SCHOLAR Study Guide

SQA Higher Physics

Unit 1: Mechanics and Properties of Matter

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SCHOLAR Study Guide Unit 1: SQA Higher Physics

1. SQA Higher Physics

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Topic 1

Vectors

Contents

1.1 Introduction

Vectors play an important role in Physics. You will be using vectors to describe a large number of different physical quantities throughout this course, so this Topic provides you with all the skills you need to use vectors. We will deal with two issues: how to add vectors, and how to find components of a vector. You will find that these two tasks appear time and time again as you progress through the course. If you pick up a good understanding of vectors now, other Physics Topics will be made a lot simpler.

1.2 Vectors and scalars

Learning Objective

To distinguish between vector and scalar quantities.

1.2.1 Distance and displacement

Consider these two situations. On Monday morning, you leave your house and walk directly to school, 500 m from your home. On Tuesday morning, instead of walking directly to school, you take a turn off the direct route to your friend's house. From there you walk to the newsagent to buy a magazine, and from there you walk to school.

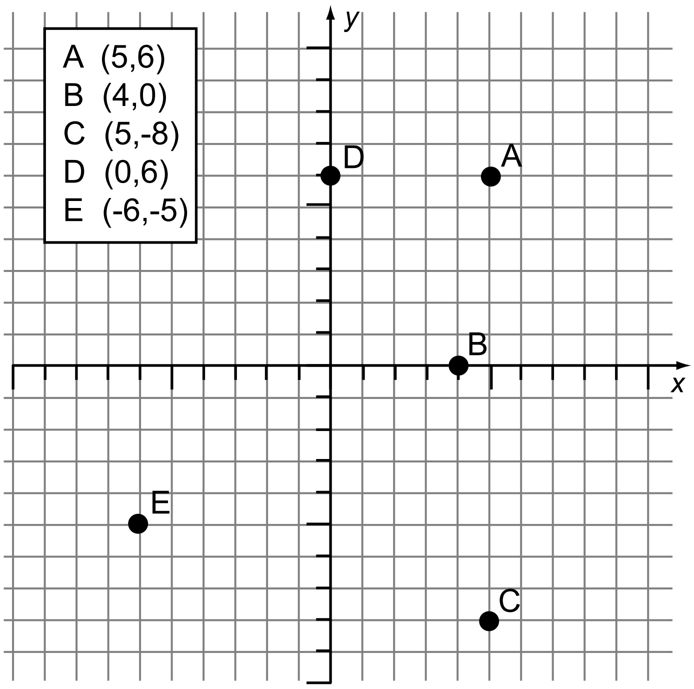
Now, in both of these cases, you have ended up a distance of 500 m from where you started, but clearly on Tuesday you have walked a lot further to get there. In Physics we distinguish between your displacement and the distance you have travelled.

The **displacement** of an object from a particular point is defined as the distance in a specified direction between that point and the object. When we are talking about displacement, we are not concerned with the distance travelled by the object, only its direct distance from the starting point. So even if your route to school on Tuesday covered 800 m, your final displacement from home is still 500 m.

Displacement is an example of a **vector** quantity. A vector is a quantity that has direction, as well as magnitude. Distance is called a **scalar** quantity; one which has magnitude but no direction.

To illustrate the difference between vectors and scalars, consider Figure 1.1.

Figure 1.1: Points at different displacements from the origin



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The coordinates of the points A, B, C, D and E are as shown. Let us now consider a direction, such as the *x*-direction. Although they are at different distances from the origin, points A and C have the same displacement in the *x*-direction (given by their *x*-coordinate). Point D has zero displacement in the *x*-direction, whilst point E has a negative *x*-displacement.

Distance and displacement

10 min

You can try this activity online or by writing the answers down on paper.

Answer the following questions which refer to Figure 1.1.

Q1: Which point has the same displacement in the *y*-direction as point A?

. . . .

Q2: Which point has a displacement of zero in the *y*-direction?

. . . .

Q3: Which point has a negative *x*-displacement *and* a negative *y*-displacement?

. . . .

Q4: Which point is at the same distance from the origin as point A, but with different *x*- and *y*-displacements?

. . . .

Two points can lie at the same distance from the origin yet have different displacements.

1.2.2 Other vector and scalar quantities

We have seen that we can make a distinction between distance and displacement, since one is a scalar quantity and the other is a vector quantity. There are also scalar and vector quantities associated with the rate at which an object is moving.

Speed is a scalar quantity. If we say an object has a speed of 10 m s^{-1}, we are not specifying a direction. The **velocity** of an object is its speed in a given direction, so velocity is a vector. We would say that an object is moving with a velocity of 10 m s^{-1} in the *x*-direction, or a velocity of 10 m s^{-1} in a northerly direction, or any other direction.

Just as with displacement, it is important to specify the direction of velocity. Suppose we throw a ball vertically upwards into the air. If we specify upwards as the positive direction of velocity, then the velocity of the ball will start with a large value, and decrease as the ball travels upwards. At the highest point of its motion, the velocity is zero for a split second before the ball starts moving downwards. When it is moving back towards the Earth, the velocity of the ball is negative, because we have defined "upwards" as the positive direction.

We have described the difference between the scalar quantities distance and speed and the vector quantities displacement and velocity. Which other physical quantities are scalar quantities, and which are vector quantities?

Acceleration is the rate of change of velocity, and so it is a vector quantity. Force is also a vector quantity. Along with displacement and velocity, acceleration and force are the most common vector quantities we will be using in this course.

Amongst the other scalar quantities you will have already met in Physics are mass and temperature. There is no direction associated with the mass or temperature of an object. Scalar quantities are combined using the normal rules of mathematics. So if you have a 5.0 kg mass, and you add a 3.5 kg mass to it, the combined mass is
5.0 + 3.5 = 8.5 kg. As we will see in the next section, it is not always so straightforward to combine two or more vectors.

1.3 Combining vectors

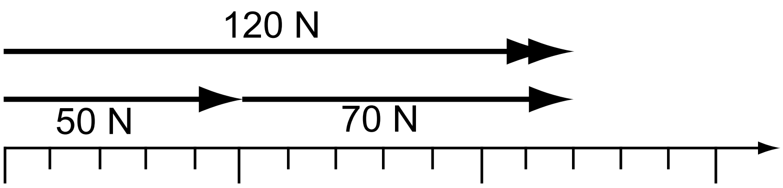
Learning Objective

To calculate the resultant when a number of vectors are combined.

What happens when we add two (or more) vectors together? We will start by looking at the simplest case, which is two vectors acting in the same direction. For example, if two men are trying to push-start a car, one may be applying a force of 50 N, the other may be applying a force of 70 N. Since the two forces are acting in the same direction, the resultant force is just the sum of the two, 50 + 70 = 120 N.

We can get the same result if we use a scale drawing, as shown in Figure 1.2. Draw the two vectors "nose-to-tail", in either order, and the resultant is equal to the total length of the two vectors, 120 N.

Figure 1.2: Collinear vectors acting in the same direction



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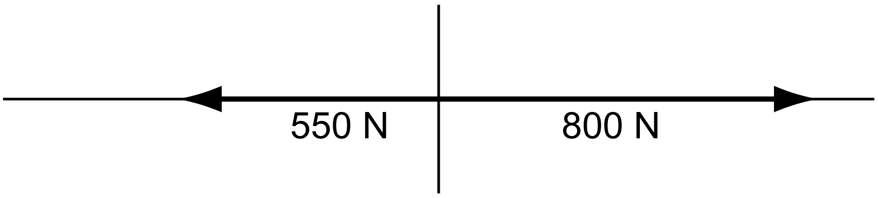
Two vectors acting in the same direction are called collinear vectors.

What about two vectors acting in opposite directions, like the opposing forces in a tug-of-war contest? Suppose one tug-of-war team pulls to the right with a force of 800 N, while the other team pulls to the left with a force of 550 N, as shown in Figure 1.3.

Figure 1.3: Two forces acting in opposite directions



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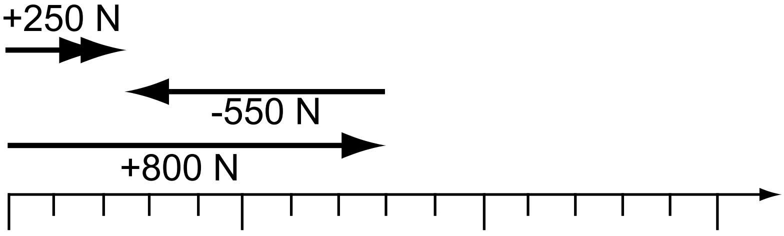
The resultant force can be found by adding the two vectors, but remember that a vector has direction as well as magnitude. Common sense tells us that the tug-of-war team pulling with the greatest force will win the contest. Acting to the right, we have forces of +800 N and -550 N, so the total force acting to the right is 800 - 550 = +250 N. This process is called finding the vector sum of the two vectors.

Again, we can use a nose-to-tail vector diagram, as in Figure 1.4. The resultant force is 250 N to the right.

Figure 1.4: Collinear vectors acting in opposite directions



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We can combine as many collinear vectors as we like by finding their vector sum.

20 min

Adding collinear vectors

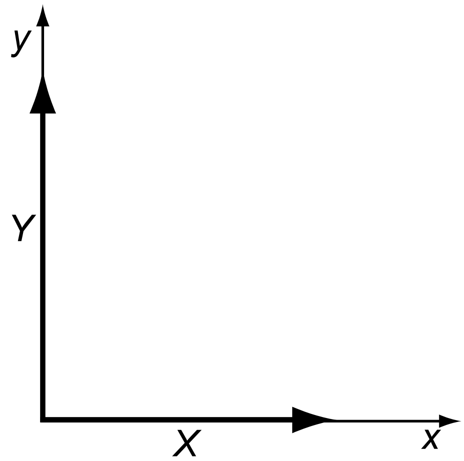
Online simulation showing how to find the resultant of two or more collinear vectors.

Full instructions are given on-screen.

The resultant of several collinear vectors can be determined by vector addition or by an accurate scale diagram.

The next case to look at is the addition of two vectors which act at right angles to each other, sometimes called rectangular, orthogonal or perpendicular vectors. We can consider the general case of a vector *X* acting in the positive *x*-direction, and a vector *Y* acting in the positive *y*-direction. Figure 1.5 shows two orthogonal vectors.

Figure 1.5: Orthogonal vectors



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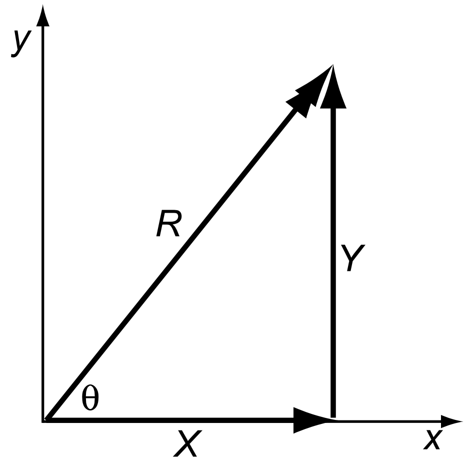
We could compare this situation to that of a man walking a certain distance to the east, who then turns and walks a further distance northwards. Again, common sense tells us his total displacement will be somewhere in a north-easterly direction, which we can find by the vector addition of the two east and north displacement vectors.

If we look at the nose-to-tail vector diagram in this case (Figure 1.6), we can see that the resultant is the hypoteneuse of a right-angled triangle whose other sides are the two vectors which we are adding. So whenever we are combining two orthogonal vectors, the resultant is vector *R*, where the magnitude of *R* is $|R|$ which is given by $|R| = \sqrt{X^2 + Y^2}$. The direction of *R* is given by the angle θ, where $\theta = \tan^{-1}\left({}^{Y}/_{X}\right)$. So we would say the the resultant is a vector of magnitude *R*, acting at an angle θ to the *x*-axis.

Figure 1.6: Resultant of two orthogonal vectors



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Crossing the river

15 min

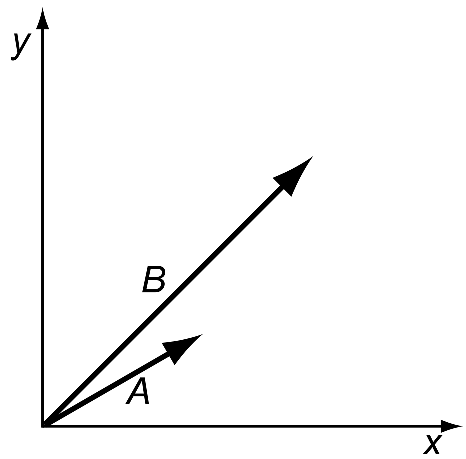
An example of two (velocity) vectors combining occurs when a boat crosses a fast-flowing river. The velocity of the boat and the velocity of the stream combine to produce a resultant velocity. Use this simulation to investigate how the two vectors add together.

Full instructions are given on-screen.

The laws of right-angled triangles can be used to determine the magnitude and direction of the resultant of two perpendicular vectors.

The general case of two vectors acting in different directions can be solved by using a scale drawing. As an example, let's consider a force *A* of magnitude 20 N, acting at 30° to the *x*-axis, and a force *B* of magnitude 40 N acting at 45° to the *x*-axis, where both forces act in the direction away from the origin. This set-up is shown in Figure 1.7

Figure 1.7: Two forces



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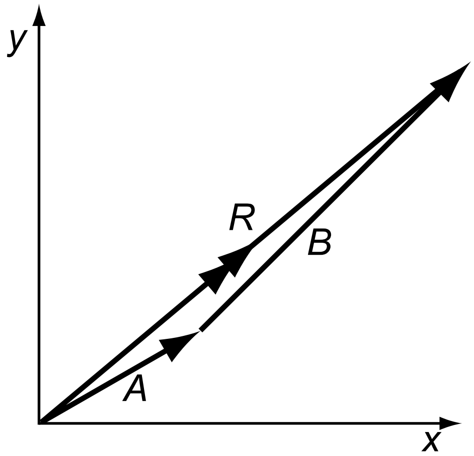
An accurate scale drawing allows us to determine the magnitude and direction of the resultant force.

Figure 1.8: Scale drawing to determine the resultant of two vectors



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In this case the scale drawing shows us that the magnitude of the resultant R is 60 N, and the direction of R (measured with a protractor) is 40° to the x-axis.

Again, we can find the resultant of any number of vectors by drawing them in scale, nose-to-tail.

Addition of vectors

15 min

This online simulation allows you to find the resultant of up to four vectors.

Full instructions are given on-screen.

An accurate scale drawing can be used to determine the magnitude and direction of the resultant of several vectors.

Quiz 1 Adding vectors

15 min

Multiple choice quiz.

First try the questions. If you get a question wrong or do not understand a question, there are 'Hints' at the the back of the book . The hints are in the same order as the questions. If you read the hint and still do not understand then ask your tutor.

Q5: Two forces are applied to an object to slide it along the floor. One force is 75 N, the other is 40 N. If the two forces act in the same direction, what is the magnitude of the total force acting on the object?

a) 0.53 N
b) 1.875 N
c) 35 N
d) 85 N

e) 115 N

. . . .

Q6: What is the resultant force when the two following forces are applied to an object: a 25 N force acting to the north, and a 55 N force acting to the south?

a) 30 N acting northwards
b) 30 N acting southwards
c) 80 N acting northwards
d) 80 N acting southwards
e) 1375 N acting northwards

. . . .

Q7: Two orthogonal forces act on an object: a 120 N force acting in the positive *x*-direction, and a 70 N force acting in the positive *y*-direction. What is the magnitude of the resultant force acting on the object?

a) 9.5 N
b) 14 N
c) 90 N
d) 139 N
e) 190 N

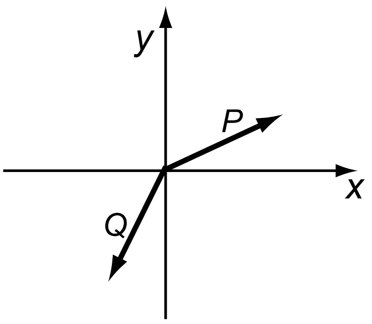
. . . .

Q8: Following on from the previous question, what is the angle between the resultant force and the *x*-axis?

a) 1.7°
b) 30°
c) 36°
d) 54°
e) 60°

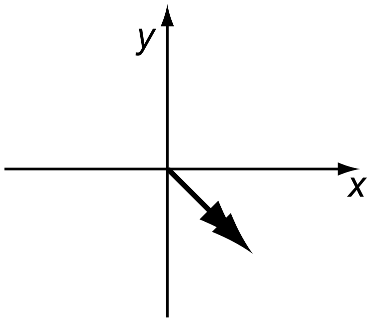
. . . .

Q9: Consider the two vectors *P* and *Q* shown in the diagram.

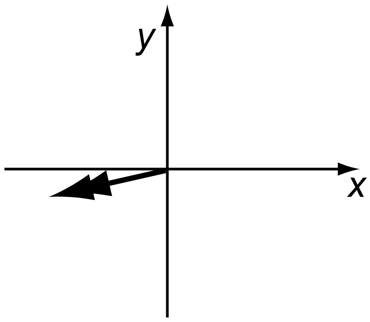


Which of the following could represent the resultant *R* of the two vectors *P* and *Q*?

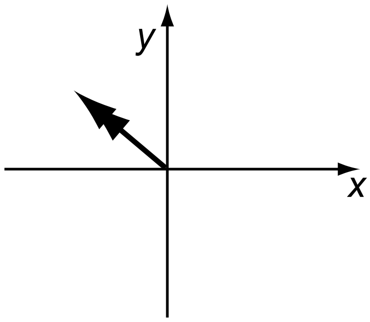
a)



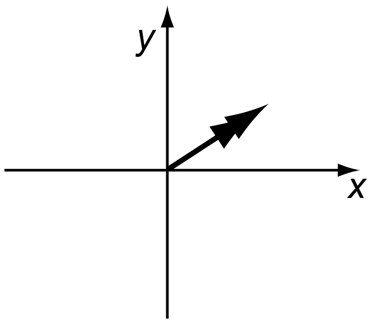
b)



c)



d)



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1.4 Components of a vector

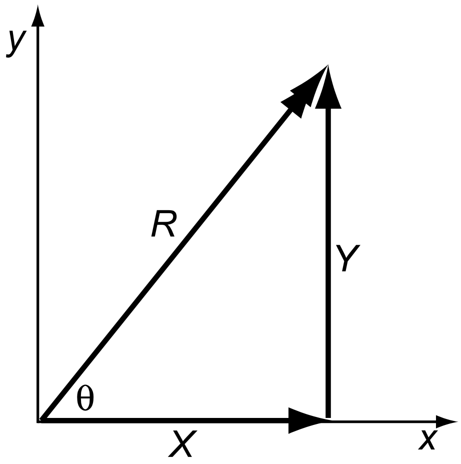
Learning Objective

To determine the orthogonal components of a vector.

Looking back to the previous section, we saw that the resultant of two orthogonal vectors could be found using the laws of right-angled triangles. It is often useful for us to do the opposite process, and work out the rectangular (or orthogonal) **components of a vector**. Let's look at the two vectors *X* and *Y* and their resultant

R, shown again in Figure 1.9.

Figure 1.9: Orthogonal components of a vector



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If we know the values of *R* and θ, we can work out the values of *X* and *Y* using the laws of right-angled triangles:

$$\sin\theta = \frac{Y}{R}$$
$$\therefore Y = R\sin\theta$$
$$\cos\theta = \frac{X}{R}$$
$$\therefore X = R\cos\theta$$

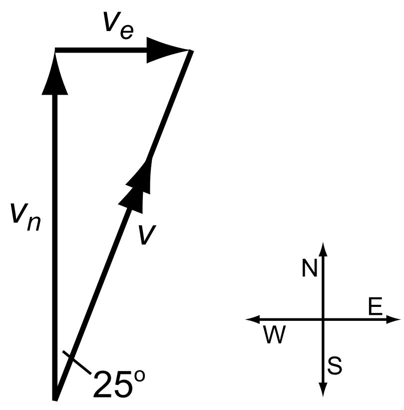
We will meet many situations in Physics where we use the orthogonal components of a vector, so it is important that you are able to carry out this process.

Example

A car is travelling at 20 m s^{-1}. A compass on the dashboard tells the driver she is travelling in a direction 25° east of magnetic north. Find the component of the car's velocity

1. in a northerly direction;
2. in an easterly direction.

Figure 1.10: Components of velocity



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1. Referring to Figure 1.10, the component v_n in the northerly direction is

$$v_n = v \times \cos 25$$
$$\therefore v_n = 20 \times 0.906$$
$$\therefore v_n = 18\,\mathrm{m\,s^{-1}}$$

2. The component v_e in the easterly direction is

$$v_e = v \times \sin 25$$
$$\therefore v_e = 20 \times 0.423$$
$$\therefore v_e = 8.5\,\mathrm{m\,s^{-1}}$$

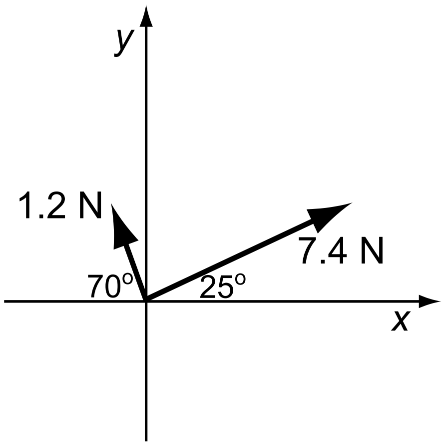
....

One final point should be noted about the components of a vector. If we are adding two or more vectors together, we can use the components of each vector. If we find the *x*- and *y*-components, say, of each vector, then these components can be easily combined as they are collinear. Adding all the *x*-components together gives us the *x*-component of the resultant vector, and adding all the *y*-components together gives us its *y*-component. This method is often easier to use than making an accurate scale drawing.

Example

The two forces shown in Figure 1.11 act on an object placed at the origin. By finding the rectangular components of the two forces, calculate the magnitude and direction of the resultant force acting on the object.

Figure 1.11: Two forces acting at the origin



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The *y*-components of the two forces both act in the positive direction, so the *y*-component R_y of the resultant is

$$R_y = (7.4 \times \sin 25) + (1.2 \times \sin 70)$$
$$\therefore R_y = 4.255\,\text{N}$$

The *x*-components of the two forces act in opposite directions, so the *x*-component R_x of the resultant is

$$R_x = (7.4 \times \cos 25) - (1.2 \times \cos 70)$$
$$\therefore R_x = 6.296\,\text{N}$$

The magnitude *R* of the resultant is

$$R = \sqrt{R_y^2 + R_x^2}$$
$$\therefore R = \sqrt{4.255^2 + 6.296^2}$$
$$\therefore R = 7.6\,\text{N}$$

Both R_x and R_y act in a positive direction, so *R* acts in the (+*x*,+*y*) direction. The angle θ between *R* and the *x*-axis is

$$\theta = \tan^{-1}\left(\frac{4.255}{6.296}\right)$$
$$\therefore \theta = 34°$$

....

Components of a vector

Online simulation showing how to find the resultant of two orthogonal vectors. The simulation can also be used to find the components of a single vector.

20 min

Full instructions are given on-screen.

Any vector can be split into orthogonal components.

---

15 min

Quiz 2 Components of a vector

Multiple choice quiz.

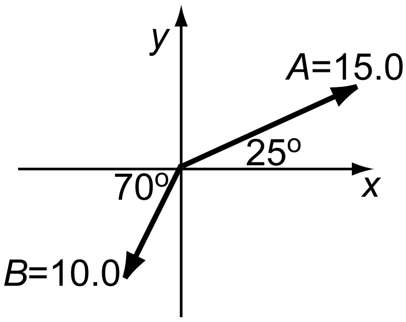
First try the questions. If you get a question wrong or do not understand a question, there are 'Hints' at the the back of the book . The hints are in the same order as the questions. If you read the hint and still do not understand then ask your tutor.

Q10: A marksman fires his gun. The bullet leaves the gun with speed 320 m s^{-1} at an angle of elevation 40°. What is the horizontal component of the bullet's velocity as it leaves the gun?

a) 8.0 m s^{-1}
b) 206 m s^{-1}
c) 245 m s^{-1}
d) 268 m s^{-1}
e) 418 m s^{-1}

....

Q11: Consider the two vectors *A* and *B* shown in the diagram.



By considering the components of each vector, what is the *y*-component of the resultant of these two vectors?

a) -10.2
b) -3.06
c) +3.06
d) +10.2
e) +15.7

....

Q12: What is the *x*-component of the resultant of the vectors *A* and *B* shown in the previous question?

a) -10.2
b) -3.06
c) +3.06
d) +10.2
e) +15.7

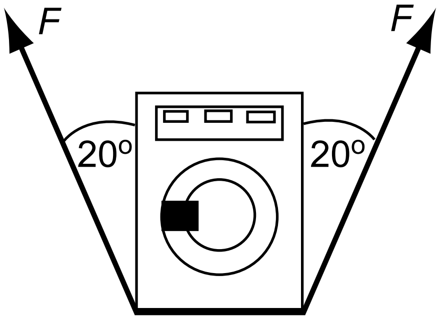
. . . .

Q13: A woman is dragging a suitcase along the floor in an airport. The strap of the suitcase makes an angle of 30° with the horizontal. If the woman is exerting a force of 96 N along the strap, what is the horizontal force being applied to the suitcase?

a) 24 N
b) 48 N
c) 62 N
d) 83 N
e) 96 N

. . . .

Q14: To carry a new washing machine into a house, two workmen place the machine on a harness. They then lift the harness by a rope attached either side. The ropes make an angle of 20° to the vertical, as shown in the diagram.



If each workman applies a force F = 340 N, what is the total *vertical* force applied to the washing machine?

a) 82 N
b) 230 N
c) 250 N
d) 320 N
e) 640 N

. . . .

1.5 Summary

By the end of this Topic you should be able to:

- distinguish between distance and displacement, and between speed and velocity;
- define and classify vector and scalar quantities;
- state what is meant by the resultant of a number of vectors;
- use scale diagrams to find the magnitude and direction of the resultant of a number of vectors;
- carry out calculations to find the rectangular (orthogonal) components of a vector.

Online assessments

Two online test are available. Each test should take you no more than 20 minutes to complete. Both tests have questions taken from all parts of the Topic.

Topic 2

Equations of motion

Contents

2.1 Introduction

If you drop a book out of the window, how long does it take to reach the ground? How fast is it travelling when it hits the ground? In this Topic we will study the vector quantities displacement, velocity and acceleration so that we can answer questions on the motion of objects. There are many practical examples that we will be able to analyse, such as the motion of cars as they accelerate or slow down.

Throughout the Topic we will be concentrating only on objects moving with constant acceleration. We will use graphs of acceleration, velocity and displacement plotted against time to give a graphical representation of the motion of an object.

We will derive three equations, known as the kinematic relationships or the equations of motion. We can use these equations to solve problems involving motion with constant acceleration. By the time you finish the Topic, you should be able to solve problems of motion in one and two dimensions using the kinematic relationships.

2.2 Acceleration

Learning Objective

To state that acceleration is the rate of change of velocity.

If the velocity of an object changes over a period of time, the object has an **acceleration**. Acceleration (*a*) is defined as the change in velocity per unit time. This can also be expressed as the rate of change of velocity. Like velocity, acceleration is a vector quantity.

$$\text{acceleration} = \frac{\text{change in velocity}}{\text{time}}$$

$$a = \frac{\Delta v}{t}$$

The units of acceleration are m s^{-2}. We can think of this as "m s^{-1} per second". If an object has an acceleration of 10 m s^{-2}, then its velocity increases by 10 m s^{-1} every second. If its acceleration is -10 m s^{-2}, then it's velocity is decreasing by 10 m s^{-1} every second. It is worth noting that an object can have a negative acceleration but a positive velocity, or vice versa.

2.3 Graphical representation

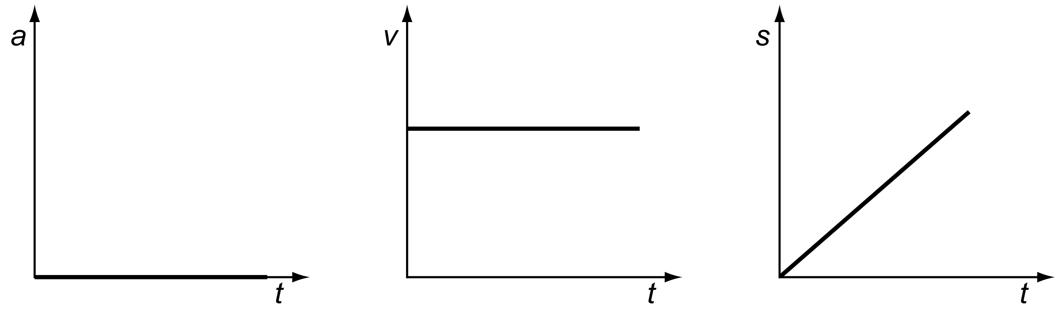
Learning Objective

To draw acceleration-time and velocity-time graphs, and use them to deduce information about the motion of an object.

In this section we will look at how motion with constant acceleration can be represented in graphical form. We can use graphs to show how the acceleration, velocity and displacement of an object vary with time.

1. Suppose a car is being driven along a straight road at constant velocity. In this case the acceleration of the car is zero at all times, whilst the velocity has a constant value.

Figure 2.1: Graphs for motion with constant velocity



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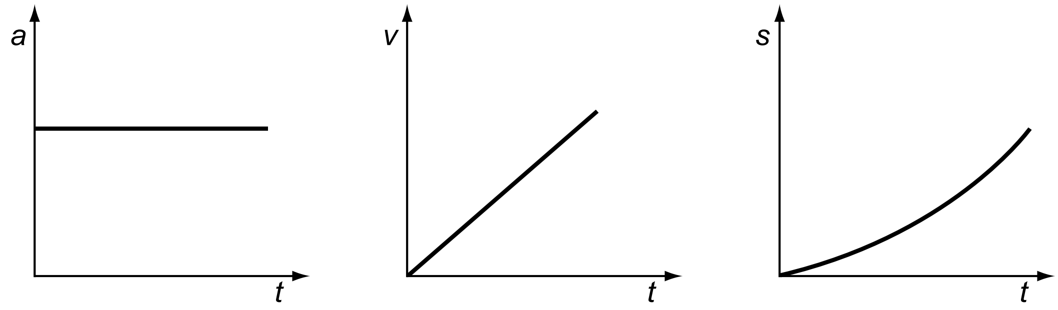
2. Suppose instead the car starts from rest, accelerating at a uniform rate of, say, 2.0 m s^{-2}. The velocity increases by 2.0 m s^{-1} every second.

Figure 2.2: Graphs for motion with constant positive acceleration



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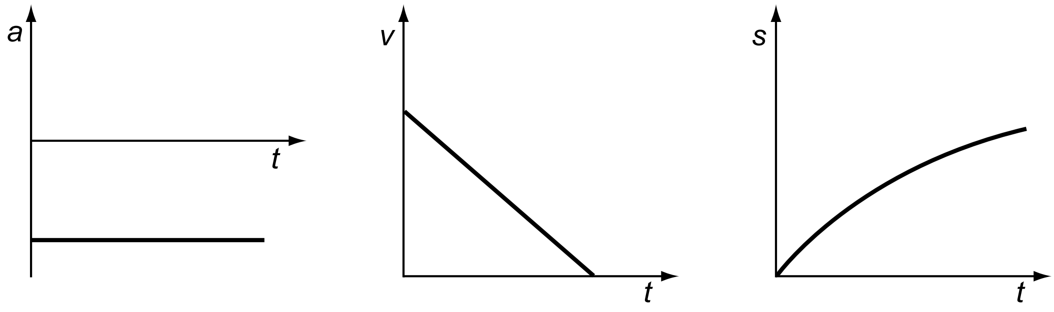
3. What if the car is travelling at a certain velocity when the brakes are applied? In this case the car may be decelerating at 2.0 m s^{-2} (an acceleration of -2.0 m s^{-2}) until it comes to rest.

Figure 2.3: Graphs for motion with constant negative acceleration



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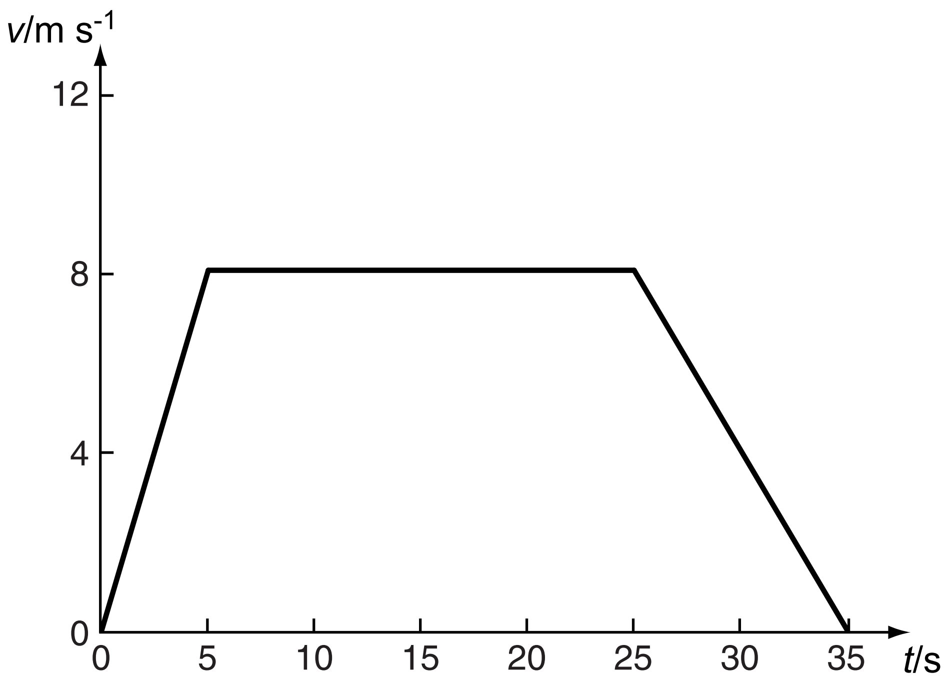
Since acceleration is the rate of change of velocity, we can work out the acceleration-time graph by studying the velocity-time graph.

Extra Help: Interpretation of displacement-time and acceleration-time graphs

Example

The graph in Figure 2.4 shows the motion of a car. The car starts from rest, accelerating uniformly for the first 5 s. It then travels at constant velocity of 8.0 m s^{-1} for 20 s, before the brakes are applied and the car comes to rest uniformly in a further 10 s.

Figure 2.4: Velocity-time graph



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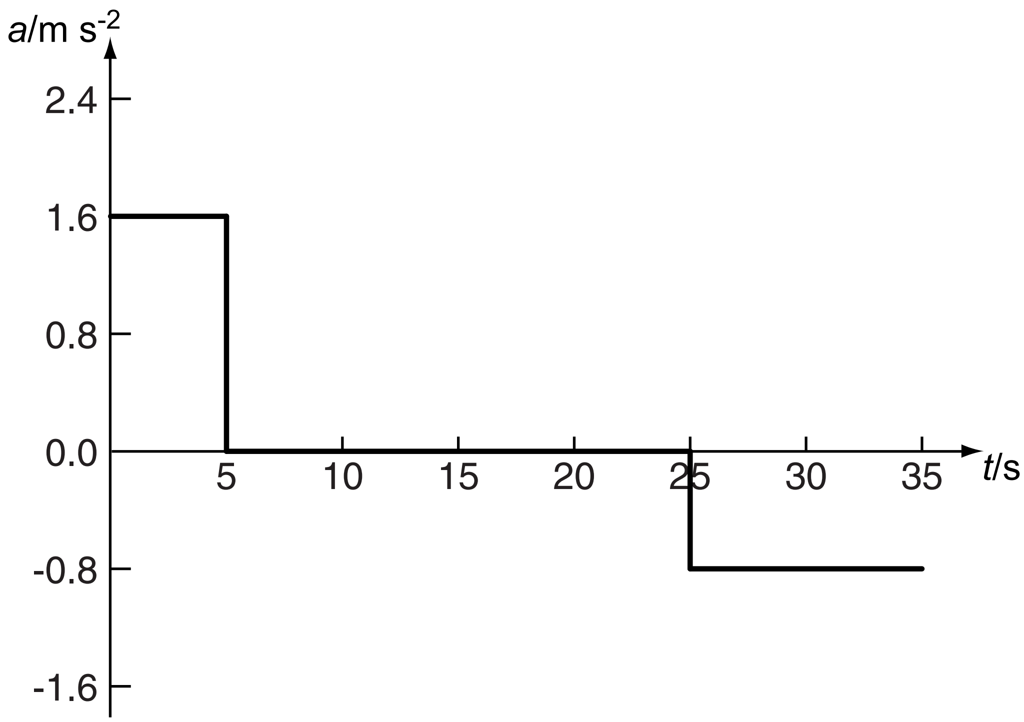
1. Using the graph in Figure 2.4, calculate the value of the acceleration of the car whilst the brakes are being applied.

2. Sketch the acceleration-time graph for the motion of the car.

1. The graph shows us that the car slows from 8.0 m s^{-1} to rest in 10 s. Its acceleration over this period is ${}^{-8.0}/_{10} = -0.80\,\mathrm{m\,s^{-2}}$.

2. The car starts from rest, and accelerates to a velocity of 8.0 m s^{-1} in 5.0 s. The acceleration *a* over this period is ${}^{8.0}/_{5.0} = 1.6\,\mathrm{m\,s^{-2}}$. For the next 20 s, the car is travelling at constant velocity, so its acceleration is zero. We have already calculated the acceleration between *t* = 25 s and *t* = 35 s. Using the values of acceleration we have just calculated, the acceleration-time graph can be plotted:

Figure 2.5: Acceleration-time graph



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Quiz 1 Acceleration

15 min

Multiple choice quiz.

First try the questions. If you get a question wrong or do not understand a question, there are 'Hints' at the the back of the book . The hints are in the same order as the questions. If you read the hint and still do not understand then ask your tutor.

Q1: An Olympic sprinter accelerates from rest to a velocity of 9.0 m s^{-1} in the first 2.5 s of a race. What is the average acceleration of the sprinter during this time?

a) 0.28 m s^{-2}
b) 0.69 m s^{-2}
c) 1.44 m s^{-2}
d) 3.6 m s^{-2}
e) 22.5 m s^{-2}

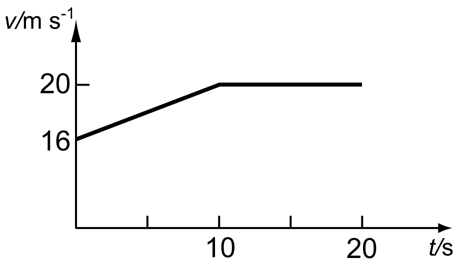
. . . .

Q2: A motorcycle is travelling at a constant velocity along a straight road. Which *one* of the following statements is true?

a) Both the acceleration and the velocity of the motorcycle have constant, positive values.
b) The velocity of the motorcycle is constant, so its acceleration is zero.
c) The velocity of the motorcycle is positive, but its acceleration is negative.
d) The acceleration of the motorcycle increases with time.
e) The acceleration of the motorcycle decreases with time.

. . . .

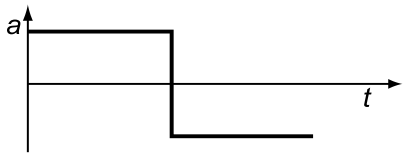
Q3: A car is being driven along a straight road. The velocity-time graph is shown below. What is the acceleration of the car between *t* = 0 and *t* = 10 s?

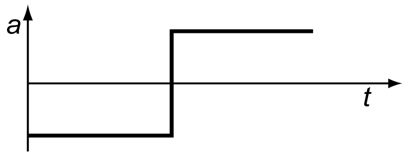


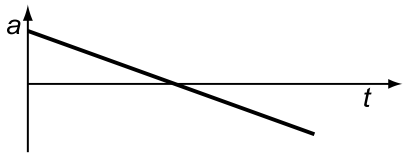
a) 0.4 m s^{-2}
b) 0.5 m s^{-2}
c) 1.6 m s^{-2}
d) 2.0 m s^{-2}
e) 2.5 m s^{-2}

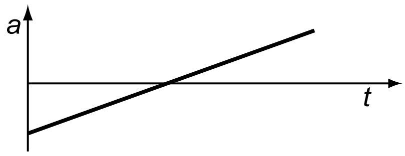
. . . .

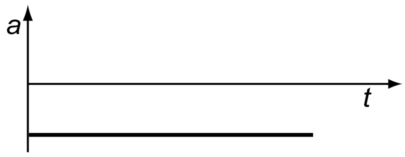
Q4: A tennis ball is thrown vertically upwards. It reaches a height of 5.0 m before falling back to the ground. Which of the following could represent the acceleration-time graph of the ball during the whole of this motion?

a) 

b) 

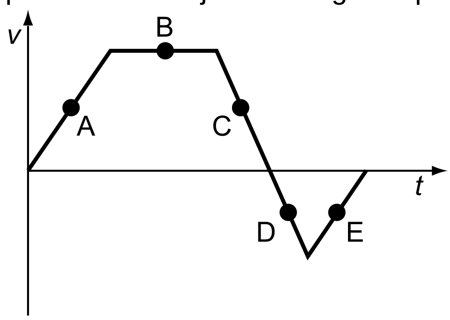
c) 

d) 

e) 

....

Q5: The following graph shows the velocity against time for a certain object. At which points is the object moving with positive acceleration?



a) A and B
b) A and E
c) C and D
d) A and C
e) B and E

....

2.4 Kinematic relationships

Learning Objective

To derive and apply the kinematic relationships.

Acceleration is defined as the rate of change of velocity. Remember that we are considering only objects which move with a uniform acceleration. If we have an object whose velocity changes from an initial value *u* to a final value *v* in time *t*, then the acceleration *a* (= rate of change of velocity) is given by

$$a = \frac{v-u}{t}$$

We can rearrange this equation:

$$\begin{aligned} a &= \frac{v-u}{t} \\ \therefore at &= v-u \\ \therefore v &= u+at \end{aligned} \tag{2.1}$$

. . . .

This is an important equation for an object moving with constant acceleration, and you will need to remember it.

If the object's velocity changes from *u* to *v* in time *t*, then the average velocity is

$$v_{ave} = \frac{u+v}{2}$$

The displacement *s* of the object in time *t* is equal to the average velocity multiplied by *t*. That is to say,

$$\begin{aligned} s &= v_{ave} \times t \\ \therefore s &= \frac{u+v}{2} \times t \end{aligned}$$

Since we know that $v = u + at$, we can substitute for *v* in the equation for *s*:

$$\begin{aligned} s &= \frac{u+v}{2} \times t \\ \therefore s &= \frac{u+(u+at)}{2} \times t \\ \therefore s &= \frac{2u+at}{2} \times t \\ \therefore s &= \frac{2ut+at^2}{2} \\ \therefore s &= ut + \tfrac{1}{2}at^2 \end{aligned} \tag{2.2}$$

. . . .

This is another important equation for you to remember.

Both of the equations we have derived have included time *t*. We will now derive an equation which does not involve *t*. To begin, let us rearrange Equation 2.1 in terms of *t*:

$$\begin{aligned} v &= u + at \\ \therefore v - u &= at \\ \therefore t &= \frac{v-u}{a} \end{aligned}$$

We can now substitute for *t* in our second equation:

$$\begin{aligned} s &= ut + \tfrac{1}{2}at^2 \\ \therefore s &= u\frac{(v-u)}{a} + \frac{1}{2}a\left(\frac{v-u}{a}\right)^2 \\ \therefore s &= u\frac{(v-u)}{a} + \frac{1}{2a}(v-u)^2 \\ \therefore 2as &= 2u(v-u) + (v-u)^2 \\ \therefore 2as &= 2uv - 2u^2 + (v^2 - 2uv + u^2) \\ \therefore 2as &= -u^2 + v^2 \\ \therefore v^2 &= u^2 + 2as \end{aligned} \tag{2.3}$$

. . . .

This is the third of the equations you need to remember.

These three equations are called the **kinematic relationships**, and we can use them to solve problems involving motion with constant acceleration. Remember that *s*, *u*, *v* and *a* are all vector quantities.

Extra Help: Using sign conventions in equations of motion and momentum calculations

Motion of a bouncing ball

15 min

What do the acceleration-time and velocity-time graphs look like for the motion of a bouncing ball?

Full instructions for this simulation, as well as solutions to the calculations, are given on-screen.

This simulation shows the graphs representing the motion of a bouncing ball. Note the sign convention that has been adopted - the upward direction is taken as positive.

Try to answer the following questions:

1. Suppose the ball is dropped from a height of 10 m. What is its velocity just before it hits the ground? (Remember the sign convention.)
2. A ball dropped from a height of 10 m rebounds with initial velocity 12 m s^{-1}.
   To what height does it rise?

The motion of a bouncing ball can be described using the kinematic relationships.

---

20 min

Horizontal Motion

On-screen simulation, in which you can work out the deceleration needed to ensure a car stops before a set of traffic lights.

Full instructions are given on-screen, as well as the solution to the calculation included in the activity.

Always use the same procedure to solve a kinematics problem in one dimension - sketch a diagram, list the data and select the appropriate kinematic relationship.

Example

A stone is dropped from the top a building which is 45 m high. If the acceleration due to gravity is 9.8 m s^{-2}, calculate

1. the time taken for the stone to reach the ground;
2. the downward velocity of the stone when it hits the ground.

From the question, we are told that the stone is dropped, so its initial velocity *u* = 0 m s^{-1}. Its acceleration (downwards) *a* is 9.8 m s^{-2} and its displacement (downwards) *s* = 45 m.

1. Given *u*, *a* and *s*, to find *t* we use the relationship $s = ut + \frac{1}{2}at^2$.

$$\begin{aligned} s &= ut + \tfrac{1}{2}at^2 \\ \therefore 45 &= 0 + \left(\tfrac{1}{2} \times 9.8 \times t^2\right) \\ \therefore 45 &= 4.9t^2 \\ \therefore t^2 &= \frac{45}{4.9} = 9.18 \\ \therefore t &= 3.0\,\text{s} \end{aligned}$$

2. To find *v* when we know the values of *u*, *a* and *s*, we use the relationship $v^2 = u^2 + 2as$

$$\begin{aligned} v^2 &= u^2 + 2as \\ \therefore v^2 &= 0^2 + (2 \times 9.8 \times 45) \\ \therefore v^2 &= 882 \\ \therefore v &= 30\,\text{m s}^{-1} \text{ to nearest whole number} \end{aligned}$$

....

To solve problems involving motion of an object in two dimensions, we usually split the motion into vertical and horizontal components. If the only acceleration is the acceleration due to gravity *g*, then there is no horizontal acceleration of the object.

20 min

Projectile motion

This simulation allows you to investigate projectile motion. You can launch a projectile with any velocity, at any angle and from any height. You can even change the value *of g, the acceleration due to gravity. Use this simulation to investigate how the initial conditions (velocity, angle, etc) affect the height, time of flight and horizontal displacement of the projectile. You can also see how the height and velocity change while the projectile is in motion.*

Full instructions are given on-screen.

By considering the vertical and horizontal components separately, the kinematic relationships can be applied to the motion of a projectile.

Example

At an athletics meeting, a long jumper takes off with a velocity of 7.8 m s^{-1} at an angle of 25° to the ground. How far does she jump?

To solve this problem, we must split the motion into horizontal and vertical components. Horizontally, her velocity is constant since there is no horizontal acceleration. We can use the simple relationship *displacement* = *velocity* x*time*, which in this case gives us

$$\begin{aligned} s_x &= u\cos\theta \times t \\ \therefore s_x &= 7.8 \times 0.9063 \times t \\ \therefore s_x &= 7.069t \end{aligned}$$

We can find the value of *t* by looking at the vertical motion, and finding the time from take-off until the jumper returns to the ground. That is to say, the time when her displacement s_y is zero again. Her initial velocity upwards is $u\sin\theta$ m s^{-1} and her acceleration upwards is -9.8 m s^{-2}.

$$\begin{aligned} s &= ut + \tfrac{1}{2}at^2 \\ \therefore 0 &= (7.8\sin 25 \times t) + \left(\tfrac{1}{2} \times -9.8 \times t^2\right) \\ \therefore 0 &= 3.296t - 4.9t^2 \\ \therefore 4.9t^2 &= 3.296t \\ \therefore 4.9t &= 3.296 \\ \therefore t &= 0.673\,\text{s} \end{aligned}$$

We can substitute this value of *t* into our equation for horizontal motion:

$$\begin{aligned} s_x &= 7.069t \\ \therefore s_x &= 7.069 \times 0.673 \\ \therefore s_x &= 4.8\,\text{m} \end{aligned}$$

....

Quiz 2 Kinematic relationships

15 min

Multiple choice quiz.

First try the questions. If you get a question wrong or do not understand a question, *there are 'Hints' at the the back of the book . The hints are in the same order as the questions. If you read the hint and still do not understand then ask your tutor.*

Useful data:

acceleration due to gravity *g*	9.8 m s^{-2}

Q6: A car accelerates from rest at a rate of 3.5 m s^{-2}. How far has the car travelled after 4.0 s?

a) 7 m
b) 28 m
c) 56 m
d) 78 m
e) 98 m

. . . .

Q7: A ball is thrown vertically upwards. What is the acceleration of the ball when it is at its maximum height?

a) 0 m s^{-2}
b) 9.8 m s^{-1} upwards
c) 9.8 m s^{-1} downwards
d) 9.8 m s^{-2} upwards
e) 9.8 m s^{-2} downwards

. . . .

Q8: In a film stunt, a car is pushed over a cliff, landing on the ground 55 m below the cliff edge. What is the vertical velocity of the car when it strikes the ground?

a) 9.8 m s^{-1}
b) 23 m s^{-1}
c) 33 m s^{-1}
d) 55 m s^{-1}
e) 145 m s^{-1}

. . . .

Q9: When the brakes are applied, a car slows from 25 m s^{-1} to 10 m s^{-1} in 3.6 s. What is the acceleration of the car?

a) -0.10 m s^{-2}
b) -0.24 m s^{-2}
c) -4.2 m s^{-2}
d) -9.7 m s^{-2}
e) -69 m s^{-2}

. . . .

Q10: A cricketer hits a ball at an angle of 75° to the ground. If the ball leaves the bat with velocity 24 m s^{-1}, what is the maximum height above the ground that the ball reaches?

a) 1.2 m

b) 2.0 m
c) 7.6 m
d) 27 m
e) 55 m

2.5 Summary

By the end of this Topic you should be able to

- state that acceleration is the rate of change of velocity;
- draw an acceleration-time graph using information obtained from a velocity time graph, for motion with constant acceleration;
- use the terms "constant velocity" and "constant acceleration" to describe motion represented in tabular or graphical form;
- show how the three kinematic relationships can be defined from the basic definitions of motion with constant acceleration:

$$v = u + at$$
$$s = ut + \frac{1}{2}at^2$$
$$v^2 = u^2 + 2as$$

- carry out calculations involving motion in one and two dimensions using these kinematic relationships.

Online assessments

Four online assessments are available. Each test should take you no longer than 20 minutes to complete. The questions in Test 1 are on acceleration and the graphical representation of kinematics. Test 2 has questions about motion in one dimension, and Test 3 covers motion in two dimensions. Test 4 has questions taken from all parts of this Topic.

Topic 3

Newton's second law, energy and power

Contents

3.1 Introduction

In the "Equations of Motion" Topic we studied the displacement, velocity and acceleration of objects. Now we turn our attention to finding out what causes an object to accelerate. To study force and acceleration, we will be applying Newton's laws of motion, concentrating mainly on the second law. The technique that we use to analyse the forces acting on an object involves drawing special diagrams called free body diagrams.

Along with free body diagrams, we will be using the technique of finding perpendicular components of a vector that we met in the "Vectors" Topic. Make sure you understand this technique before you start work on this Topic. We will also be using the kinematic relationships, so have a look through your notes on the "Equations of Motion" if you cannot remember how to use these relationships.

3.2 Newton's laws of motion

Learning Objective

To state and apply Newton's laws of motion.

3.2.1 Newton's first and third laws

Although we will be concentrating on Newton's second law of motion, it is worthwhile having a quick reminder of the first and third laws.

Newton's first law of motion tells us what happens to an object when the net force acting on it is zero. The velocity of an object remains constant if the net force that acts on it is zero. That is to say, a stationary object will remain stationary, and an object travelling with velocity *v* will continue to travel with velocity *v* - moving in the same direction and with the same speed. An object is said to be in equilibrium if the net force acting on it is zero.

Newton's third law of motion states that if one body exerts a force on a second body, then the second body will exert an equal and opposite force on the first body. So if you place a book on a desk top, the weight of the book acts downwards on the desk. The desk exerts an equal and opposite force (the "normal reaction force") upwards on the book.

3.2.2 Newton's second law of motion

Newton's second law of motion can be stated as: "A net force acting on an object of mass *m* will cause the mass to accelerate in the direction of that force, with the acceleration proportional to the force." This law can be summed up in the equation

$$F = ma \tag{3.1}$$

. . . .

The unit of force, the **newton** (N), is defined using Equation 3.1 as the force that, when applied to an object of mass 1 kg, will cause the object to accelerate at 1 m s^{-2}. From Equation 3.1, we can see that 1 N is equivalent to 1 kg m s^{-2}. Remember that force and acceleration are both vector quantities. Newton's second law tells us that the acceleration is in the same direction as the net force.

Example

A horizontal force of 24 N is applied to a 4.0 kg object which is at rest on a smooth table top.

1. Calculate the acceleration of the object.
2. How far does the object slide along the table top in 1.0 s?

1. Using Equation 3.1,

$$F = ma$$
$$\therefore a = \frac{F}{m}$$
$$\therefore a = \frac{24}{4.0}$$
$$\therefore a = 6.0\,\text{m s}^{-2}$$

2. Now we have *a* = 6.0 m s^{-2}, *u* = 0 m s^{-1}, *t* = 1.0 s and we want to find the displacement *s*. We will use the kinematic relationship $s = ut + \frac{1}{2}at^2$.

$$s = ut + \tfrac{1}{2}at^2$$
$$\therefore s = 0 + \left(\tfrac{1}{2} \times 6.0 \times 1.0^2\right)$$
$$\therefore s = 3.0\,\text{m}$$

. . . .

Quiz 1 Newton's second law

15 min

Multiple choice quiz.

First try the questions. If you get a question wrong or do not understand a question, there are 'Hints' at the the back of the book . The hints are in the same order as the questions. If you read the hint and still do not understand then ask your tutor.

Q1: 1 N is equivalent to

a) 1 kg m s^{-1}
b) 1 m kg^{-1} s^{-1}
c) 1 kg m s^{-2}
d) 1 kg m^{-1} s^{-2}
e) 1 m s kg^{-2}

. . . .

Q2: A 60 N force is applied to an object of mass 4.0 kg. What is the acceleration of the object?

a) 0.067 m s^{-2}

b) 15 m s^{-2}
c) 64 m s^{-2}
d) 225 m s^{-2}
e) 240 m s^{-2}

....

Q3: Two men are pushing a broken-down car. One applies a force of 130 N, the other a force of 180 N, with both forces acting in the same direction. If the mass of the car is 1700 kg, what is the car's acceleration?

a) 0.091 m s^{-2}
b) 0.18 m s^{-2}
c) 5.5 m s^{-2}
d) 11 m s^{-2}
e) 14 m s^{-2}

....

Q4: A force *F* applied to a mass *m* causes an acceleration *a*. What is the new acceleration if the force is doubled and the mass is halved?

a) *a*/4
b) *a*/2
c) *a*
d) 2*a*
e) 4*a*

....

Q5: A 50 N force is applied to a stationary object of mass 8.0 kg. What is the velocity of the object after the force has been applied for 4.0 s?

a) 0.64 m s^{-1}
b) 2.56 m s^{-1}
c) 6.25 m s^{-1}
d) 25 m s^{-1}
e) 100 m s^{-1}

....

3.3 Free body diagrams

Learning Objective

To use free body diagrams to analyse the forces acting on an object.

The first step that you should take when trying to solve a problem involving several forces acting on an object is to sketch a free body diagram. A free body diagram is a diagram showing *all the forces* acting on an object. Imagine the object in isolation, so your diagram only includes that object, and then draw in all the forces that are acting on it.

In this part of the Topic we will see how free body diagrams are used to solve problems of objects in equilibrium (where the net force acting on an object is zero) and dynamics (where the object is accelerating).

3.3.1 Objects in equilibrium

When an object is in equilibrium, there is no net force acting in a particular direction, and the acceleration of the object is zero. If we analyse the components of all the forces acting on the object, their vector sum *in any direction* will be zero. We often look at the orthogonal (perpendicular) components of all the forces, such as the components acting parallel and perpendicular to a surface.

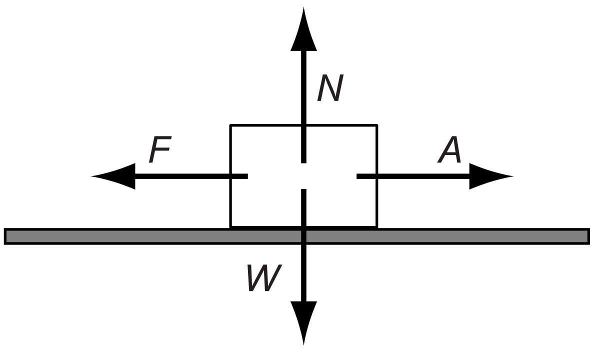
Example

A portable television of mass 5.0 kg rests on a horizontal table. A force of 65 N applied horizontally to the television causes it to slide at a constant velocity across the table top. Calculate

1. the normal reaction force which the table exerts on the television;
2. the (horizontal) frictional force acting on the television.

There are four forces acting on the television. Its weight *W* acts downwards, the normal reaction force *N* acts upwards, the applied force *A* acts horizontally, and the frictional force *F* acts horizontally in the opposite direction to *A*. We can show all these forces on a free body diagram such as Figure 3.1

Figure 3.1: Free body diagram



....

1. Vertically (perpendicular to the surface), the television is in equilibrium, since it is not moving in a vertical direction, so the forces acting upwards equal the forces acting downwards. Hence

$$N = W$$
$$\therefore N = m \times g$$
$$\therefore N = 5.0 \times 9.8$$
$$\therefore N = 49\,\text{N}$$

2. Horizontally (parallel to the surface), the television is also in equilibrium since it is

moving with constant velocity, so the net horizontal force must be zero.

$$F = A$$
$$\therefore F = 65\,\text{N}$$

....

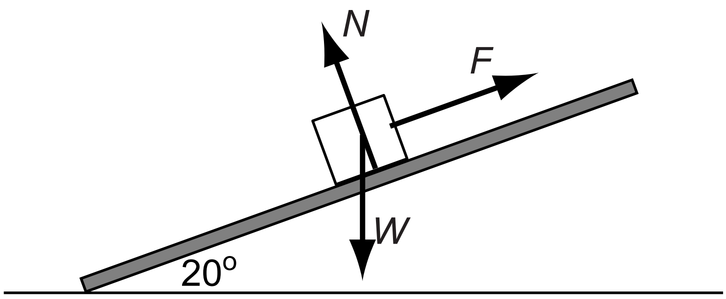
That example was probably straightforward enough for you to solve without needing a diagram, but a free body diagram can prove invaluable in more complex situations.

Example

Suppose the table is now lifted at one end, so that the 5.0 kg television is now resting on a surface inclined at 20° to the horizontal. The only forces acting on the television are its weight, the normal reaction force of the table top, and the frictional force acting parallel to the slope. If the television is stationary, calculate the magnitude of the frictional force.

Let us label the three forces as *W* for the weight, *N* for the normal reaction force and *F* for the frictional force, which acts up the slope to prevent the television sliding down the slope. These forces are shown in Figure 3.2

Figure 3.2: Television resting on a tilted table



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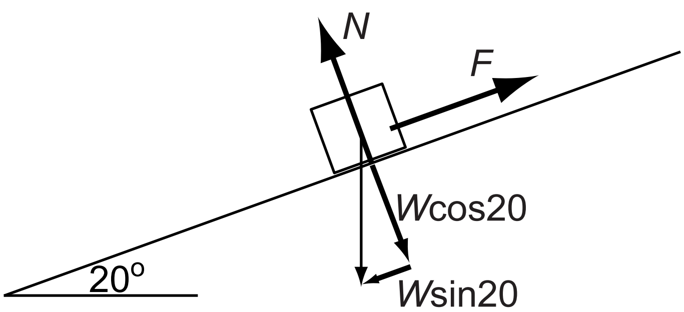
To solve this problem, we will consider the components of *W*, *N* and *F* acting parallel and perpendicular to the slope. Whilst *N* has no component parallel to the slope and *F* has no component perpendicular to the slope, *W* acts at an angle to the slope. The free body diagram (Figure 3.3) should clarify the situation.

Figure 3.3: Free body diagram of the television on a tilted table



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Make sure that you understand why the component of *W* acting parallel to the slope is *W*sin20 and the component acting perpendicular to the slope is *W*cos20. We can now calculate *F* by analysing the components acting parallel to the table top.

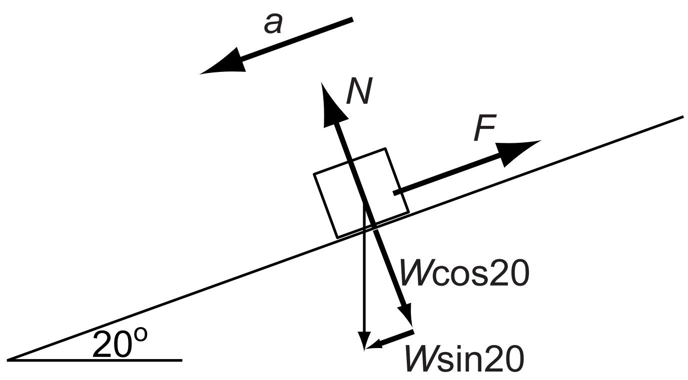
$$\begin{aligned} F &= W\sin 20 \\ \therefore F &= mg\sin 20 \\ \therefore F &= 5.0 \times 9.8 \times 0.342 \\ \therefore F &= 17\,\mathsf{N} \end{aligned}$$

. . . .

3.3.2 Objects undergoing acceleration

If an object is accelerating, there must be a net force acting on it in the direction of the acceleration. Let us continue with the example of the television (mass 5.0 kg) resting on a tilted table. If the frictional force *F* between the television and the table top is only 8.0 N, then an analysis of the forces acting parallel to the slope show a net force acting down the slope. Again, we use a free body diagram as shown in Figure 3.4.

Figure 3.4: Free body diagram of the television sliding down the slope



. . . .

We have an acceleration *a* acting down the slope. The free body diagram shows us that the net force acting in this direction is $W\sin 20 - F$. So in this case, from Newton's second law:

$$(W\sin 20 - F) = ma$$

We can solve this equation to find the acceleration *a*:

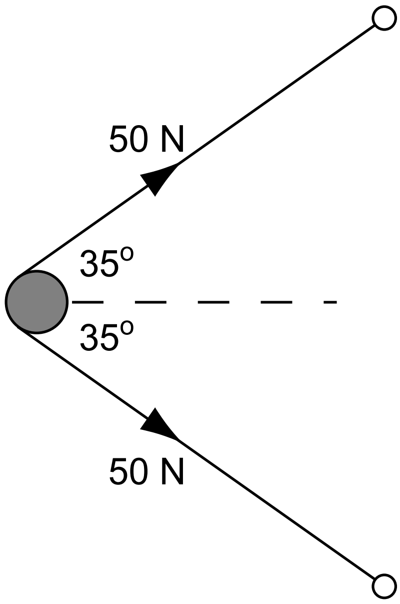
$$\begin{aligned} (W\sin 20 - F) &= ma \\ \therefore a &= \frac{(W\sin 20 - F)}{m} \\ \therefore a &= \frac{(5.0 \times 9.8 \times 0.342) - 8.0}{5.0} \\ \therefore a &= \frac{8.758}{5.0} \\ \therefore a &= 1.8\,\mathsf{m\,s}^{-2} \end{aligned}$$

So we have used a free body diagram along with Newton's second law to determine the acceleration. We can apply this technique to solve any problem in this Topic.

Example

Figure 3.5 shows a catapult, just about to launch a pebble.

Figure 3.5: The catapult just before the pebble is released



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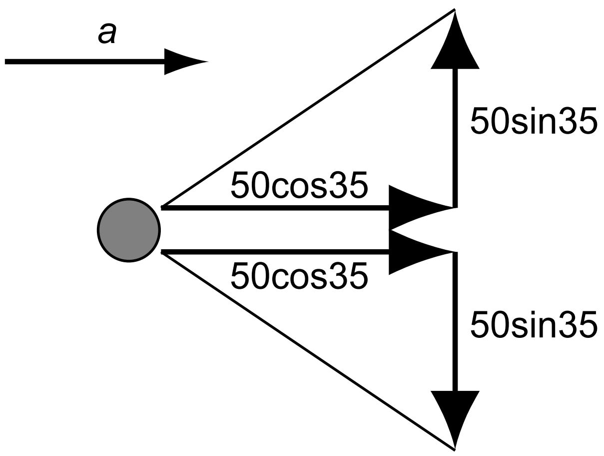
The mass of the pebble is 100 grams. The tension in the catapult elastic is 50 N. At the instant the pebble is released, what is

1. the force acting on the pebble?
2. the acceleration of the pebble?

We start by drawing a free body diagram.

Figure 3.6: Free body diagram for the catapult



. . . .

1. As we have drawn it in Figure 3.6, there is a net force acting to the right, causing an acceleration *a*. By taking the components acting in this direction, we can calculate the net force *F*.

$$
\begin{aligned}
F &= (50\cos 35) + (50\cos 35)\\
\therefore F &= 2\times 50\times 0.819\\
\therefore F &= 82\,\text{N}
\end{aligned}
$$

2. Now we have a value for the net force, we can use *F* = *ma* to find the acceleration.

$$
\begin{aligned}
F &= ma\\
\therefore a &= \frac{F}{m}\\
\therefore a &= \frac{82}{0.1}\\
\therefore a &= 820\,\text{m s}^{-2}
\end{aligned}
$$

. . . .

World's strongest man

20 min

At the World's Strongest Man competition, one of the events involves the strongmen pulling a truck. Use your knowledge of Newton's laws and free body diagrams to find out which contestant will win the truck pulling competition.

The acceleration of an object can be calculated using free body diagrams and applying Newton's second law.

Mass on a slope

20 min

Use this simulation to investigate how the acceleration of an object down a slope is affected by the mass of the object and the angle of the slope.

Using a free body diagram, the forces acting on an object can be analysed to calculate its acceleration.

Quiz 2 - Free body diagrams

20 min

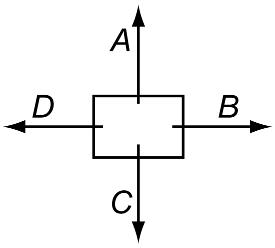
Multiple choice quiz.

First try the questions. If you get a question wrong or do not understand a question, there are 'Hints' at the the back of the book . The hints are in the same order as the questions. If you read the hint and still do not understand then ask your tutor.

Useful data:

acceleration due to gravity *g*	9.8 m s^{-2}

Q6: Four forces *A*, *B*, *C* and *D* act on an object as shown.



The magnitudes of the forces are: A = 10 N, B = 20 N, C = 10 N and D = 40 N. In which direction is the object accelerating?

a) In the direction of *A*
b) In the direction of *B*
c) In the direction of *C*
d) In the direction of *D*
e) The object is in equilibrium and is not accelerating.

....

Q7: A 15 kg crate is sliding down a slope at constant velocity. If the slope is inclined at 25° to the horizontal, calculate the magnitude of the frictional force acting on the crate.

a) 62 N
b) 69 N
c) 133 N
d) 147 N
e) 3700 N

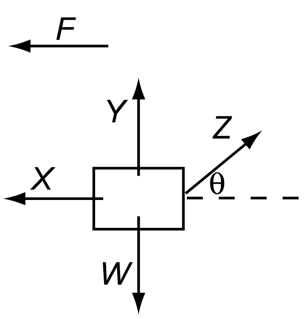
....

Q8: On a building site, a 5.0 kg block is being lowered on the end of a rope. If the downward acceleration of the block is 1.6 m s^{-2}, what is the force of the block on the rope?

a) 8.0 N
b) 16 N
c) 41 N
d) 47 N
e) 57 N

....

Q9: The following diagram relates to the next two questions.



Four forces *W*, *X*, *Y* and *Z* act on an object. *X* has magnitude 35 N. *Z* has magnitude 40 N, and acts at an angle θ = 40°. What is the net force *F* acting in the direction shown?

a) -5.0 N
b) -3.8 N
c) 1.4 N
d) 4.4 N
e) 9.3 N

. . . .

Q10: Referring to the diagram in the previous question, the magnitude of *W* is 60 N. If the net force is zero in the direction of *W*, what is the magnitude of *Y*?

a) 0 N
b) 13 N
c) 20 N
d) 29 N
e) 34 N

. . . .

3.4 Energy and power

Learning Objective

To calculate kinetic energy, potential energy and power in various situations.

3.4.1 Energy

Work is done whenever a force is applied to move an object through any distance. The energy *E* used in applying a force *F* over a distance *s* can be summed up in the equation

$$\begin{gathered}\text{work done} = \text{force} \times \text{distance}\\ E = F \times s\end{gathered} \tag{3.2}$$

. . . .

Energy (or work done) is a scalar quantity, and is measured in joules J.

The **kinetic energy** of an object is the energy that the object possesses due to its motion. An object of mass *m* moving with speed *v* has kinetic energy given by

$$KE = \frac{1}{2}mv^2 \tag{3.3}$$

. . . .

We can think of kinetic energy as the work that would have to be done to bring the object to rest.

The other sort of energy we must consider is **potential energy**. This is energy stored in an object due its position, its shape or its state. For example, if you stretch an elastic band, potential energy is stored in the band. When you release the band, that energy is used to restore the band to its natural shape - snapping your fingers in the process! If you lift a book up from the floor and place it on a table, you have increased the gravitational potential energy of the book. This potential energy is converted into kinetic energy if you nudge the book off the table and it falls back to the floor.

If you raise an object of mass *m* through a vertical height *h*, the increase in the potential energy of the object is given by Equation 3.4.

$$PE = mgh \tag{3.4}$$

....

g is the gravitational field strength. We can apply the principle of **conservation of energy** to solve some problems involving vertical motion.

Example

A toy rocket is projected vertically upwards from the ground, with an initial velocity of 30 m s^{-1}. Show that the maximum height reached by the rocket is 46 m by using

1. kinematic relationships;
2. conservation of energy.

1. Using the kinematic relationships, we must find the displacement of the rocket at the instant when its velocity is momentarily zero, just before it starts to fall back to Earth. We have an initial (upwards) velocity *u* = 30 m s^{-1}, final velocity *v* = 0 m s^{-1} and acceleration *a* = -*g* m s^{-2}, so we will use the kinematic relationship $v^2 = u^2 + 2as$.

$$\begin{aligned} v^2 &= u^2 + 2as \\ \therefore s &= \frac{v^2 - u^2}{2a} \\ \therefore s &= \frac{-30^2}{2 \times (-9.8)} \\ \therefore s &= 46 \text{ m} \end{aligned}$$

2. At the instant the rocket is projected, its potential energy is zero, and all the energy is kinetic energy. At its maximum height *h*, when the velocity is momentarily zero, all the kinetic energy is converted to potential energy.

$$\begin{aligned} KE_{ground} &= PE_h \\ \therefore \tfrac{1}{2}mv^2 &= mgh \\ \therefore \tfrac{1}{2}v^2 &= gh \\ \therefore h &= \frac{v^2}{2g} \\ \therefore h &= \frac{30^2}{2 \times 9.8} \\ \therefore h &= 46 \text{ m} \end{aligned}$$

....

Projectile motion

15 min

We return to the motion of a projectile, using one of the simulations you should have used in the previous Topic. Use the simulation to investigate how the kinetic energy and potential energy of a projectile vary throughout its motion.

The total energy (the sum of kinetic and potential energies) of a projectile is constant.

3.4.2 Power

Power is defined as the rate at which work is being done.

$$P = \frac{E}{t} \tag{3.5}$$

. . . .

Power is measured in watts (W), where 1 W is equivalent to 1 J s^{-1}. If a total of 2000 J of work is done over a period of 40 s, then the average power over that period of time is 50 W.

We can calculate the rate at which work is being done when a force is applied to move an object. For example, suppose you are pushing a heavy object across the floor at a uniform velocity, doing work against friction. If a force *F* is being applied to move the object at velocity *v*, then the power *P* (the rate at which work is being done) is given by the formula

$$P = F \times v \tag{3.6}$$

. . . .

Quiz 3 - Energy and power

15 min

Multiple choice quiz.

First try the questions. If you get a question wrong or do not understand a question, there are 'Hints' at the the back of the book . The hints are in the same order as the questions. If you read the hint and still do not understand then ask your tutor.

Useful data:

acceleration due to gravity *g*	9.8 m s^{-2}

Q11: A 5.0 kg object is travelling with a uniform velocity of 12 m s^{-1}. What is the kinetic energy of the object?

a) 30 J
b) 150 J
c) 180 J
d) 360 J
e) 1800 J

. . . .

Q12: A roof tile drops 10 m from the top of a building onto the ground. Calculate the speed of the tile just before it hits the ground.

a) 9.9 m s^{-1}
b) 14 m s^{-1}
c) 44 m s^{-1}
d) 98 m s^{-1}
e) 196 m s^{-1}

. . . .

Q13: A steady force of 1200 N is required to slide a 25 kg crate across the floor at a constant velocity of 2.5 m s^{-1}. At what rate is work being done in moving the crate?

a) 63 W
b) 78 W
c) 120 W
d) 3000 W
e) 7.5 x 10^4 W

. . . .

Q14: A projectile of mass 8.0 kg is launched with velocity 18 m s^{-1}, at an angle of 30° to the ground. What is the kinetic energy of the object when it is at its maximum height?

a) 0 J
b) 320 J
c) 970 J
d) 1300 J
e) 7800 J

. . . .

3.5 Summary

By the end of this Topic you should be able to:

- define the newton;
- state Newton's second law of motion;
- use the relationship $F = ma$ to solve problems;
- use free body diagrams to analyse the forces acting on an object;
- carry out calculations involving work done, potential energy, kinetic energy and power.

Online assessments

Three online assessments are available. Each test should take you no longer than 20 minutes to complete. The questions in Test 1 are on Newton's second law of motion. Test 2 has questions involving the use of free body diagrams, and Test 3 has questions taken from all parts of this Topic.

Topic 4

Momentum and impulse

Contents

4.1 Introduction

So far in Mechanics we have considered the motion of objects, and what happens when a force is applied to an object. In this Topic we will combine some ideas that we have already met to investigate what happens when two objects interact.

To analyse a collision, we need to introduce a new physical quantity called momentum, which depends on the mass and velocity of the object. We will see that using momentum allows us to predict the velocities of objects after they have been involved in collisions or explosions.

4.2 Momentum

Learning Objective

To define momentum.

Throughout this Topic we are going to be concerned with the **momentum** of different objects. We will be dealing with objects moving in straight lines. The linear momentum *p* of an object of mass *m* moving with velocity *v* is given by Equation 4.1.

$$p = mv \tag{4.1}$$

....

Momentum is a vector quantity, having both magnitude and direction. The units of *p* are kg m s^{-1}.

4.3 Collisions

Learning Objective

To state and apply the law of conservation of momentum.
To show an understanding of the difference between elastic and inelastic collisions.

4.3.1 Conservation of momentum

To get yourself thinking about collisions and momentum, try the "Newton's cradle" simulation. While you change the parameters in this simulation, think about the momentum of the colliding balls.

15 min

Newton's cradle

This Activity is an online simulation of a piece of equipment you may already be familiar with - Newton's cradle.

Full instructions are given on-screen.

In any collision, momentum is conserved.

When two objects collide, the law of **conservation of momentum** states that the total momentum is conserved, so long as no external force acts on the two objects. That is to say, the vector sum of the momentum of the two objects before the collision is equal to the vector sum of their momentum after the collision. If you look back at the "Newton's cradle" simulation you will see that momentum is being conserved in every collision - the same number of balls move with the same velocity before and after a collision.

Throughout this Topic, we will only be concerned with objects moving in one dimension. A more thorough treatment would show us that momentum is conserved in any direction.

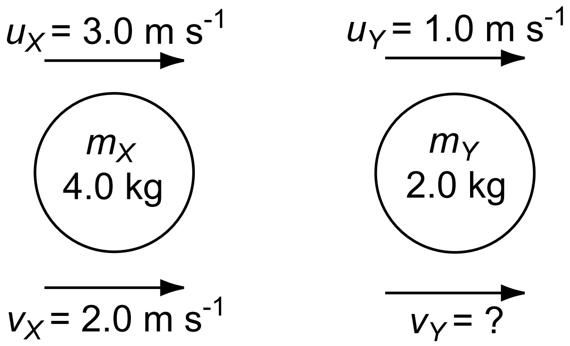
To clarify matters, the labelling convention used throughout this Topic is as follows: for a collision between two objects of masses m_1 and m_2, their velocities before the collision are labelled u_1 and u_2 respectively. After the collision, m_1 has velocity v_1 and m_2 has velocity v_2. The law of conservation of momentum can then be summarised in Equation 4.2.

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2 \tag{4.2}$$

....

Remember that momentum and velocity are vector quantities. All velocities are measured in the same direction, and therefore may be assigned a positive or negative value. A sketch diagram is often useful. Consider two situations involving collisions between two spheres X and Y, with m_X = 4.0 kg and m_Y = 2.0 kg. Suppose X is moving at 3.0 m s^{-1} and Y is moving at 1.0 m s^{-1}. Both spheres are moving in the same direction. After they collide, X continues in the same direction with velocity
2.0 m s^{-1}. What is the new velocity of Y? A sketch diagram of this situation is shown in Figure 4.1.

Figure 4.1: Sketch diagram of the two colliding spheres



....

Using the diagram, we can straight away write down and solve the conservation of momentum equation.

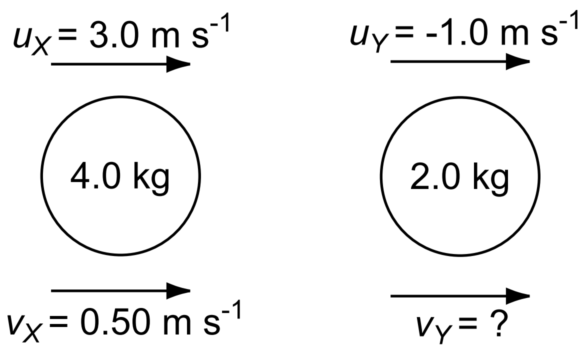
$$
\begin{aligned}
m_X u_X + m_Y u_Y &= m_X v_X + m_Y v_Y \\
\therefore (4.0 \times 3.0) + (2.0 \times 1.0) &= (4.0 \times 2.0) + (2.0 \times v_Y) \\
\therefore 14 &= 8.0 + 2v_Y \\
\therefore v_Y &= 3.0\,\mathrm{m\,s^{-1}}
\end{aligned}
$$

So after the collision, *Y* has velocity 3.0 m s^{-1} in its original direction.

Suppose instead the two spheres were moving *towards* each other, with *X* travelling at 3.0 m s^{-1} and *Y* moving at 1.0 m s^{-1}. If *X* has a velocity after the collision of 0.50 m s^{-1} in the same direction as it was originally moving, what is the new velocity of *Y*?

Again, a sketch diagram like the one shown in Figure 4.2 is useful.

Figure 4.2: Sketch diagram for two spheres moving towards each other



Note that this time sphere *Y* has a positive velocity to the left, or a negative velocity to the right. Using the labelling shown in Figure 4.2 should ensure you don't make a mistake with the signs of the velocities in the conservation of momentum equation:

$$
\begin{aligned}
m_X u_X + m_Y u_Y &= m_X v_X + m_Y v_Y \\
\therefore (4.0 \times 3.0) + (2.0 \times -1.0) &= (4.0 \times 0.50) + (2.0 \times v_Y) \\
\therefore 10 &= 2.0 + 2v_Y \\
\therefore v_Y &= 4.0\,\mathrm{m\,s^{-1}}
\end{aligned}
$$

After this collision, *Y* is travelling to the right with velocity 4.0 m s^{-1}.

5 min

Example - Collision between two spheres

Paper-based calculation.

A sphere *A* of mass 2.0 kg rolls into a stationary sphere *B* of mass 3.0 kg. The velocity of *A* before the collision is 5.0 m s^{-1} and its velocity after the collision is 0.50 m s^{-1}. Sketch a diagram like the one in Figure 4.1 and hence calculate the velocity of *B* after the collision.

A sketch diagram showing the correct velocity vectors is useful in solving problems involving collisions.

4.3.2 Inelastic collisions

As well as studying the momentum of objects colliding with one another, we can also consider their kinetic energy. If you look at the first example in this Topic, the kinetic energy of sphere *X* before the collision is

$$\begin{aligned} KE_X &= \tfrac{1}{2}m_X u_X^2 \\ \therefore KE_X &= \tfrac{1}{2} \times 4.0 \times 3.0^2 \\ \therefore KE_X &= 18\,\text{J} \end{aligned}$$

Using a $KE_Y = {}^1\!/_2 mu^2$ calculation, sphere *Y* has kinetic energy 1.0 J before the collision. (Make sure you can perform this calculation.) So the total kinetic energy before the collision is 18 + 1.0 = 19 J.

After the collision, the kinetic energy of sphere *X* is $KE_X = {}^1\!/_2 mv_X^2$ and kinetic energy of sphere *Y* is $KE_Y = {}^1\!/_2 mv_Y^2$. If you carry out the calculations, you should find the new kinetic energy of *X* is 8.0 J and the new kinetic energy of *Y* is 9.0 J. The total kinetic energy after the collision is 8.0 + 9.0 = 17 J. So the *total* energy has been reduced by 2.0 J, from 19 J to 17 J.

A collision in which kinetic energy is not conserved is called an **inelastic collision**.

One type of inelastic collision is one in which the two objects stick together after colliding. In this case the two objects have the same velocity after the collision, and the conservation of momentum equation becomes

$$m_1u_1 + m_2u_2 = (m_1 + m_2)\,v \tag{4.3}$$

. . . .

In fact, a collision in which the two objects stick together or coalesce is called a *totally inelastic* collision. Any other collision in which kinetic energy is not conserved is called inelastic.

Ballistic pendulum

20 min

This simulation brings together several branches of Physics that you have studied already - momentum, kinetic energy and potential energy.

Full instructions are given on-screen, as well as worked solutions to the calculations.

In an inelastic collision, momentum is conserved but kinetic energy is not.

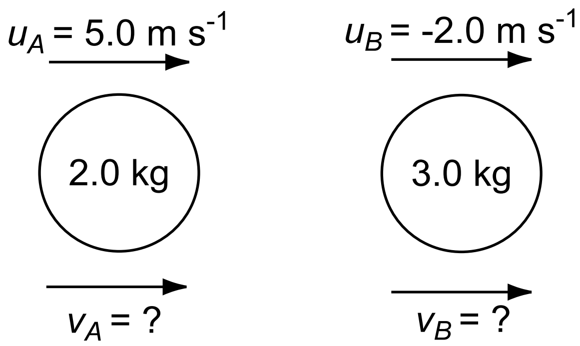
4.3.3 Elastic collisions

An **elastic collision** between two objects is one in which both momentum and kinetic energy are conserved. Since two quantities are conserved, we can write two "before and after" equations to solve problems involving elastic collisions.

Example

Let us consider two spheres *A* (2.0 kg) and *B* (3.0 kg). Suppose they are rolling towards each other, with *A* travelling at 5.0 m s^{-1} and *B* travelling at 2.0 m s^{-1}. Find the velocities of *A* and *B* if they undergo an elastic collision.

Figure 4.3: Sketch diagram for elastic collision



....

The conservation of momentum equation is

$$\begin{aligned} m_A u_A + m_B u_B &= m_A v_A + m_B v_B \\ \therefore (2 \times 5) + (3 \times -2) &= (2 \times v_A) + (3 \times v_B) \\ \therefore 4 &= 2v_A + 3v_B \\ \therefore v_A &= 2 - \tfrac{3}{2} v_B \end{aligned}$$

The conservation of kinetic energy equation is

$$\begin{aligned} \tfrac{1}{2} m_A u_A^2 + \tfrac{1}{2} m_B u_B^2 &= \tfrac{1}{2} m_A v_A^2 + \tfrac{1}{2} m_B v_B^2 \\ \therefore m_A u_A^2 + m_B u_B^2 &= m_A v_A^2 + m_B v_B^2 \\ \therefore \left(2 \times 5^2\right) + \left(3 \times -2^2\right) &= \left(2 \times v_A^2\right) + \left(3 \times v_B^2\right) \\ \therefore 62 &= 2v_A^2 + 3v_B^2 \end{aligned}$$

We can substitute for v_A in this equation.

$$62 = 2\left(2 - \tfrac{3}{2} v_B\right)^2 + 3v_B^2$$

This expression can be expanded and simplified, leading to a quadratic equation in v_B.

$$5v_B^2 - 8v_B - 36 = 0$$

Make sure you can perform the expansion and simplification to arrive at this stage. The solutions to this quadratic equation are v_B = -2.0 m s^{-1} and v_B = 3.6 m s^{-1}. The first solution can be discarded, since this solution corresponds to sphere A initially starting to the right of B. In this case, the spheres move apart and do not collide. The solution v_B = 3.6 m s^{-1} can be substituted back into the conservation of momentum equation to give the final answers: v_A = -3.4 m s^{-1} and v_B = 3.6 m s^{-1}.

. . . .

Quiz 1 Momentum

15 min

Multiple choice quiz.

First try the questions. If you get a question wrong or do not understand a question, there are 'Hints' at the the back of the book . The hints are in the same order as the questions. If you read the hint and still do not understand then ask your tutor.

Q1: Ball *M* (mass 0.500 kg) rolls at 3.00 m s^{-1} into ball *N*, which is stationary and has mass 0.900 kg. If the collision brings *M* to rest, what is the velocity of *N* immediately after the collision?

a) 0.120 m s^{-1}
b) 0.250 m s^{-1}
c) 1.08 m s^{-1}
d) 1.67 m s^{-1}
e) 6.75 m s^{-1}

. . . .

Q2: A 2500 kg train carriage travelling at 10 m s^{-1} runs into the back of a stationary carriage of mass 4000 kg. If the carriages couple together, what is their velocity just after impact?

a) 3.8 m s^{-1}
b) 6.2 m s^{-1}
c) 6.3 m s^{-1}
d) 5.0 m s^{-1}
e) 10 m s^{-1}

. . . .

Q3: In any inelastic collision between two objects,

a) kinetic energy is conserved but momentum is not.
b) both objects must coalesce after impact.
c) momentum and kinetic energy are both conserved.
d) one object must be stationary before the collision.
e) momentum is conserved but kinetic energy is not.

. . . .

Q4: An object is travelling with momentum 100 kg m s^{-1} and kinetic energy 80 J. What is the velocity of the object?

a) 0.89 m s^{-1}

b) 1.3 m s^{-1}
c) 1.6 m s^{-1}
d) 7.9 m s^{-1}
e) 63 m s^{-1}

. . . .

Q5: Two spheres, one of mass 4.0 kg and the other of mass 1.0 kg, collide head-on. Each is moving at 3.0 m s^{-1} before the collision. If the collision is perfectly elastic, what is the speed of the 4.0 kg sphere immediately after the collision?

a) 0.60 m s^{-1}
b) 1.8 m s^{-1}
c) 3.0 m s^{-1}
d) 3.2 m s^{-1}
e) 6.6 m s^{-1}

. . . .

4.4 Impulse

Learning Objective

To carry out calculations involving momentum, force and impulse.

Can we link the work we have done on momentum in this Topic to Newton's laws of motion? Clearly if two objects collide, at the moment of impact each object exerts a force on the other, and Newton's third law tells us that these forces are equal in magnitude and opposite in direction.

Let us look at the force acting on one of the objects in the collision. Newton's second law tells us that the force *F* acting on it causes an acceleration, given by the equation $F = ma$. In Topic 2, we defined acceleration by the equation $a = {}^{\Delta v}/_{t}$.

$$\begin{aligned} F &= ma \\ \therefore F &= m\frac{\Delta v}{t} \\ \therefore F &= \frac{\Delta p}{t} \end{aligned} \tag{4.4}$$

. . . .

So force is the rate of change of momentum. Rearranging Equation 4.4,

$$\Delta p = F \times t$$

The quantity *F* x *t* is called the **impulse**. You should be able to state that impulse is the change of momentum. Impulse has units of N s or kg m s^{-1}.

Example

A car driver trying to reverse into a parking space crashes into a wall. The car, mass

1700 kg, is travelling at 2.50 m s^{-1} as it hits the wall. What is the impulse acting on the car?

Impulse is defined as the change in momentum:

$$
\begin{aligned}
impulse &= mv - mu \\
\therefore impulse &= (1700 \times 0) - (1700 \times 2.5) \\
\therefore impulse &= -4250\,\text{kg m s}^{-1}
\end{aligned}
$$

In this case the impulse is negative as the final momentum of the car is less than its momentum before the collision.

. . . .

This example highlights one of the practical applications of momentum and impulse, which is in the design of car bumpers and "crumple zones." With safety in mind, these parts of a car are designed to give way in a collision. If a car is driven into a wall, the car comes to a halt pretty quickly, with the occupants of the car continuing to move at the same speed. The crumple zone means that the collision is not one between two inflexible objects - because the car is designed to crumple, the collision is spread out in time, resulting in a smaller force acting on the people in the car.

"Test your strength" contest

15 min

This on-screen simulation shows a fairground "Test Your Strength" machine - a machine in which impulse and momentum play important parts.

Full instructions are given on-screen, as well as worked solutions to the calculations.

Impulse is the change in momentum of an object.

Quiz 2 Impulse

15 min

Multiple choice quiz.

First try the questions. If you get a question wrong or do not understand a question, there are 'Hints' at the the back of the book . The hints are in the same order as the questions. If you read the hint and still do not understand then ask your tutor.

Q6: The velocity of a 640 kg racehorse increases from 12 m s^{-1} to 17 m s^{-1}. What is the impulse on the racehorse?

a) 204 kg m s^{-1}
b) 3200 kg m s^{-1}
c) 7680 kg m s^{-1}
d) 10880 kg m s^{-1}
e) 18560 kg m s^{-1}

. . . .

Q7: Newton's second law can be expressed as

a) force = impulse.
b) impulse = rate of change of momentum.
c) force = rate of change of momentum.
d) impulse = mass x acceleration.
e) force = impulse x time.

. . . .

Q8: A horizontal force of 30 N acts on a 0.40 kg mass at rest on a smooth table top. What is the momentum of the mass after 2.0 s?

a) 6.0 kg m s^{-1}
b) 12 kg m s^{-1}
c) 24 kg m s^{-1}
d) 30 kg m s^{-1}
e) 60 kg m s^{-1}

. . . .

Q9: Impulse has the same dimensions as

a) momentum.
b) force.
c) velocity.
d) acceleration.
e) energy.

. . . .

Q10: The brakes of a car are applied for 4.0 s, reducing the velocity of the car from 20 m s^{-1} to 8.0 m s^{-1}. If the mass of the car is 1600 kg, what is the magnitude of the force exerted by the brakes?

a) 33 N
b) 530 N
c) 3200 N
d) 4800 N
e) 8000 N

. . . .

4.5 Explosions

When a gun is fired, the bullet shoots out of the barrel, and the marksman feels a "kick" or recoil from the gun. This is another example of conservation of momentum. Before the gun is fired, the gun and the loaded bullet are both stationary. Immediately after it is fired, the bullet has momentum in the direction it is travelling. To keep the total momentum at zero, the gun must recoil in the opposite direction.

Rockets and jet engines are two more practical examples of the conservation of momentum in explosions. Let us look first at the rocket. The fuel and oxidant are both stored on the rocket, and a portion of these are used up every unit of time. They are expelled at high speed from the rear of the rocket as exhaust gases, causing an increase in the forward momentum of the rocket.

A jet engine takes in cool air at its front end. This air is compressed, and used in the engine to support the burning of the engine fuel. Again, the air and the exhaust gases are expelled from the rear of the engine, resulting in an increase in the forward momentum of the aeroplane.

Quiz 3 Explosions

15 min

Multiple choice quiz.

First try the questions. If you get a question wrong or do not understand a question, there are 'Hints' at the the back of the book . The hints are in the same order as the questions. If you read the hint and still do not understand then ask your tutor.

Q11: Two blocks A (1.5 kg) and B (2.5 kg) are at rest on a smooth horizontal surface. A light spring is compressed between A and B. When the blocks are released, A moves to the left at 4.0 m s^{-1}. What is the speed of B to the right?

a) 0.42 m s^{-1}
b) 1.1 m s^{-1}
c) 2.4 m s^{-1}
d) 4.0 m s^{-1}
e) 6.7 m s^{-1}

. . . .

Q12: A gun fires a bullet of mass 0.020 kg at 480 m s^{-1}. If the mass of the gun is 0.80 kg, what is the recoil velocity of the gun?

a) 0.083 m s^{-1}
b) 0.24 m s^{-1}
c) 4.8 m s^{-1}
d) 7.7 m s^{-1}
e) 12 m s^{-1}

. . . .

Q13: A nucleus of mass *m* splits into two particles, one of mass 0.25*m*, the other of mass 0.75*m*. If the less massive particle has speed *v*, what is the speed of the other particle, travelling in the opposite direction?

a) $^{v}/_{3}$
b) $^{2v}/_{3}$
c) $^{3v}/_{4}$
d) $^{4v}/_{3}$
e) $^{3v}/_{2}$

. . . .

Q14: At the bowling alley, a woman of mass 45 kg is using a 5.0 kg ball. She runs up at 2.0 m s^{-1} and bowls the ball at 10 m s^{-1}. What is her velocity immediately after releasing the ball?

a) 0.67 m s^{-1}
b) 0.89 m s^{-1}
c) 0.91 m s^{-1}
d) 1.1 m s^{-1}
e) 3.3 m s^{-1}

. . . .

Q15: When a cannon fires a 10 kg cannonball at 30 m s^{-1}, its recoil velocity is 6.0 m s^{-1}. What is the recoil velocity when a 15 kg cannonball is fired at 24 m s^{-1}?

a) 1.2 m s^{-1}
b) 6.0 m s^{-1}
c) 7.2 m s^{-1}
d) 31 m s^{-1}
e) 36 m s^{-1}

. . . .

4.6 Summary

By the end of this Topic you should be able to:

- state that momentum is the product of mass and velocity;
- state that momentum is conserved when two objects moving in one dimension interact, in the absence of external forces;
- state that momentum and kinetic energy are conserved in an elastic collision, but only momentum is conserved in an inelastic collision;
- carry out calculations involving collisions or explosions in one dimension;
- apply the law of conservation of momentum to interactions of two objects in one dimension to show that the changes of momentum of each object are equal in magnitude and opposite in direction;
- apply the law of conservation of momentum to interactions of two objects in one dimension to show that the forces acting on each object are equal in magnitude and opposite in direction;
- state that *impulse* = *force* x *time* = *change of momentum*, and carry out calculations using these relationships.

Online assessments

Three online assessments are available. Each test should take you no longer than 20 minutes to complete. The questions in Test 1 are on conservation of momentum and collisions. Test 2 has questions involving impulse and explosions, and Test 3 has questions taken from all parts of this Topic.

Topic 5

Density and Pressure

Contents

5.1 Introduction

This Topic introduces the concepts of density and pressure. We will compare the densities of typical solids, liquids and gases. We will also look at how the pressure on an object changes when it is placed in a fluid (a liquid or a gas).

A good understanding of the concepts presented in this Topic will be very useful in many other branches of physics, especially the laws controlling the behaviour of gases.

5.2 Density

Learning Objective

To define density, and to carry out calculations involving density.

5.2.1 Mass, volume and density

If a friend says to you that "Lead is heavier than wood", what does she mean? Certainly if you have two identical blocks, one made of lead and the other of wood, then the lead block has a greater mass. On the other hand, a large plank of wood has a greater mass than a pea-sized piece of lead.

The statement your friend should have made to you is that "For equal volumes of the two materials, a piece of lead has a greater mass than a piece of wood". The idea that equal volumes of different materials can have different mass means that we need to introduce a physical quantity called **density**. The density ρ of a substance is its mass per unit volume, measured in kg m^{-3}. For example, water has a density of 1000 kg m^{-3}, meaning that 1 m^3 of water has mass 1000 kg. Density is a scalar quantity. The relationship between density ρ, mass *m* and volume *V* is given in Equation 5.1.

$$\rho = \frac{m}{V} \tag{5.1}$$

. . . .

Example

Copper has density 8.96 x 10^3 kg m^{-3}. What is the mass of a solid copper sphere of volume 5.50 x 10^{-4} m^3?

We can rearrange Equation 5.1 before inserting the data:

$$\rho = \frac{m}{V}$$
$$\therefore m = \rho \times V$$
$$\therefore m = \left(8.96 \times 10^{3}\right) \times \left(5.50 \times 10^{-4}\right)$$
$$\therefore m = 4.93\,\text{kg}$$

. . . .

5.2.2 Densities of solids, liquids and gases

Table 5.1 gives the densities of some common substances.

Table 5.1: Densities of different substances

Substance	**Density** (kg m^{-3})
Aluminium	2700
Copper	8960
Ice	920
Silver	10500
Sea Water	1020
Water	1000
Air	1.29
Carbon Dioxide	1.98
Oxygen	1.43

. . . .

For the most part, solids are more dense than liquids, which in turn are more dense than gases. The reason is that the atoms are packed together most closely in a solid. At the other extreme, atoms and molecules in a gas have relatively large distances between them, so the number of individual atoms in a volume of gas is much less than in the same volume of a solid or a liquid.

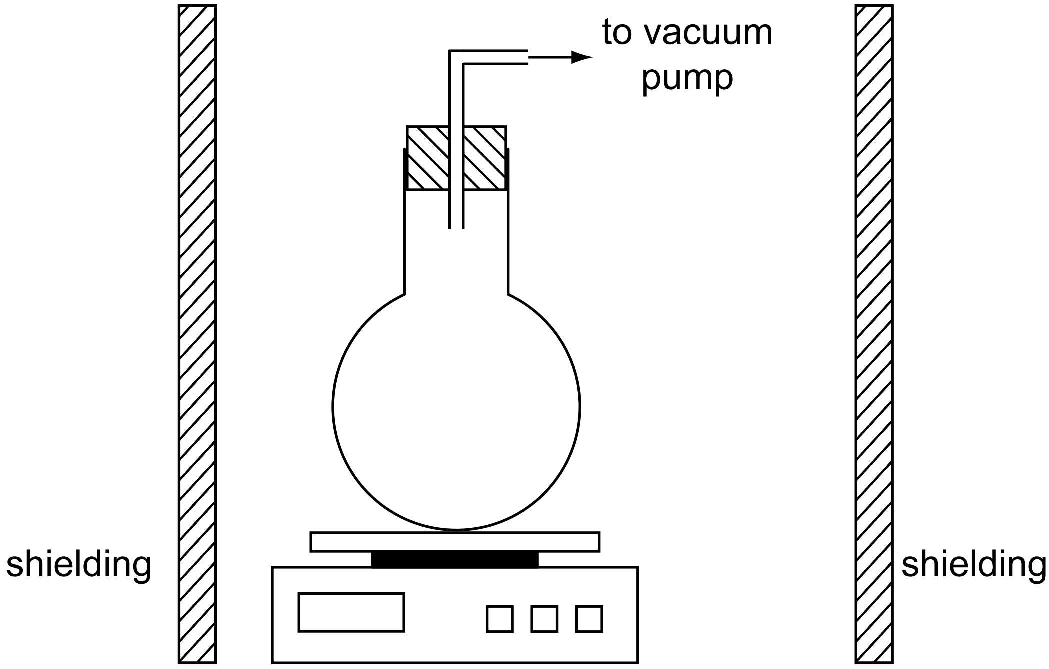
There are some exceptions to the general rule. Wood has a density of around 500 kg m^{-3}, which is why a piece of wood can float on water. Interestingly, Table 5.1 shows that the density of ice is slightly less than the density of water, which is why an iceberg floats on water. Generally speaking, a liquid is around 1000 times more dense than a gas.

5.2.3 Measuring the density of air

We can perform a straightforward experiment in the lab to measure the density of air. The principles of the experiment are fairly simple. If we measure the mass of a flask full of air, and then measure the mass of the flask with all the air removed, the difference between the two masses must be the mass of air which had filled the container. If we then measure the volume of the flask, we can use the values of mass and volume to calculate the density using Equation 5.1.

The experimental arrangement is shown schematically in Figure 5.1

Figure 5.1: Schematic diagram of the experimental measurement of the density of air



. . . .

The round-bottomed flask is connected to a vacuum pump, and is placed on a sensitive balance. The mass of the flask is recorded while it is still full of air. The vacuum pump is then switched on and the flask is evacuated. Once all the air in the flask has been removed, the new mass can be recorded. Note that throughout the experiment, the flask *must* be kept within the safety shields. There is always a danger that an evacuated flask could implode, so the experiment should never be undertaken without the safety shields in place.

Once the two masses have been recorded, air can be *slowly* returned to the flask until it is full again. To measure the volume of the flask, it can be filled with water. If this water is carefully poured into a measuring cylinder, the volume of water tells us the volume of the inside of the flask, and hence the volume of air whose mass has been measured. The density of air is then

$$\rho_{air} = \frac{m_{full\,flask} - m_{evacuated\,flask}}{V} \tag{5.2}$$

. . . .

5.3 Pressure

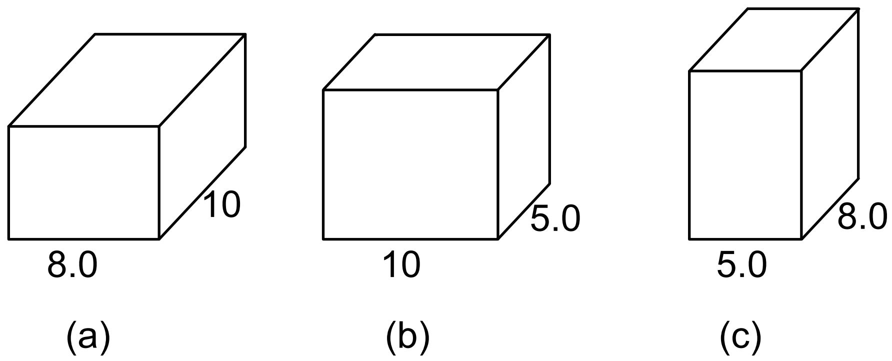
What do we mean by "pressure"? Is it any different to "force"?

The answer to the second question is "yes" as pressure also takes into account the area over which a force is acting. We define **pressure** as the force acting per unit area. Pressure is a scalar quantity, measured in N m^{-2}. Remember that force is a vector quantity, so this is another difference between force and pressure. An alternative unit for pressure is the pascal (Pa) which is equivalent to measuring in N m^{-2}, so 1 Pa = 1 N m^{-2}. The following worked example should help you understand the difference between pressure and force.

Example

The metal block shown in Figure 5.2 has mass 2.0 kg and sides of lengths 5.0 cm, 8.0 cm and 10 cm. Calculate the pressure the block exerts on a table top due to its weight, when it is placed on each of its faces.

Figure 5.2: Metal block resting on each different face



. . . .

The weight of the block (= *mg*) does not change, but it acts over a different area in each case.

(a)

$$p = \frac{F}{A}$$
$$\therefore p = \frac{2.0 \times 9.8}{0.08 \times 0.10}$$
$$\therefore p = 2.4 \times 10^3 \text{ Pa}$$

(b)

$$p = \frac{F}{A}$$
$$\therefore p = \frac{2.0 \times 9.8}{0.05 \times 0.10}$$
$$\therefore p = 3.9 \times 10^3 \text{ Pa}$$

(c)

$$p = \frac{F}{A}$$
$$\therefore p = \frac{2.0 \times 9.8}{0.05 \times 0.08}$$
$$\therefore p = 4.9 \times 10^3 \text{ Pa}$$

So in (c), where the area over which the force is exerted is the smallest, the pressure is the greatest.

. . . .

15 min

Quiz 1 Density and pressure

Multiple choice quiz.

First try the questions. If you get a question wrong or do not understand a question, there are 'Hints' at the the back of the book . The hints are in the same order as the questions. If you read the hint and still do not understand then ask your tutor.

Useful data:

acceleration due to gravity *g*	9.8 m s^{-2}
density of water	1000 kg m^{-3}
density of sea water	1020 kg m^{-3}

Q1: Comparing a typical liquid of density ρ_l to a typical gas of density ρ_g, what is the order of magnitude of the ratio ${}^{\rho_l}/_{\rho_g}$?

a) 1000
b) 100
c) 10
d) 0.01
e) 0.001

. . . .

Q2: What volume of sea water has the same mass as 1.00 x10^{-2} m^3 of fresh water?

a) 9.60 x 10^{-5} m^3
b) 1.04 x 10^{-4} m^3
c) 9.80 x 10^{-3} m^3
d) 1.00 x 10^{-2} m^3
e) 1.02 x 10^{-2} m^3

. . . .

Q3: Which of the following is equivalent to 1 Pa?

a) 1 N m^{-1}
b) 1 N kg^{-1}
c) 1 kg m^{-3}
d) 1 N m^{-2}
e) 1 kg N^{-2}

. . . .

Q4: In a physics research lab, the excess pressure inside a chamber is 6.0 x 10^6 Pa. A viewing window on the chamber wall has area 3.2 x 10^{-5} m^2. What is the force exerted on the window?

a) 5.3 x 10^{-12} N
b) 190 N
c) 5.7 x 10^6 N
d) 6.3 x 10^6 N

e) 1.9×10^{11} N

. . . .

Q5: Three industrial compressors have the following specifications: Compressor X can exert a force F over an area A, compressor Y can exert a force $3F$ over an area $2A$ and compressor Z can exert a force $5F$ over an area $4A$. Starting with the greatest, place the compressors in order of the pressure they exert.

a) X, Y, Z
b) Y, X, Z
c) X, Z, Y
d) Z, Y, X
e) Y, Z, X

. . . .

5.4 Fluids and buoyancy

Learning Objective

To calculate the pressure in a fluid.
To explain buoyancy in terms of pressure difference.

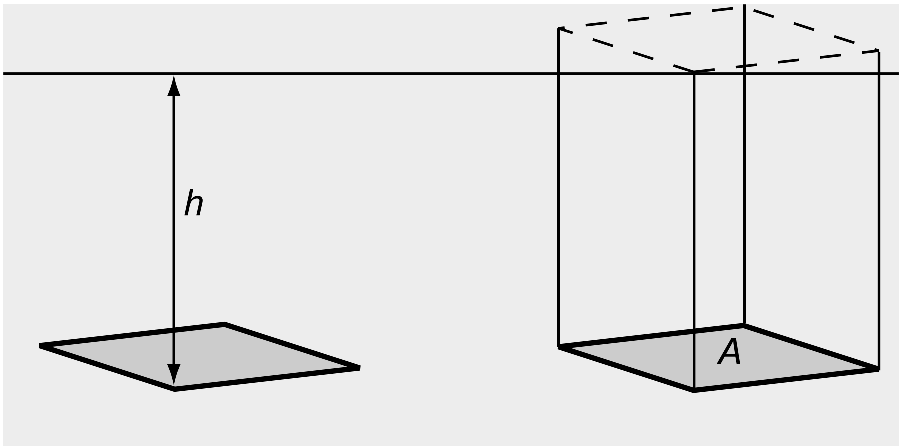
We conclude this Topic by looking at pressure and density in a fluid - a liquid or a gas. An object immersed in a fluid has a pressure exerted on it due to the weight of the fluid. We will quantify this pressure, and also see how pressure leads to a buoyancy force.

5.4.1 Pressure in a fluid

Submarines are built to withstand the huge pressure that is exerted on them when they dive to the bottom of the ocean. This pressure is caused by the enormous volume of water directly above the submarine. As the submarine moves to a greater depth, so the volume (and hence the weight) of water above it increases, and therefore the pressure increases too.

To try to quantify this pressure, let us imagine a thin plate of area A, which is placed horizontally in a fluid at a distance h below the surface. This situation is shown in Figure 5.3.

Figure 5.3: Thin plate immersed in a fluid



....

The volume of fluid directly above the plate is *h* x *A*. If the fluid has density ρ then the mass of fluid directly above the plate is $hA\rho$. The force *F* acting directly downwards on the plate is therefore

$$F = hA\rho g$$

We can now find the pressure *p* acting downwards on the plate.

$$\begin{aligned} p &= \frac{F}{A} \\ \therefore p &= \frac{hA\rho g}{A} \\ \therefore p &= h\rho g \end{aligned} \tag{5.3}$$

....

Equation 5.3 gives us the pressure at a depth *h* in a fluid of density ρ. In some textbooks you may find this referred to as the hydrostatic pressure. Since pressure is the force acting per unit area, Equation 5.3 gives us the pressure exerted on any object placed at that depth in the fluid. This pressure always acts at right angles to the surface of an object placed in a fluid. So for the plate in Figure 5.3, the pressure on the upper surface of the plate is acting downwards, and the pressure on the lower surface of the plate is acting upwards.

Example

The deep end of a swimming pool has depth 3.0 m. What is the pressure due to the water on someone diving to the bottom of the pool?

Assuming the water in the pool has density 1000 kg m^{-3}, we can use Equation 5.3.

$$\begin{aligned} p &= h\rho g \\ \therefore p &= 3.0 \times 1000 \times 9.8 \\ \therefore p &= 2.9 \times 10^4 \text{ Pa} \end{aligned}$$

....

The pressure at the bottom of the swimming pool may seem like a very large number, but you should bear in mind that the pressure exerted on you by the air in the atmosphere ("atmospheric pressure") is a little over 10^5 Pa. So the extra pressure is only a fraction of atmospheric pressure.

To be accurate, the pressure calculated using Equation 5.3 is only accurate if the fluid is stationary. The

pressure in a fluid which is flowing is calculated by different means, which are beyond the scope of this course.

Pressure in a fluid

Online simulation showing how the pressure in a fluid depends on the depth in the fluid and the density of the fluid.

15 min

Full instructions are given on-screen.

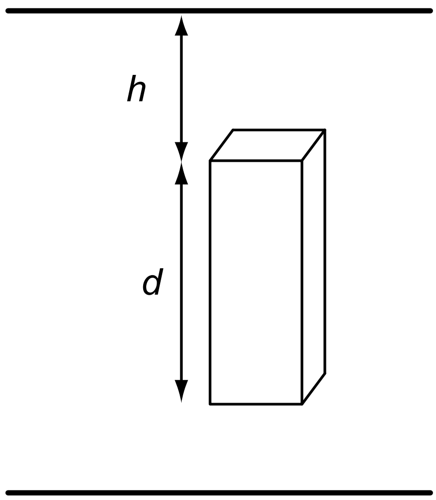
The pressure in a fluid is proportional to both the depth at which the pressure is being measured and the density of the fluid.

5.4.2 Buoyancy

What is the force that acts on an object to make it float when it is placed in a fluid? Where does this force come from? We can answer these questions by considering the effect known as buoyancy.

Consider Figure 5.4, in which a solid object of length *d* has been immersed in a fluid.

Figure 5.4: Object placed in a fluid



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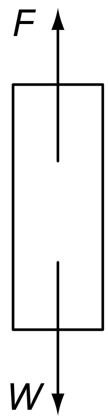
The top end of the object is at a depth *h* in the fluid, hence the force due to the pressure acts down (at right angles to the surface) on the top of the object. This pressure is $p_T = h\rho g$. The other end of the object is at a depth (*h*+*d*), and the force due to the pressure acts upwards on this surface of the object. This upward pressure is

$p_B = (h + d)\,\rho g$. Therefore the net upward pressure is

$$p = p_B - p_T$$
$$\therefore p = (h + d)\,\rho g - h\rho g$$
$$\therefore p = d\rho g$$

This net upwards pressure means that there is a force, called the **buoyancy force**, acting upwards on an object immersed in a fluid. This force is sometimes called the upthrust, which should help you remember in which direction it acts. The fact that an upthrust is acting does not necessarily mean an object will float, however. Figure 5.5 shows a free body diagram of the object in the fluid.

Figure 5.5: Free body diagram of an object in a fluid



....

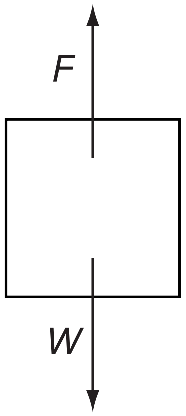
The buoyancy force *F* acts upwards, whilst the weight *W* acts downwards. If $W > F$, then the object will accelerate downwards. If $F > W$, the object accelerates upwards.

Example

A wooden block of mass 0.50 kg is held under water. When the block is released, the buoyancy force acting on it is 6.0 N. Calculate the magnitude and direction of the acceleration of the block at the instant it is released.

As usual, a free body diagram such as Figure 5.6 will come in useful:

Figure 5.6: Free body diagram for the immersed block



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The weight *W* of the block = *mg* = 4.9 N, so the free body diagram shows us that the net force acting on the block is *upwards*. Applying Newton's second law:

$$F - W = ma$$
$$\therefore a = \frac{F - W}{m}$$
$$\therefore a = \frac{6.0 - 4.9}{0.50}$$
$$\therefore a = 2.2\,\mathrm{m\,s^{-2}}$$

....

Hot air balloon

15 min

Online simulation

Full instructions are given on-screen, along with answers to the calculations.

A free body diagram should be used whenever you are performing calculations involving buoyancy.

Quiz 2

15 min

Multiple choice quiz.

First try the questions. If you get a question wrong or do not understand a question, there are 'Hints' at the the back of the book . The hints are in the same order as the questions. If you read the hint and still do not understand then ask your tutor.

Useful data:

acceleration due to gravity *g*	9.8 m s^{-2}
density of water	1000 kg m^{-3}
density of sea water	1020 kg m^{-3}

Q6: What is the pressure due to the water at a depth of 30 m in a freshwater lake?

a) 3.3×10^2 Pa
b) 3.1×10^3 Pa
c) 2.0×10^4 Pa
d) 4.0×10^4 Pa
e) 2.9×10^5 Pa

. . . .

Q7: At what depth in a freshwater lake is the hydrostatic pressure equal to 2.50×10^5 Pa?

a) 2.55 m
b) 25.5 m
c) 255 m
d) 2550 m
e) 2.55×10^4 m

. . . .

Q8: A submarine is at a depth of 40 m in the ocean. What is the *increase* in pressure if it moves to a depth of 55 m?

a) 1.5×10^4 Pa
b) 5.6×10^4 Pa
c) 1.5×10^5 Pa
d) 4.0×10^5 Pa
e) 5.5×10^5 Pa

. . . .

Q9: A polystyrene block of mass 0.10 kg is held beneath the surface of a lake by a force applied vertically downwards. The buoyancy force acting on the block is 1.5 N. What vertical force is required to keep the block stationary?

a) 0.52 N
b) 0.98 N
c) 1.5 N
d) 1.6 N
e) 2.5 N

. . . .

5.5 Summary

By the end of this Topic you should be able to:

- state that density is mass per unit volume;
- carry out calculations involving density, mass and volume;

- describe the principals of a method of measuring the density of air;
- state and explain the relative magnitudes of the densities of typical solids, liquids and gases;
- state that pressure is the force acting per unit area on a surface, at right angles to that surface, and perform calculations involving pressure, force and area;
- state that a pressure of one pascal (1 Pa) is equivalent to one newton per square metre (1 N m^{-2});
- state that the pressure *p* at a point in a fluid at rest is given by the formula

$$p = h\rho g$$

- carry out calculations involving pressure, density and depth;
- explain the buoyancy force in terms of the pressure difference between the top and bottom of an object immersed in a fluid.

Online assessments

Two online test are available. Each test should take you no more than 20 minutes to complete. Both tests have questions taken from all parts of the Topic.

Topic 6

Gas Laws

Contents

6.1 Introduction

In this final Mechanics Topic, we will be studying the behaviour of gases. The theory that is used to explain how gases behave is called the kinetic theory. The Topic begins with a discussion of kinetic theory, before we go on to investigate the relationships between the pressure, volume and temperature of a gas.

6.2 Kinetic theory

Learning Objective

To describe the kinetic model of a gas.

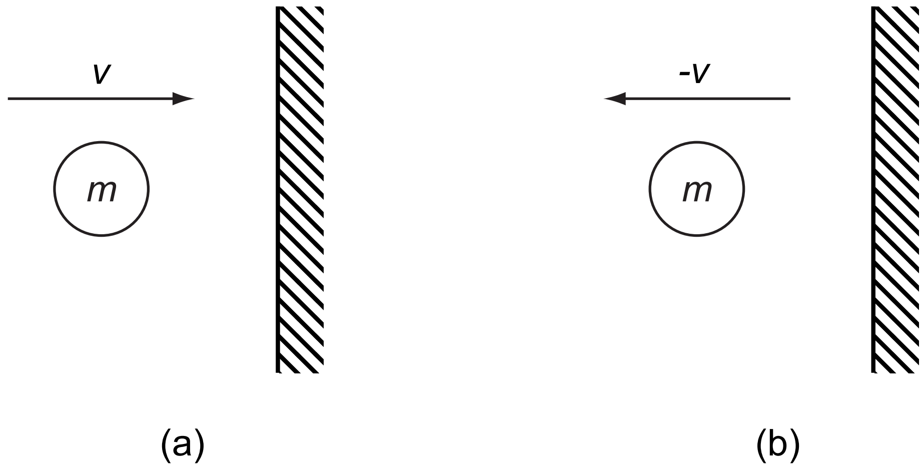
6.2.1 Pressure exerted by a gas

The "Density and Pressure" Topic introduced the idea that a fluid exerts a pressure on any object which is immersed in it. In this Topic we will explain why this pressure is exerted. Since this Topic is concerned with the behaviour of gases, then we will not be discussing liquids in this explanation.

To understand the pressure of a gas, we use a concept called **kinetic theory**. In this theory, we picture a gas as being a collection of molecules which are free to move in any direction. We assume this gas is in a container with solid, rigid walls. The molecules are all moving at different speeds in random directions. We assume that a molecule will move in a straight line until it collides with either another gas molecule or the walls of the container.

Let us picture a gas molecule colliding with a wall of the container. To keep things as simple as possible, we will consider a molecule moving horizontally, colliding head-on with the wall, as shown in Figure 6.1.

Figure 6.1: A gas molecule (a) before, and (b) after colliding with the container wall



....

In the kinetic theory, we assume the collision is perfectly elastic, so a molecule travelling

with velocity v before the collision rebounds with velocity $-v$ after the collision. If the mass of the molecule is m, then the impulse (= change in momentum) is
$mv - (-mv) = 2mv$. Each time a molecule collides with a wall of the container, it exerts a force on the wall. Thus with molecules continuously bombarding the container walls, the gas exerts a pressure on the container.

6.2.2 Temperature and energy

Another part of the kinetic model that we must consider concerns the energy of gas molecules. If a gas gets hotter, what happens to the molecules of the gas?

The energy of a gas increases if its temperature increases - there is more energy in a container of hot gas than an identical container of colder gas. Looking at the gas molecules, more energy per molecule means that the average kinetic energy of a gas is greater in a hot gas than a cold gas, meaning that the average speed of molecules is greater the hotter the gas is.

6.3 The behaviour of gases

Learning Objective

To state and apply the laws governing the pressure, volume and temperature of a gas.

6.3.1 The kelvin temperature scale

You should be familiar with the celsius or centrigrade temperature scale, where water freezes at 0 °C and boils at 100 °C. The SI unit of temperature is the kelvin K.

On the kelvin scale, water freezes at 273 K and boils at 373 K. (Note that a temperature in kelvin is written as, say, 300 K and not 300 °K.) Like the celsius scale, the difference between the boiling and freezing points of water is 100 units on the kelvin scale, which means that a temperature *difference* of 1 K is equal to a temperature difference of 1 °C. So the size of a temperature unit is the same on both scales. The scales differ in the point at which zero is defined. We will see later that 0 K is equivalent to -273.15 °C.

6.3.2 Pressure and volume

Consider a fixed mass of gas in a container, such as the airtight cylinder shown in Figure 6.2. What happens to the pressure of the gas if, whilst keeping the temperature constant, we change the volume by pushing in the piston?

Figure 6.2: A fixed mass of gas at constant temperature being reduced in volume

. . . .

As the volume decreases, so the pressure increases. We can explain this in terms of the kinetic model. We have seen how the pressure exerted by the gas on its container arises from the collisions of molecules with the container walls. If we have the same number of molecules in a smaller container, they will hit the walls more frequently. If you are not sure why, then consider the situation where there is just one molecule of gas in the container, bouncing back and forth in a horizontal direction with a constant speed.

Figure 6.3: A molecule moving in one dimension in containers of different volumes

. . . .

If we reduce the volume of the container by moving the two sides closer together, then the molecule will clearly spend less time between collisions, since the distance between the walls is less. If there are more collisions per second, then the pressure will be higher.

To formalise this law, we can state that for a fixed mass of gas at constant temperature,

$$\begin{aligned} pV &= \text{constant} \\ \text{or } p &\propto \frac{1}{V} \end{aligned} \tag{6.1}$$

. . . .

So if the gas in the cylinder in Figure 6.2 has initial pressure and volume p_1 and V_1, the pressure and volume after the piston has been moved are p_2 and V_2, where

$$p_1V_1 = p_2V_2 \tag{6.2}$$

. . . .

This gas law is sometimes known as Boyle's law, after the Irish physicist Robert Boyle (1627 - 1691).

Example

A fixed mass of gas in a chamber of volume 2.0 x 10^{-4} m^3 has pressure 1.0 x 10^5 Pa. Calculate the new pressure of the gas if it is compressed at constant temperature to a volume of 1.2 x 10^{-4} m^3.

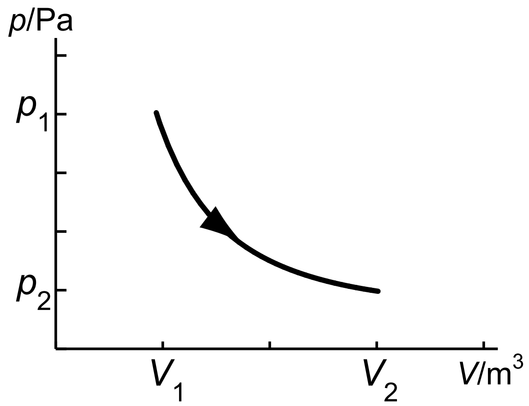
Using Equation 6.2

$$\begin{aligned}
p_1V_1 &= p_2V_2\\
\therefore p_2 &= \frac{p_1V_1}{V_2}\\
\therefore p_2 &= \frac{1.0\times10^5\times2.0\times10^{-4}}{1.2\times10^{-4}}\\
\therefore p_2 &= 1.7\times10^5\text{ Pa}
\end{aligned}$$

....

We often show changes to the condition of a gas on a graph of pressure against volume, commonly known as a "*p*-*V* graph". A *p*-*V* graph for the expansion of a fixed mass of gas at constant temperature is shown in Figure 6.4. The gas shown here is expanding from an initial state (p_1, V_1) to a final state (p_2, V_2).

Figure 6.4: Graph for the expansion of a gas at constant temperature



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Quiz 1 Kelvin scale and the *p-V* law

15 min

Multiple choice quiz.

First try the questions. If you get a question wrong or do not understand a question, there are 'Hints' at the the back of the book . The hints are in the same order as the questions. If you read the hint and still do not understand then ask your tutor.

Q1: What is the equivalent of 45 °C measured on the kelvin scale?

a) 228 K
b) 318 K
c) 328 K
d) 418 K
e) 428 K

....

Q2: A liquid in a beaker is heated, so that it's temperature *increases* by 25 °C. What is the *increase* in temperature of the liquid in kelvin?

a) 25 K

b) 248 K
c) 298 K
d) 348 K
e) Need to know the actual temperatures before and after the liquid has been heated.

. . . .

Q3: Which *one* of the following statements about the kinetic model is true?

a) All the molecules are moving in the same direction.
b) All the molecules are moving with the same speed.
c) The size of each molecule increases if the temperature increases.
d) Molecules undergo elastic collisions with the container walls.
e) The pressure depends on the thickness of the container walls.

. . . .

Q4: The pressure in an airtight cylinder increases from 8.0×10^4 Pa to 1.0×10^5 Pa. If the volume of the cylinder was originally 2.0×10^{-4} m^3 and the gas remains at constant temperature, what is the new volume of the cylinder?

a) 1.6×10^{-4} m^3
b) 2.5×10^{-4} m^3
c) 4.0×10^{-4} m^3
d) 4.0×10^{-3} m^3
e) 6.3×10^{-3} m^3

. . . .

Q5: A fixed amount of gas is compressed at constant temperature to a quarter of its original volume. If the gas was initially at pressure *p*, what is the new pressure of the gas?

a) $p/4$
b) $p/2$
c) p
d) $2p$
e) $4p$

. . . .

6.3.3 Temperature and pressure

We will now look at the relationship between the temperature and pressure of a fixed mass of gas, with the volume of the container kept constant.

According to the kinetic model, the average speed of the gas molecules increases with increasing temperature. The hotter the gas, the faster the gas molecules are moving. We saw earlier in Figure 6.1 that when a molecule collides with a wall of a container, the impulse exerted on the wall depends on the speed of the molecule. So the faster the gas molecules are moving, the greater the pressure exerted by the gas on the walls of the container.

For a fixed mass of gas at fixed volume,

$$p \propto T \tag{6.3}$$

....

Remember, the temperature must be given in K, not °C. Suppose we have a gas trapped in a rigid container, at pressure p_1 and temperature T_1. If we heat the gas to temperature T_2 without changing the volume of the container, the new pressure inside the container is p_2, where

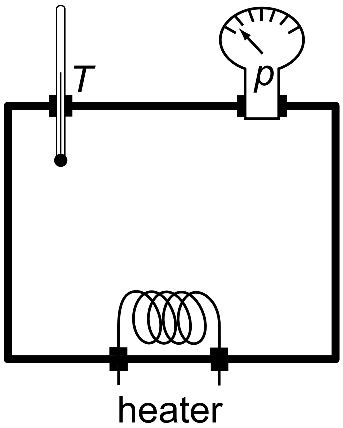
$$\frac{p_1}{T_1} = \frac{p_2}{T_2} \tag{6.4}$$

....

Example

Figure 6.5 shows a rigid container in which a gas at 15 °C is at a pressure of 1.0 x 10^5 Pa. What is the new pressure if the gas is heated to 100 °C?

Figure 6.5: A gas in a rigid container



....

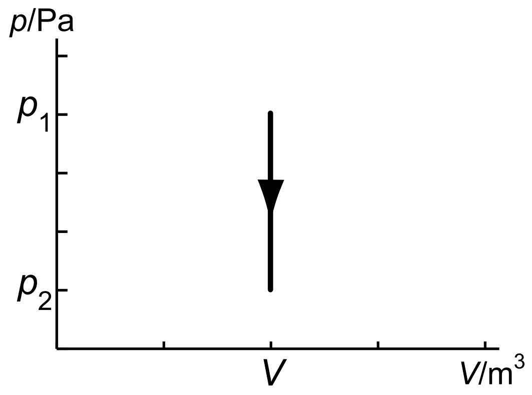
Before we can solve this problem, we must convert the temperatures to kelvin. The initial temperature is (15+273) = 288 K, and the final temperature is (100+273) = 373 K. Now we can use Equation 6.4.

$$\begin{aligned}
\frac{p_1}{T_1} &= \frac{p_2}{T_2} \\
\therefore p_2 &= \frac{p_1}{T_1} \times T_2 \\
\therefore p_2 &= \frac{1.0 \times 10^5 \times 373}{288} \\
\therefore p_2 &= 1.3 \times 10^5 \text{ Pa}
\end{aligned}$$

....

On a *p*-*V* graph, a change in which the volume remains constant is represented by a straight line, as shown in Figure 6.6. In the diagram, the temperature of the gas is decreasing, so that the pressure decreases from p_1 to p_2. The volume remains fixed at a value V.

Figure 6.6: Gas cooling down while its volume remains constant

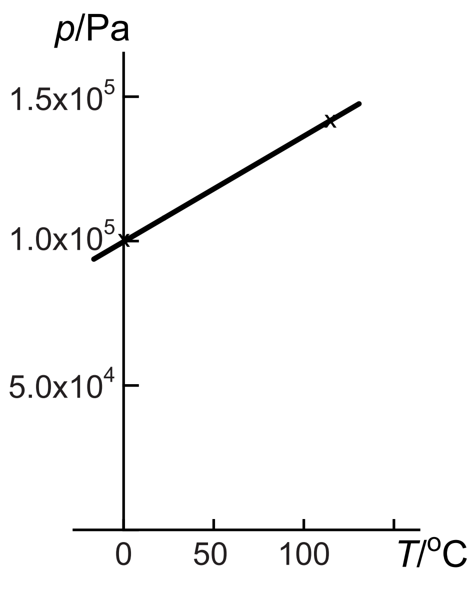


....

6.3.4 Absolute zero

A fixed mass of gas in an airtight rigid container can be used as an accurate thermometer. Such a thermometer is called a constant volume gas thermometer. To calibrate this thermometer, we measure the pressure when the container is placed in melting ice at 0 °C (273 K). The pressure is then recorded when the container is placed in a steam jacket at 100 °C (373 K). A calibration graph can then be plotted.

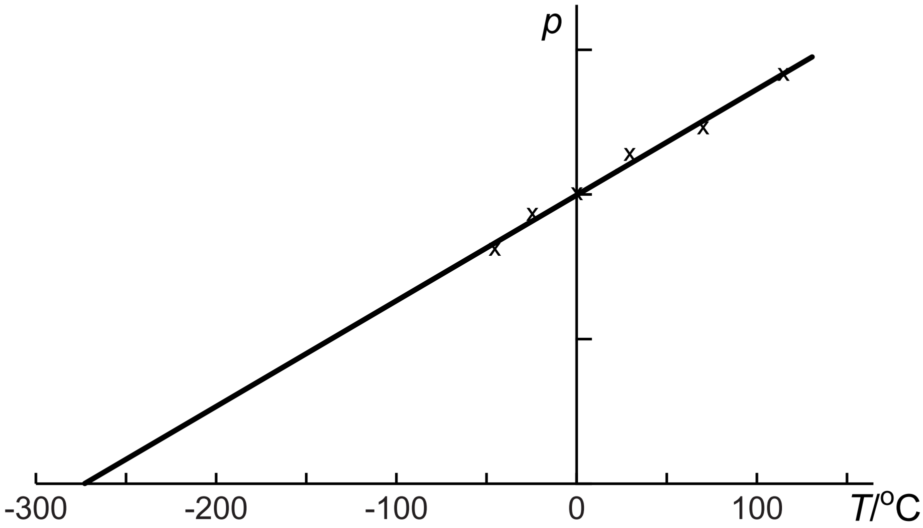
Figure 6.7: Calibration of a constant volume gas thermometer



....

With only two points on the graph (Figure 6.7), we assume a straight line, in accordance with Equation 6.3. In fact, if we use the constant volume gas thermometer along with a calibrated thermometer, we do see a linear relationship between pressure and temperature as shown in Figure 6.8.

Figure 6.8: Extended graph of pressure against temperature



. . . .

If we extend the straight line graph to the point where p = 0 Pa, the line intercepts the temperature axis at T = -273 °C. In fact, no matter what gas we are using inside the constant volume gas thermometer, the calibration graph always intercepts the temperature axis at T = -273 °C, which is equivalent to 0 K. (To be strictly accurate, the interception point is at -273.15 °C.)

This temperature is called **absolute zero**, since it is impossible to reduce the temperature of any material to 0 K. The coldest temperatures that have been created in recent experiments have been fractions of a kelvin above absolute zero. Because 0 K is the value of absolute zero, a temperature measured on the kelvin scale is sometimes referred to as the absolute temperature.

We have discussed the temperature of a gas as being related to the kinetic energy of the gas molecules. If we remove some of this energy, the kinetic energy of the molecules decreases (their average speed decreases) and the gas cools down. At absolute zero, the kinetic energy of the molecules is zero. The temperature cannot get any lower as there is no more energy to be removed.

You should note that the graph in Figure 6.8 has been extended well beyond the experimental points. A real gas will liquify and eventually freeze as it is cooled down.

The constant volume gas thermometer

20 min

Online simulation.

Full instructions are given on-screen. You will need graph paper to plot out the data given in the simulation.

For a fixed mass of gas at constant volume, the pressure is proportional to the temperature.

6.3.5 Volume and temperature

Let us look now at the situation where the pressure of a gas remains constant and the volume and temperature are allowed to vary. What is the relationship between these two quantities?

Consider a fixed mass of gas trapped in a cylinder with a frictionless piston. If the gas is heated, the average speed of the gas molecules increases. This means the impulse exerted on the piston when gas molecules collide with it increases. It also means the number of collisions per second will increase. The increased impulse and rate of collisions act to push out the piston, increasing the volume inside the cylinder and hence the volume of the gas.

Once the piston has been pushed back, the rate of collisions will decrease, as molecules now have further to travel between collisions with the piston or the walls. Thus the pressure remains the same - the effects of greater impulse and lower rate of collision balance each other out. The overall effect is that heating a gas causes it to expand, and the relationship between volume and temperature is

$$V \propto T \tag{6.5}$$

. . . .

If a fixed mass of gas at constant pressure changes from a volume V_1 and temperature T_1 to volume V_2 and temperature T_2, the relationship between these quantities is

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \tag{6.6}$$

. . . .

Equation 6.6 is known as Charles' law after the French scientist Jacques Charles (1746 - 1823).

Example

Suppose a gas at temperature T is trapped in a cylinder with a frictionless piston. If the cylinder has volume 4.0 x 10^{-5} m^3, what is the new volume the gas occupies if the kelvin temperature is doubled while its pressure remains constant?

We can solve this problem using Equation 6.6 without needing to know the actual values of the initial and final temperatures.

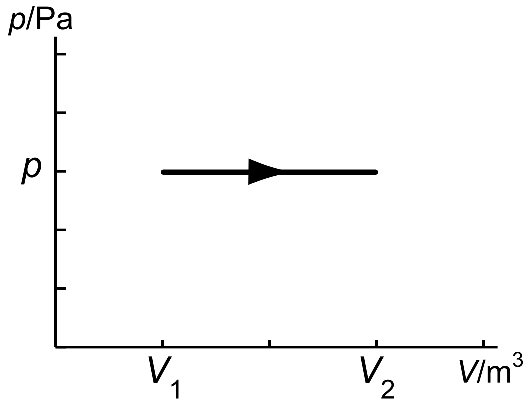
$$\begin{aligned}
\frac{V_1}{T_1} &= \frac{V_2}{T_2} \\
\therefore V_2 &= \frac{V_1}{T_1} \times T_2 \\
\therefore V_2 &= \frac{4.0 \times 10^{-5} \times 2T_1}{T_1} \\
\therefore V_2 &= 8.0 \times 10^{-5}\ \text{m}^3
\end{aligned}$$

Again, it is important to remember that the temperature must be expressed in kelvin.

. . . .

We can show a change in which a fixed mass of gas remains at constant pressure on a *p*-*V* graph. Figure 6.9 shows a gas expanding from volume V_1 to volume V_2 whilst its pressure remains equal to *p*.

Figure 6.9: Graph of a gas expanding at constant pressure



. . . .

6.3.6 A general gas equation

Now that we have seen what happens when two of the three quantities pressure, volume and temperature are allowed to vary, we will look at the more general case in which all three quantities can change.

For a fixed mass of gas, the general relationship is

$$pV \propto T \tag{6.7}$$

. . . .

It should be clear that if we ensure that one of the quantities in Equation 6.7 remains fixed, then we end up with one of the *p*-*V*, *p*-*T* or *V*-*T* relationships that was introduced earlier.

If a gas changes from an initial state p_1, V_1, T_1 to a final state p_2, V_2, T_2 then the equation linking these quantities is

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \tag{6.8}$$

. . . .

Example

An air bubble at the bottom of the sea occupies a volume of 4.00 x 10^{-6} m^3. At this depth, the pressure is 2.50 x10^7 Pa and the temperature is 275 K. Calculate the volume of the bubble when it has risen to a point where the pressure is 5.00 x 10^6 Pa and the temperature is 280 K, assuming the bubble contains a fixed mass of air.

To solve this problem, we use the general gas equation (Equation 6.8).

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$
$$\therefore V_2 = \frac{p_1}{p_2}\frac{T_2}{T_1}V_1$$
$$\therefore V_2 = \frac{2.50 \times 10^7}{5.00 \times 10^6} \times \frac{280}{275} \times 4.00 \times 10^{-6}$$
$$\therefore V_2 = 2.04 \times 10^{-5}\ \mathrm{m}^3$$

. . . .

15 min

Quiz 2 Gas laws

Multiple choice quiz.

First try the questions. If you get a question wrong or do not understand a question, there are 'Hints' at the the back of the book . The hints are in the same order as the questions. If you read the hint and still do not understand then ask your tutor.

Q6: A constant volume gas thermometer measures a pressure of 9.7 x 10^4 Pa at the freezing point of water. If the pressure is 2.8 x 10^4 Pa at the boiling point of nitrogen, what temperature does this thermometer give for the boiling point of nitrogen?

a) 3.5 K
b) 10 K
c) 79 K
d) 108 K
e) 945 K

. . . .

Q7: The pressure of a gas is proportional to its kelvin temperature only if

a) the density and kinetic energy of the gas are constant.
b) the kinetic energy and volume of the gas are constant.
c) the volume and potential energy of the gas are constant.
d) the mass and kinetic energy of the gas are constant.
e) the mass and volume of the gas are constant.

. . . .

Q8: A gas in an expandable container occupies a volume of 4.0 x 10^{-5} m^3 when its temperature is 15 °C. What volume does it occupy when its temperature is 40 °C?

a) 1.5 x 10^{-5} m^3
b) 3.7 x 10^{-5} m^3

c) 4.3×10^{-5} m^3
d) 6.5×10^{-5} m^3
e) 1.1×10^{-4} m^3

. . . .

Q9: Which one of the following statements is *false*?

a) A *p*-*T* graph for a constant volume gas thermometer can be extended to find the temperature of absolute zero.
b) Absolute zero is the minimum temperature for a gas, not a solid or a liquid.
c) The value of absolute zero is equivalent to -273.15 °C.
d) No material can have a temperature less than absolute zero.
e) At absolute zero, the kinetic energy of any molecule is zero.

. . . .

Q10: The general gas equation can be expressed as

a) $\frac{pV}{T} =$ constant
b) $\frac{pT}{V} =$ constant
c) $\frac{VT}{p} =$ constant
d) $\frac{V}{pT} =$ constant
e) $pVT =$ constant

. . . .

6.4 Gas laws and the kinetic model

Learning Objective

To explain the gas laws in terms of the kinetic model.

Before we end this Topic, it will be worthwhile to recap what we have learnt about the behaviour of gases, and how the gas laws can be explained in terms of the kinetic model.

The kinetic model tells us that a gas exerts pressure on the walls of its container due to elastic collisions between gas molecules and the container walls. If we make the container larger, and keep the mass and temperature of the gas fixed, then the average time a molecule spends between collisions with the walls will increase, since the molecule has to travel further between collisions. There will be fewer collisions per unit time, and hence the **pressure decreases when the volume increases**.

The average speed of the gas molecules depends on the temperature of the gas. If the temperature increases, the average speed of the molecules increases. This means that the impulse exerted by a molecule when it collides with a container wall increases, and hence the pressure increases too. So for a fixed mass of gas held at constant volume, the **pressure increases when the temperature increases**.

An increase in temperature increases the impulse exerted on the container walls by a molecule. If the gas is in an expandable container, then this increased impulse will push back the walls of the container, until the effect of increased impulse is balanced by the increased time between collisions, and hence the decreased number of collisions per second. Thus if we have a fixed mass of gas at constant pressure, the **volume increases when the temperature increases**.

In the general case when pressure, volume and temperature can all vary, then Equation 6.7 applies.

6.5 Summary

By the end of this Topic you should be able to:

- carry out calculations to convert temperatures in °C to K, and vice versa;
- describe how the kinetic model accounts for the pressure of a gas;
- state that the pressure of a fixed mass of gas at constant temperature is inversely proportional to its volume;
- state that the pressure of a fixed mass of gas at constant volume is proportional to its temperature (in kelvin);
- state that the volume of a fixed mass of gas at constant pressure is proportional to its temperature (in kelvin);
- explain what is meant by the absolute zero of temperature;
- explain the pressure-volume, pressure-temperature and volume-temperature laws in terms of the kinetic model;
- carry out calculations involving the pressure, volume and temperature of a fixed mass of gas using the general gas equation.

Online assessments

Three online test are available. Each test should take you no more than 20 minutes to complete. Each test has questions taken from all parts of the Topic.

Glossary

absolute zero

The minimum temperature theoretically attainable, 0 K, equivalent to -273.15 °C.

acceleration

The rate of change of velocity. Acceleration is a vector quantity.

buoyancy force

The upwards force (sometimes called the upthrust) acting on an object immersed in a fluid, due to the difference in pressure (and hence force) acting on its upper and lower surfaces.

components of a vector

Two vectors which act at right angles, the vector sum of which is the original vector.

conservation of energy

Energy cannot be created or destroyed, only converted from one form to another.

conservation of momentum

When two or more objects interact, the total momentum is conserved, in the absence of external forces.

density

The density ρ of a substance is its mass per unit volume, measured in kg m^{-3}.

displacement

A specified distance from a fixed point, in a specified direction. Displacement is a vector quantity.

elastic collision

A collision in which both momentum and kinetic energy are conserved.

impulse

The change of momentum of an object, equal to the product of the force acting on the object and the time over which the force acts.

inelastic collision

A collision in which momentum is conserved but kinetic energy is not.

kinematic relationships

A set of equations which describe the motion of an object moving with constant acceleration.

kinetic energy

The energy of an object due to its motion.

kinetic theory

A theory that explains the physical properties of a gas by considering the gas to be made up of a number of molecules moving with random speeds, undergoing elastic collisions with each other and with the walls of the container.

momentum

The product of the mass of an object and its velocity. Momentum is a vector quantity, measured in kg m s^{-1}.

newton

One newton is the force that, when applied to an object of mass 1 kg, will cause the object to accelerate at a rate of 1 m s^{-2} in the direction of the applied force.

potential energy

The energy stored in an object due to its position, its shape or its state.

pressure

The force acting per unit area, measured in N m^{-2} or pascal (Pa).

scalar

A physical quantity which has magnitude but no direction.

vector

A physical quantity which has direction as well as magnitude.

velocity

The rate of change of displacement. Velocity is a vector quantity.

Hints for activities

Topic 1: Vectors

Quiz 1 Adding vectors

Hint 1:

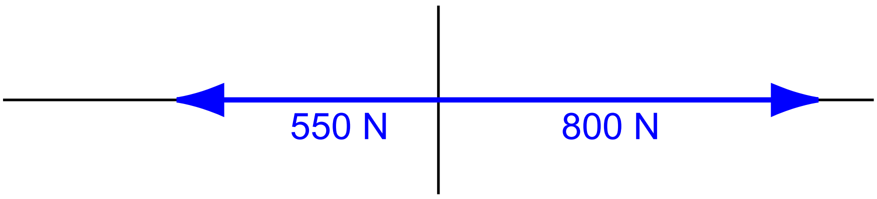
These two forces are collinear, and in the same direction - see the section titled Combining vectors.

Hint 2:

These two forces are collinear but in opposite directions - see the section titled Combining vectors.

Hint 3:

Orthogonal means that the vectors are at right angles to each other - see the following image or refer to the section titled Combining vectors - use Pythagoras theorem to find the magnitude of resultant.



Two forces acting in opposite directions

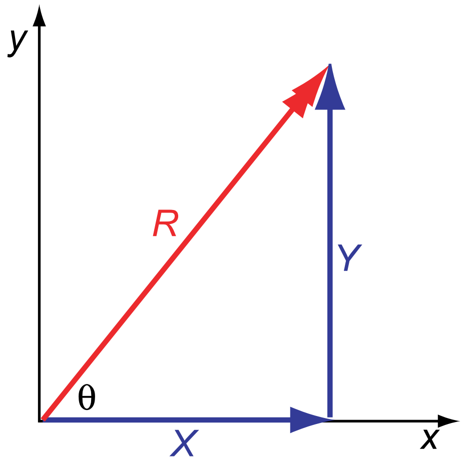
Hint 4: Draw a sketch of the forces - then use the tangent to find the angle between the x axis and the resultant.

Hint 5: Draw a sketch of the vector sum - remember 'nose-to-tail' - the resultant points from the start of the first vector to the end of the second vector.

Quiz 2 Components of a vector

Hint 1:

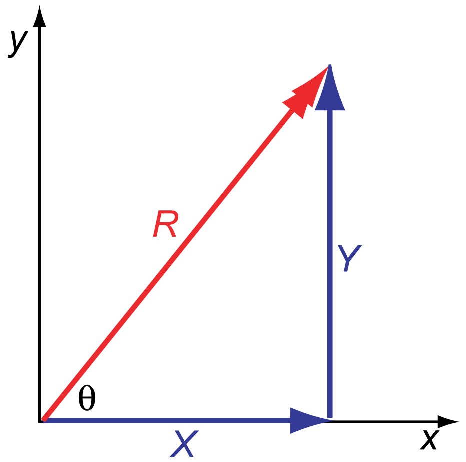
The angle of elevation θ is between the vector and the horizontal - see the following image or refer to the section titled Components of a vector - so the horizontal component of velocity = $v \cos \theta$.



Orthogonal components of a vector

Hint 2:

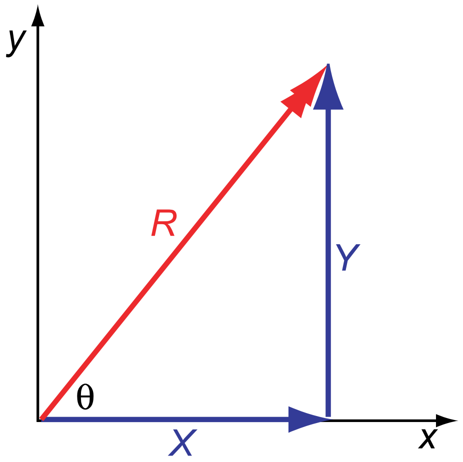
First find the y-component of each vector - if you are not sure whether to use the sine or cosine see the following image or refer to the section titled Components of a vector. Then add the y-components - don't forget about positive and negative directions.



Orthogonal components of a vector

Hint 3:

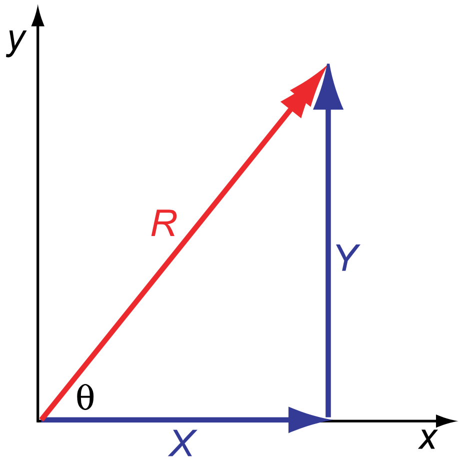
First find the x-component of each vector - if you are not sure whether to use the sine or the cosine see the following image or refer to the section titled Components of a vector. Then add the x-components - don't forget about positive and negative directions



Orthogonal components of a vector

Hint 4:

The angle θ given is between the vector and the horizontal - see the image below or refer to the section titled Components of a vector - the horizontal component of the force = F cos θ



Orthogonal components of a vector

Hint 5:

The angle θ given is between the vector and the vertical - refer to the section titled Components of a vector - the vertical component of each force = F cos θ. The angles and the forces are equal - so the total vertical force is double the vertical component of one force.

Components of velocity

Topic 2: Equations of motion

Quiz 1 Acceleration

Hint 1:

The words *'from rest'* mean the initial velocity = 0 m s^{-1}.

Hint 2:

See the definition of *acceleration* in the section titled Acceleration

Hint 3:

Use the graph to get the values of u and v for the first 10 s of the motion.

Hint 4:

Does the unbalanced force acting on the tennis ball change?

Hint 5:

Positive acceleration occurs where the *v-t* graph has a positive gradient - if you are not sure about gradients see the section in Higher Mathematics titled Gradients of straight lines.

Horizontal Motion

Hint 1:

Quiz 2 Kinematic relationships

Hint 1:

What is the initial velocity of the car? Use the second equation of motion.

Hint 2:

On screen simulation exploring projectile motion.

Hint 3:

Pushed over a cliff means the initial vertical velocity of the car = 0 m s^{-1}. Use the third equation of motion.

Hint 4:

Remember v is always the final velocity - substituting v and u incorrectly is wrong physics.

Hint 5:

First find the vertical component of the initial velocity - the maximum height is the vertical displacement when the vertical velocity = 0 m s^{-1}.

Topic 3: Newton's second law, energy and power

Quiz 1 Newton's second law

Hint 1:

In all Physics relationships, units are equivalent on both sides of an equation - apply this to $F = ma$.

Hint 2:

This is a straight application of $F = ma$.

Hint 3:

First find the resultant force - see Section titled Combining vectors.

Hint 4:

In $F = ma$, substitute $m_1 = {}^1/_2 m$ and $F_1 = 2F$.

Hint 5:

First use Newton's Second law to find *a*, then use one of the equations of motion. What does the word '*stationary*' tell you about the initial velocity of the object?

Mass on a slope

Hint 1:

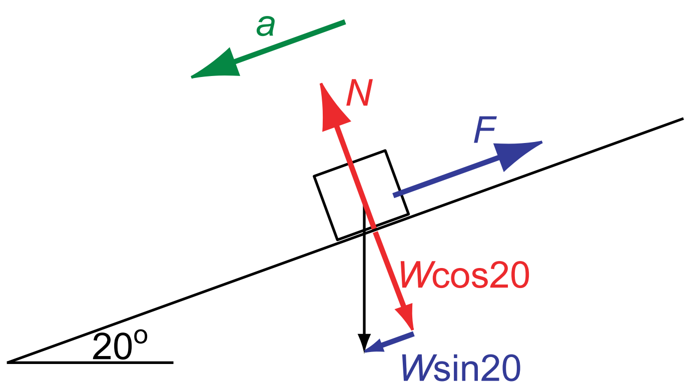
Quiz 2 - Free body diagrams

Hint 1:

Find the vertical resultant and the horizontal resultant separately.

Hint 2:

The velocity of the crate is constant - what does this tell you about the forces acting on the crate? If you are not sure how to calculate the component of the weight of the crate acting down the slope, see the following image or refer to the section titled Objects undergoing acceleration



Free body diagram of the television sliding down the slope

Hint 3:

First use $F = ma$ to find the unbalanced vertical force acting on the block - then consider all the vertical forces to find the tension in the rope

Hint 4:

Find the component of *Z* parallel to *X* - then do the vector sum - be careful with positive and negative signs.

Hint 5:

Find the component of *Z* parallel to *Y* - then do the vector sum - be careful with positive and negative signs.

Quiz 3 - Energy and power

Hint 1:

Use the equation

$$KE = \frac{1}{2}mv^2$$

or refer to the section titled Energy.

Hint 2:

The potential energy of the tile is converted to kinetic energy as it falls - see the following equations or refer to the section titled Energy.

$$KE = \frac{1}{2}mv^2$$

$$PE = mgh$$

Hint 3:

Use the equation

$$P = F \times v$$

or refer to the section titled Power

Hint 4:

Long method: find the maximum height - then find the potential energy at that height - subtract this from the initial kinetic energy.

Shorter method: find the horizontal component of the initial velocity of the projectile. At the maximum height the velocity of the projectile is horizontal so you can use this value in the kinetic energy equation to find the kinetic energy at the maximum height."

Topic 4: Momentum and impulse

Quiz 1 Momentum

Hint 1:

All of ball *M*'s momentum is transferred to ball *N*.

Hint 2:

The carriages have the same velocity after the collision.

Hint 3:

See the section titled Inelastic collisions, for the definition of an inelastic collision

Hint 4:

Refer to these equations

$$KE = \frac{1}{2}mv^2$$

and

$$p = mv$$

or look at the the sections titled Energy and Momentum.

Hint 5:

Let the velocities of spheres after the collision be v_1 and v_2.

Use conservation of momentum to find a linear equation in v_1 and v_2 .

Use conservation of kinetic energy to find a quadratic equation inv_1 and v_2.

Use simultaneous equations to find the values of v_1 and v_2.

You will get two possible answers - one of these is not physically possible as it requires the 1·0 kg sphere to pass through the 4·0 kg sphere.

"Test your strength" contest

Hint 1:

Hint 2:

Quiz 2 Impulse

Hint 1:

See section titled Impulse for a definition of impulse

Hint 2:

See section titled Impulse

Hint 3:

The words '*at rest*' mean the initial momentum is 0 kg m s^{-1}.

Hint 4:

Quantities that have the same dimensions have units which are equivalent.

Hint 5:

The impulse *Ft* is equal to the change in momentum of the car.

Quiz 3 Explosions

Hint 1:

The magnitude of the momentum of B is equal to the magnitude of the momentum of A.

Hint 2:

The magnitude of the recoil momentum of the gun is equal to the momentum of the bullet.

Hint 3:

Which particle is faster after the explosion - the less massive particle or the more massive particle?

Hint 4:

The combined mass of the woman plus the ball before she lets go of the ball is 50 kg. Use this to find the total momentum before the explosion.

Hint 5:

Use the data given in the first sentence to find the mass of the cannon - then use this with the data given in the second sentence.

Topic 5: Density and Pressure

Quiz 1 Density and pressure

Hint 1:

See the examples given in the following table, or look up values in a data book. An 'order of magnitude' is an approximation to the nearest power of 10.

Substance	**Density** (kg m^{-3})
Aluminium	2700
Copper	8960
Ice	920
Silver	10500
Sea Water	1020
Water	1000
Air	1.29
Carbon Dioxide	1.98
Oxygen	1.43

Densities of different substances

Hint 2:

For an equal mass, the higher the density the lower the volume.

Hint 3:

In all Physics relationships, units are equivalent on both sides of an equation - apply this to

$$p = \frac{F}{A}$$

.

Hint 4:

The word '*excess*' means the amount by which the pressure inside the chamber is greater than the pressure outside the chamber.

Hint 5:

Be careful substituting for force and area - and remember to start with the greatest pressure.

Hot air balloon

Hint 1:

Quiz 2

Hint 1:

Refer to this equation

$$\begin{aligned} p &= \frac{F}{A} \\ \therefore p &= \frac{hA\rho g}{A} \\ \therefore p &= h\rho g \end{aligned}$$

. Or look at the section titled Pressure in a fluid.

Hint 2:

Refer to this equation

$$\begin{aligned} p &= \frac{F}{A} \\ \therefore p &= \frac{hA\rho g}{A} \\ \therefore p &= h\rho g \end{aligned}$$

. Or look at the section titled Pressure in a fluid.

Hint 3:

Did you notice that the question asks for the **increase** in pressure?

Hint 4:

Remember that the polystyrene block has weight. Be careful with the directions of the buoyancy force and the weight.

Topic 6: Gas Laws

Quiz 1 Kelvin scale and the p-V law

Hint 1:

See the section titled The kelvin temperature scale.

Hint 2:

Do not confuse '*temperature*' with '*increase in temperature*'.

Hint 3:

"Re-read Section titled Pressure exerted by a gas - pay particular attention to the text just after the figure.

Hint 4:

Refer to this equation

$$p_1V_1 = p_2V_2$$

or look at the the section titled Pressure and volume.

Hint 5:

Substitute V_2 = 0.25 V_1 in

$$p_1V_1 = p_2V_2$$

or look at the the section titled Pressure and volume

Quiz 2 Gas laws

Hint 1:

Look at

$$\frac{p_1}{T_1} = \frac{p_2}{T_2}$$

or refer to the section titled Temperature and pressure

Hint 2:

The conditions that apply to the gas laws are very important - see the start of the section titled Temperature and pressure for the conditions that apply to this law.

Hint 3:

The words '*expandable container*' imply that the pressure remains constant.

Hint 4:

Eliminate the statements that you know are definitely true and check the sections titled Temperature and pressure, Absolute zero and Volume and temperature for the statements you are not sure of - when you have eliminated four true statements the one remaining statement is false!.

Hint 5:

See the section titled A general gas equation

Answers to questions and activities

1 Vectors

Distance and displacement (page 3)

Q1: D

Q2: B

Q3: E

Q4: E

Quiz 1 Adding vectors (page 8)

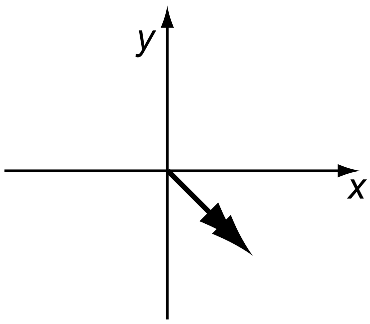
Q5: e) 115 N

Q6: b) 30 N acting southwards

Q7: d) 139 N

Q8: b) 30°

Q9: a)



Quiz 2 Components of a vector (page 14)

Q10: c) 245 m s^{-1}

Q11: b) -3.06

Q12: d) +10.2

Q13: d) 83 N

Q14: e) 640 N

2 Equations of motion

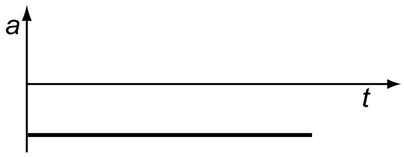
Quiz 1 Acceleration (page 22)

Q1: d) 3.6 m s^{-2}

Q2: b) The velocity of the motorcycle is constant, so its acceleration is zero.

Q3: a) 0.4 m s^{-2}

Q4: e)



Q5: b) A and E

Motion of a bouncing ball (page 25)

1. Starting from a height of 10 m, the data is *u* = 0 m s^{-1}, *a* = -9.8 m s^{-2}, *s* = -10 m and *v* is unknown. The kinematic relationship that should be used is $v^2 = u^2 + 2as$

$$
\begin{aligned}
v^2 &= u^2 + 2as \\
\therefore v^2 &= 0 + (2 \times -9.8 \times -10) \\
\therefore v^2 &= 196 \\
\therefore v &= -14\,\text{m s}^{-1}
\end{aligned}
$$

   Note that we have taken the *negative* square root - if the ball is moving downwards, its velocity is negative in the sign convention we are using.

2. Now the ball is projected upwards with *u* = 12 m s^{-1}, *v* = 0 m s^{-1}, *a* = -9.8 m s^{-2} and *s* is unknown.

$$
\begin{aligned}
v^2 &= u^2 + 2as \\
\therefore 0 &= 12^2 + (2 \times -9.8 \times s) \\
\therefore 19.6s &= 144 \\
\therefore s &= 7.3\,\text{m}
\end{aligned}
$$

Horizontal Motion (page 26)

Quiz 2 Kinematic relationships (page 27)

Q6: b) 28 m

Q7: e) 9.8 m s^{-2} downwards

Q8: c) 33 m s^{-1}

Q9: c) -4.2 m s^{-2}

Q10: d) 27 m

3 Newton's second law, energy and power

Quiz 1 Newton's second law (page 33)

Q1: c) 1 kg m s^{-2}

Q2: b) 15 m s^{-2}

Q3: b) 0.18 m s^{-2}

Q4: e) 4*a*

Q5: d) 25 m s^{-1}

Mass on a slope (page 39)

Quiz 2 - Free body diagrams (page 39)

Q6: d) In the direction of *D*

Q7: a) 62 N

Q8: c) 41 N

Q9: d) 4.4 N

Q10: e) 34 N

Quiz 3 - Energy and power (page 43)

Q11: d) 360 J

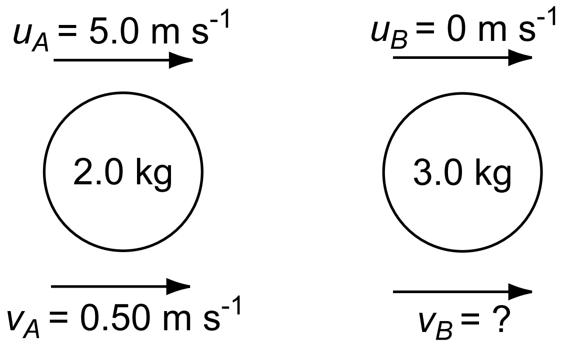
Q12: b) 14 m s^{-1}

Q13: d) 3000 W

Q14: c) 970 J

4 Momentum and impulse

Example - Collision between two spheres (page 48)



Using the law of conservation of momentum, the total momentum before the collision is equal to the total momentum after the collision.

$$\begin{aligned}(m_A u_A) + (m_B u_B) &= (m_A v_A) + (m_B v_B)\\ \therefore (2 \times 5) + (3 \times 0) &= (2 \times 0.5) + (3 \times v_B)\\ \therefore 10 &= 1 + 3v_B\\ \therefore 3v_B &= 9\\ \therefore v_B &= 3.0\,\mathrm{m\,s^{-1}}\end{aligned}$$

Quiz 1 Momentum (page 51)

Q1: d) 1.67 m s^{-1}

Q2: a) 3.8 m s^{-1}

Q3: e) momentum is conserved but kinetic energy is not.

Q4: c) 1.6 m s^{-1}

Q5: a) 0.60 m s^{-1}

"Test your strength" contest (page 53)

Quiz 2 Impulse (page 53)

Q6: b) 3200 kg m s^{-1}

Q7: c) force = rate of change of momentum.

Q8: e) 60 kg m s^{-1}

Q9: a) momentum.

Q10: d) 4800 N

Quiz 3 Explosions (page 55)

Q11: c) 2.4 m s^{-1}

Q12: e) 12 m s^{-1}

Q13: a) $^{v}/_{3}$

Q14: d) 1.1 m s^{-1}

Q15: c) 7.2 m s^{-1}

5 Density and Pressure

Quiz 1 Density and pressure (page 64)

Q1: a) 1000

Q2: c) 9.80×10^{-3} m^3

Q3: d) 1 N m^{-2}

Q4: b) 190 N

Q5: e) Y, Z, X

Hot air balloon (page 69)

Quiz 2 (page 69)

Q6: e) 2.9×10^5 Pa

Q7: b) 25.5 m

Q8: c) 1.5×10^5 Pa

Q9: a) 0.52 N

6 Gas Laws

Quiz 1 Kelvin scale and the p-V law (page 77)

Q1: b) 318 K

Q2: a) 25 K

Q3: d) Molecules undergo elastic collisions with the container walls.

Q4: a) 1.6 x 10^{-4} m^3

Q5: e) $4p$

Quiz 2 Gas laws (page 84)

Q6: c) 79 K

Q7: e) the mass and volume of the gas are constant.

Q8: c) 4.3 x 10^{-5} m^3

Q9: b) Absolute zero is the minimum temperature for a gas, not a solid or a liquid.

Q10: a) $\frac{pV}{T} =$ constant