix: Units, prefixes and scientific notation

Quiz questions (page 169)
Q1: c) 1.3
Q2: d) 39 J
Q3: c) 40.9 J


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