HIGHER PHYSICS

UNIT 2 - ELECTRICITY and ELECTRONICS

4) CAPACITORS

You must be able to:

State that the charge (Q) on 2 parallel conducting plates is directly proportional to the potential difference between the plates.

Describe the principles of a method to show that the potential difference across a capacitor is directly proportional to the charge on its plates.

State that capacitance is the ratio of charge to potential difference.

State that the unit of capacitance is the farad and that 1 farad is 1 coulomb per volt.

Carry out calculations using $C = Q / V$.

Explain why work must be done to charge a capacitor.

State that the work done to charge a capacitor is given by the area under the graph of charge against potential difference.

State that the energy stored in a capacitor is given by $E = 1/2\ QV = 1/2\ CV^2 = 1/2\ Q^2/C$ and carry out calculations using these equations.

Draw qualitative graphs of current against time and of voltage against time for the charge and discharge of a capacitor in a d.c. circuit containing a resistor and capacitor in series.

Carry out calculations involving voltage and current in capacitor-resistor circuits.

State and explain the relationship between current and frequency in resistive and capacitive circuits.

Describe the principles of a method to show how current varies with frequency in resistive and capacitive circuits.

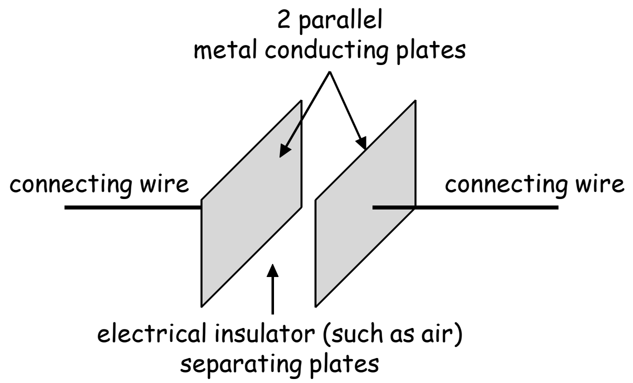
Describe and explain some possible functions of a capacitor, e.g., storing energy and blocking d.c. while passing a.c.

Capacitors - Capacitance, Charge and Potential Difference

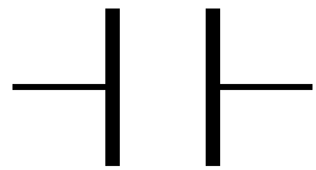
Capacitors store **electric charge**.

This ability to store **electric charge** is known as **capacitance**.

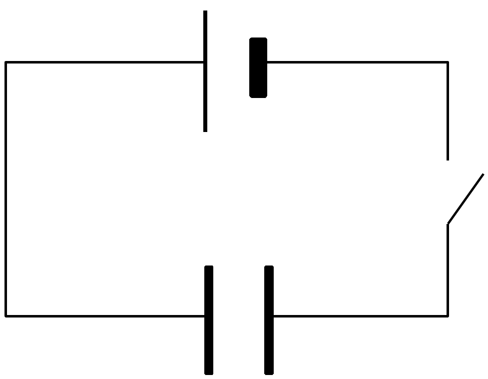
A simple **capacitor** consists of **2 parallel metal conducting plates** separated by an **electrical insulator** such as **air**.



Hence the **circuit symbol** for a **capacitor**:



To **charge** a capacitor, we connect a **battery** (or **d.c. power supply**) across its conducting plates:



Label the circuit diagram, opposite.

When the switch is closed, the capacitor **"charges up"**.

- **Electric charge** is stored on the conducting plates.
- A **potential difference** is created between the conducting plates which becomes equal to the **battery/supply voltage**.

The **higher** the **potential difference** (**V**) between the conducting plates, the greater the **charge** (**Q**) stored on the plates.

The **charge** (**Q**) stored on the 2 parallel conducting plates is **directly proportional to** the **potential difference** (**V**) between between the plates.

$$Q \propto V$$

$$\therefore Q = \text{constant} \times V$$

$$\therefore \text{constant} = \frac{Q}{V}$$

The **constant**, which equals the **"ratio of charge to potential difference"**, is called the **capacitance** of the capacitor.

$$C = \frac{Q}{V}$$

- C – capacitance of capacitor/ farads (F)
- Q – charge stored on capacitor plates/ coulombs (C)
- V – potential difference between capacitor plates/ volts (V)

1 farad = 1 coulomb per volt

($1\ F = 1\ C\ V^{-1}$)

Note about the Farad

The farad is a very large unit - Too large for the practical capacitors used in our household electronic devices (televisions, radios, etc).

These practical capacitors have smaller "sub-units":

microfarads (F).....$\times 10^{-6}$ F   nanofarads (nF).....$\times 10^{-9}$ F   picofarads (pF).....$\times 10^{-12}$ F

In each case, calculate the **capacitance** of the capacitor:

- charge stored on capacitor plates = 2.5×10^{-5} C
- potential difference between capacitor plates = 10 V

- charge stored on capacitor plates = 1.6×10^{-8} C
- potential difference between capacitor plates = 4.0 V

- charge stored on capacitor plates = 4.5×10^{-11} C
- potential difference between capacitor plates = 5.0 V

In each case, calculate the **charge** stored on the plates of the capacitor:

- capacitance of capacitor = 1.2   F
- potential difference between capacitor plates = 100 V

- capacitance of capacitor = 5.5 nF
- potential difference between capacitor plates = 12 V

- capacitance of capacitor = 6.2 pF
- potential difference between capacitor plates = 9.0 V

In each case, calculate the **potential difference** between the plates of the capacitor:

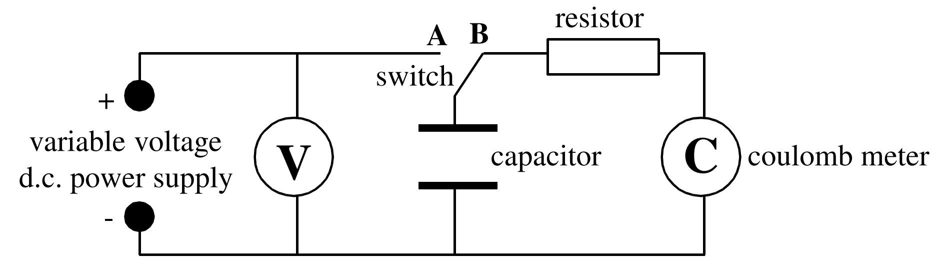
- charge stored on capacitor plates = 8.4×10^{-6} C
- capacitance of capacitor = 2.0   F

- charge stored on capacitor plates = 3.6×10^{-9} C
- capacitance of capacitor = 1.3 nF

- charge stored on capacitor plates = 9.0×10^{-12} C
- capacitance of capacitor = 4.5 pF

Experiment to show that the potential difference (V) between the conducting plates of a capacitor is directly proportional to the charge (Q) stored on the plates

This circuit can be used to determine the relationship connecting the **potential difference between the conducting plates of a capacitor** and the **charge stored on the plates.**



1) At the start of the experiment, the switch is set at position B - This means the capacitor is not connected to the variable voltage d.c. power supply - It is NOT charged.

2) The variable voltage d.c. power supply is set at a fixed voltage.

3) The switch is moved to position A - This causes the capacitor to "charge up".

- What does the reading on the voltmeter show? ________________
____
- How do we know when the capacitor has been "fully charged"? ______
____

4) When the capacitor has been "fully charged", the switch is moved back to position B - This causes the capacitor to discharge (release its stored charge) through the resistor and coulomb meter.

- What does the reading on the coulomb meter show? ____________
____

5) This process is repeated for different voltage settings of the variable voltage d.c. power supply.

6) A line graph of **"charge stored on the capacitor plates" versus "potential difference between the capacitor plates"** is plotted.

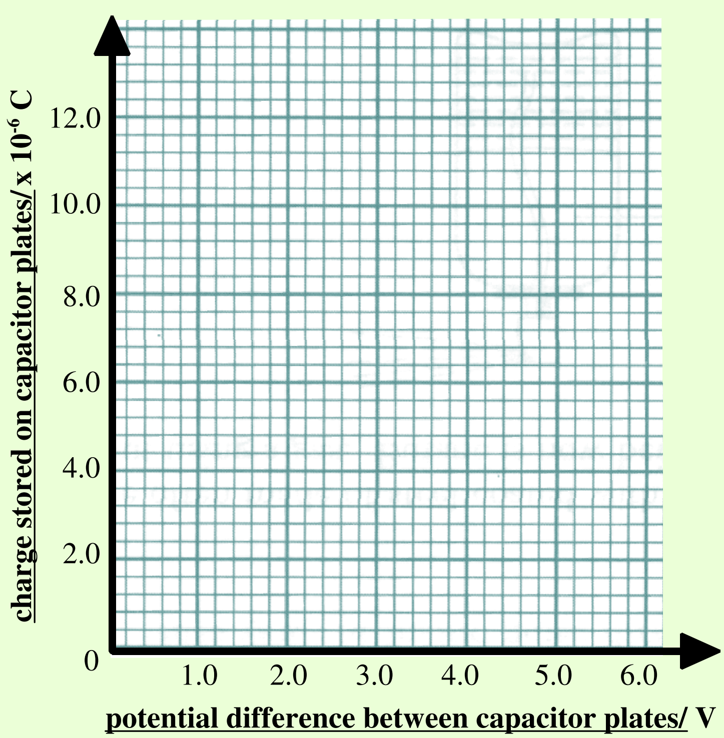
These results were obtained from an experiment like the one detailed above:

potential difference between capacitor plates/ V	charge stored on capacitor plates/ x 10^{-6} C
1.0	2.0
2.0	4.0
3.0	6.0
4.0	8.0
5.0	10.0
6.0	12.0

Plot the results on the graph.

- What does the graph show? _____
____
____
____

- Calculate the capacitance of the capacitor used for this experiment.

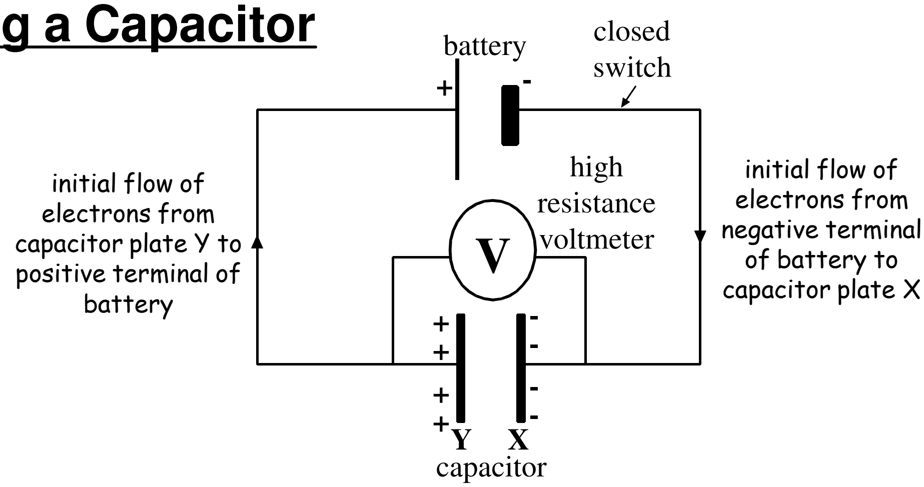


Work Done Charging a Capacitor

This circuit represents the **charging of a capacitor**.

When the switch is closed, negatively charged electrons flow from the negative terminal of the battery and build up on **plate X** of the capacitor - So **plate X becomes negatively charged**.

As a result, negatively charged electrons on **plate Y** of the capacitor are repelled and travel through the wire to the the positive terminal of the battery - So **plate Y becomes positively charged**.



- How can you tell when electrons have stopped flowing between the battery and capacitor? ____

This creates a **potential difference** (**V**) between the capacitor plates. This **potential difference** increases until it becomes equal to the **battery voltage**, when the flow of electrons stops.

NO ELECTRONS TRAVEL THROUGH THE INSULATING MATERIAL (AIR) BETWEEN THE CAPACITOR PLATES.

To push electrons onto the negatively charged capacitor plate, the battery must do work against the potential difference between the capacitor plates.

WORK MUST BE DONE TO CHARGE A CAPACITOR.

We have learned that **work done (W) = charge (Q) × potential difference (V)**.

When a **charge** (Q) is placed on the plates of an uncharged capacitor, the **potential difference** between the capacitor plates changes (increases) - So we must use the ***average*** value for the **potential difference** between the plates in the above equation.

- Before charging..... potential difference between capacitor plates = 0 volts
- After charging....... potential difference between capacitor plates = V volts

∴ Average potential difference between capacitor plates = $\frac{0 + V}{2}$ = 1/2 V volts

Substituting into the above equation:

$W = Q \times 1/2\ V$

$= 1/2\ Q\ V$

When a charge (Q) is placed on the plates of an uncharged capacitor:

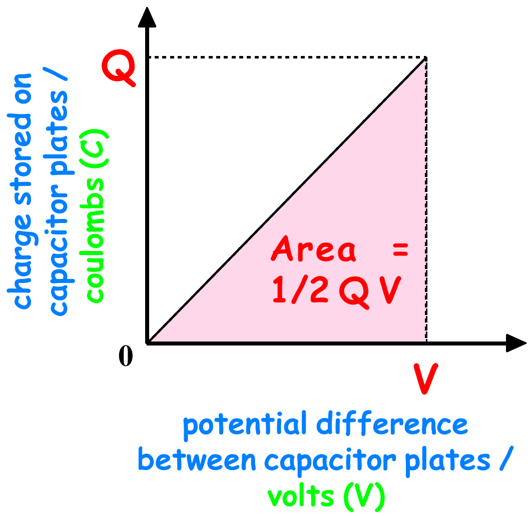
$$W = 1/2\ Q\ V$$

- W: work done by battery or power supply when fully charging capacitor/ joules (J)
- Q: charge stored on capacitor plates when fully charged/ coulombs (C)
- V: potential difference between capacitor plates when fully charged/ volts (V)

Work Done Charging a Capacitor = Area Under QV Graph

$W = 1/2\ Q\ V$

- W: work done by battery or power supply when fully charging capacitor/ joules (J)
- Q: charge stored on capacitor plates when fully charged/ coulombs (C)
- V: potential difference between capacitor plates when fully charged/ volts (V)



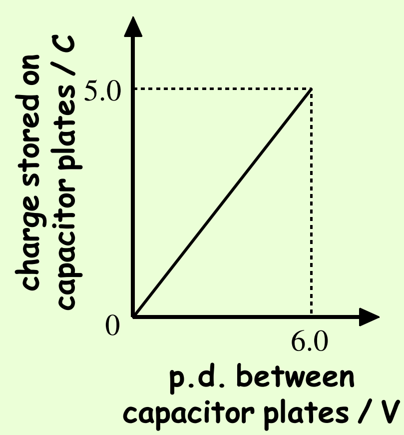
Area under QV charging graph for a capacitor = 1/2 Q V

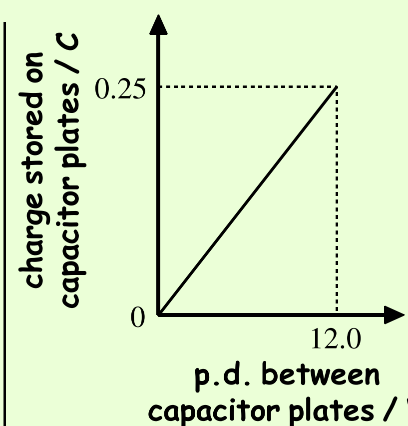
Conclusion

The work done to charge a capacitor = the area under the QV charging graph for the capacitor.

The QV charging graphs for 2 different capacitors are shown below.

In each case, calculate the **work done** by the battery/power supply which fully charged the capacitor:





Energy Stored in a Capacitor

Work done by a battery/power supply in **"charging"** a capacitor is stored as **electrical potential energy** in an **electric field** which exists between the charged capacitor plates.

This **electrical potential energy** is released when the capacitor is **discharged**, e.g., by connecting both plates of the capacitor to a light bulb.

$$E = 1/2\,Q\,V = 1/2\,C\,V^2 = 1/2\,\frac{Q^2}{C}$$

E = energy stored in capacitor/ joules (J)

Q = charge stored on capacitor plates/ coulombs (C)

V = potential difference between capacitor plates/ volts (V)

C = capacitance of capacitor/ farads (F)

$E = 1/2\, C V^2$ Problems

In each case, calculate the **energy** stored in the capacitor:

- capacitance of capacitor = 2.9   F
- potential difference between capacitor plates = 1.5 V

- capacitance of capacitor = 1.2 nF
- potential difference between capacitor plates = 3.5 V

- capacitance of capacitor = 5.5 pF
- potential difference between capacitor plates = 9.0 V

In each case, calculate the **capacitance** of the capacitor:

- energy stored in capacitor = 1.6×10^{-5} J
- potential difference between capacitor plates = 2.0 V

- energy stored in capacitor = 2.0×10^{-9} J
- potential difference between capacitor plates = 0.5 V

- energy stored in capacitor = 1.8×10^{-11} J
- potential difference between capacitor plates = 3.0 V

In each case, calculate the **potential difference** between the plates of the capacitor:

- energy stored in capacitor = 4.0×10^{-6} J
- capacitance of capacitor = 0.5   F

- energy stored in capacitor = 4.5×10^{-9} J
- capacitance of capacitor = 0.25 nF

- energy stored in capacitor = 2.5×10^{-11} J
- capacitance of capacitor = 2.0 pF

$E = 1/2\ Q\ V$ Problems

In each case, calculate the **energy** stored in the capacitor:

- charge stored on capacitor plates = 1.5 C
- potential difference between capacitor plates = 6.0 V

- charge stored on capacitor plates = 2.5×10^{-5} C
- potential difference between capacitor plates = 12 V

- charge stored on capacitor plates = 4.2×10^{-9} C
- potential difference between capacitor plates = 4.0 V

In each case, calculate the **charge** stored on the plates of the capacitor:

- energy stored in capacitor = 5.0 J
- potential difference between capacitor plates = 2.0 V

- energy stored in capacitor = 1.3×10^{-3} J
- potential difference between capacitor plates = 0.5 V

- energy stored in capacitor = 7.5×10^{-5} J
- potential difference between capacitor plates = 1.5 V

In each case, calculate the **potential difference** between the plates of the capacitor:

- energy stored in capacitor = 1.6 J
- charge stored on capacitor plates = 0.5 C

- energy stored in capacitor = 1.8×10^{-3} J
- charge stored on capacitor plates = 6.0×10^{-3} C

- energy stored in capacitor = 4.4×10^{-7} J
- charge stored on capacitor plates = 2.2×10^{-8} C

$E = 1/2 \frac{Q^2}{C}$ Problems

In each case, calculate the **energy** stored in the capacitor:

- charge stored on capacitor plates = 0.50 C
- capacitance of capacitor = 5.0   F

- charge stored on capacitor plates = 3.0×10^{-3} C
- capacitance of capacitor = 4.5 nF

- charge stored on capacitor plates = 1.6×10^{-5} C
- capacitance of capacitor = 2.2 pF

In each case, calculate the **charge** stored on the plates of the capacitor:

- energy stored in capacitor = 1.8 J
- capacitance of capacitor = 3.5   F

- energy stored in capacitor = 3.2 J
- capacitance of capacitor = 5.5 nF

- energy stored in capacitor = 0.05 J
- capacitance of capacitor = 7.2 pF

In each case, calculate the **capacitance** of the capacitor:

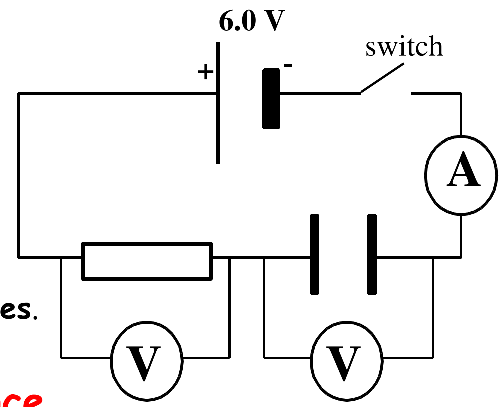
- charge stored on capacitor plates = 1.5×10^{-6} C
- energy stored in capacitor = 2.8 J

- charge stored on capacitor plates = 4.8×10^{-9} C
- energy stored in capacitor = 0.36 J

- charge stored on capacitor plates = 7.5×10^{-12} C
- energy stored in capacitor = 1.2 J

Voltage-Time Graphs for a Charging Capacitor

As you work through this page, you should refer to the description of how a capacitor charges - Page 5.



This electric circuit can be used to investigate the charging of a **capacitor**.

(The **resistor** is present to set the value of the ***maximum current*** which can flow).

Current starts to flow ***immediately the switch is closed***.

In the circuit, the **capacitor** and **resistor** are connected in **series**.
This means that, at any time:

potential difference (voltage) across capacitor + **potential difference (voltage) across resistor** = **battery/supply voltage**

- At the instant the switch is closed (time = 0.0 s), the capacitor is not charged
  - The potential difference (voltage) across it = 0.0 volts.

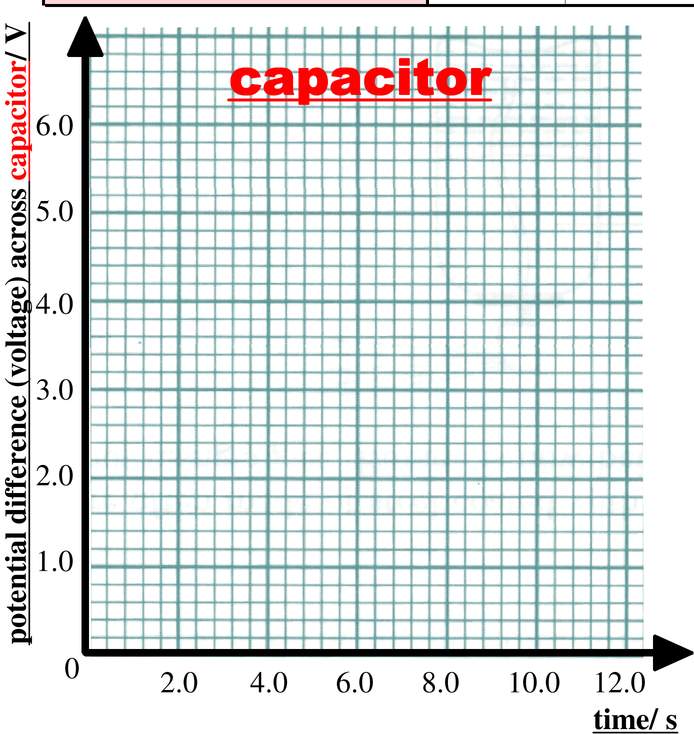
  ∴ the potential difference across the resistor = 6.0 volts (battery voltage).
- As time passes, the potential difference (voltage) across the capacitor increases

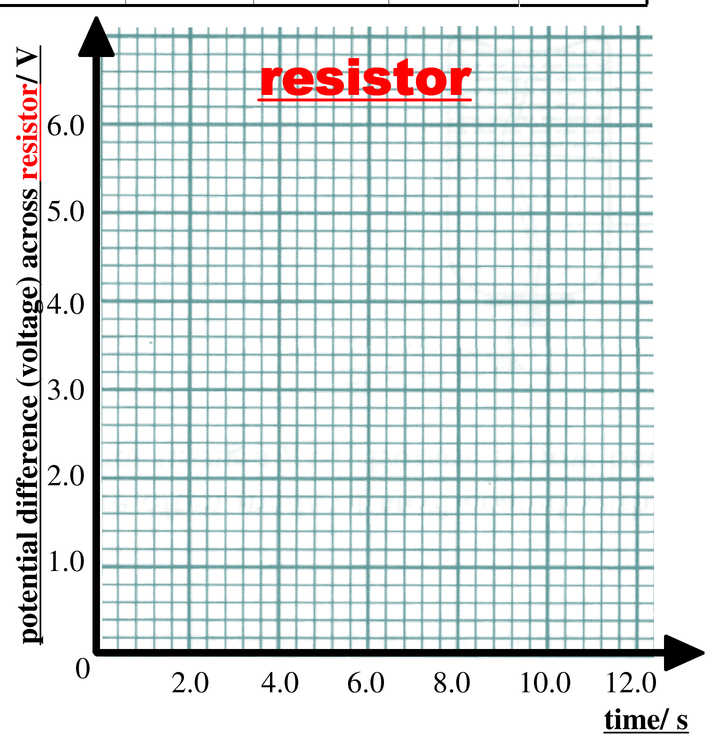
  ∴ the potential difference across the resistor decreases.
- After a certain time, the capacitor will become "fully charged"
  - The potential difference (voltage) across it will be 6.0 volts (battery voltage)

  ∴ the potential difference across the resistor = 0.0 volts.

Using the information above, complete the table then draw the ***potential difference (voltage) versus time*** graphs for the **capacitor** and the **resistor**:

Time/ seconds	0.0	2.0	4.0	6.0	8.0	10.0	12.0
Potential difference (voltage) across *capacitor* / volts		2.6	4.0	5.0	5.6	6.0	
Potential difference (voltage) across *resistor* / volts							





Current-Time Graph for a Charging Capacitor

The **resistor** in the circuit sets the value of the **maximum current** which can flow.

At any instant during the **charging process**, the size of the **current** flowing depends on the **potential difference** across the resistor at that instant and the **resistance** of the resistor.

According to Ohm's Law:

$$\text{Circuit current at any instant/ A} = \frac{\text{potential difference (voltage) across resistor/ V}}{\text{resistance of resistor/ }\Omega}$$

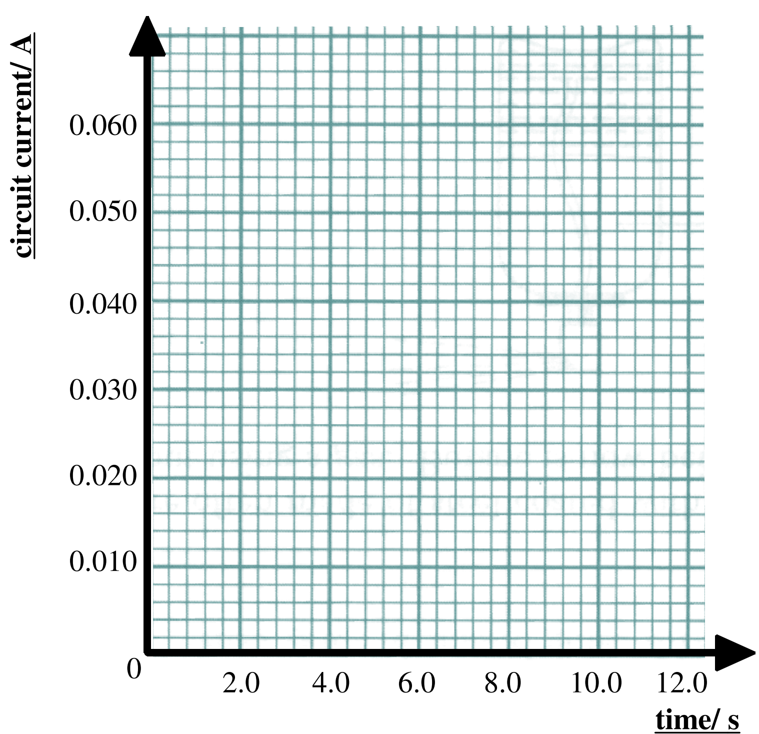
1) Copy the values for the second and third rows of this table from the previous page.

2) Calculate the value for the **circuit current** (using the above equation) at each given time.

[IN THIS CIRCUIT, THE RESISTOR HAS A VALUE OF 100 Ω].

Time/ seconds	0.0	2.0	4.0	6.0	8.0	10.0	12.0
Potential difference (voltage) across *capacitor* / volts		2.6	4.0	5.0	5.6	6.0	
Potential difference (voltage) across *resistor* / volts							
Circuit current/ amperes							

3) Draw the ***circuit current versus time*** graph for the charging capacitor.



- When does the **circuit current** have its **maximum value**? ______________________________
____
- Describe how the **circuit current** changes as the capacitor charges: ____________________
____
____

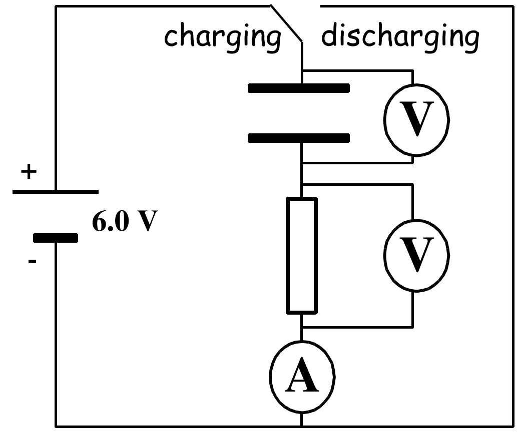
Current-Time and Voltage-Time Graphs for a Discharging Capacitor

This electric circuit can be used to investigate the **discharging of a capacitor.**

(The **resistor** is present to set the value of the ***maximum current*** which can flow).

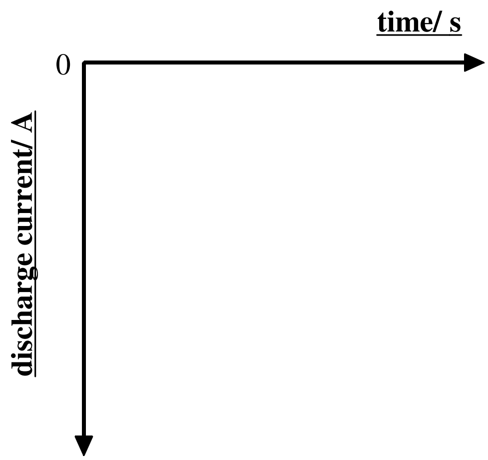
The **capacitor** is **"fully charged"** - No current is flowing.

The **capacitor** will **discharge** and **current** will start to flow ***immediately the switch is moved to the right*** - **Electrons** will flow from the **bottom capacitor plate**, through the resistor and ammeter to the **top capacitor plate**, until the **potential difference** (**voltage**) between the plates becomes **zero**, when no more electrons will flow - The **current** will be **zero.**

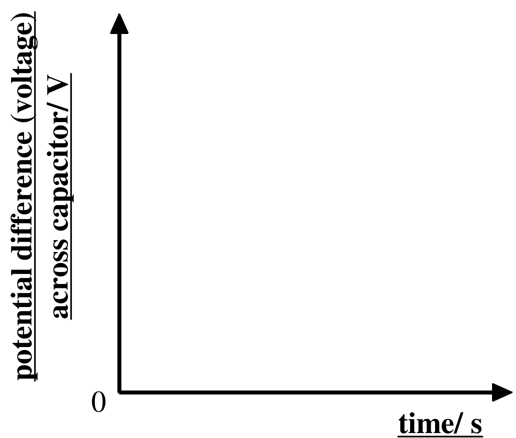


- The **discharge current** decreases from a maximum value of $\frac{\text{battery voltage}}{\text{resistance of resistor}}$ to **zero**.

The **discharge current** flows in the opposite direction to the charging current, so it is common to draw the **discharge current-time** graph in this form:

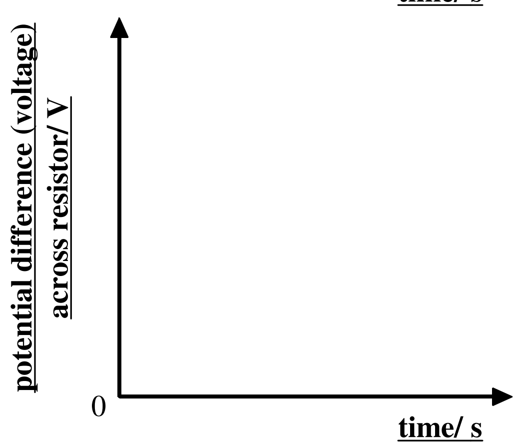


- The **potential difference** (**voltage**) across the **capacitor** decreases from the **battery voltage** to **zero**.

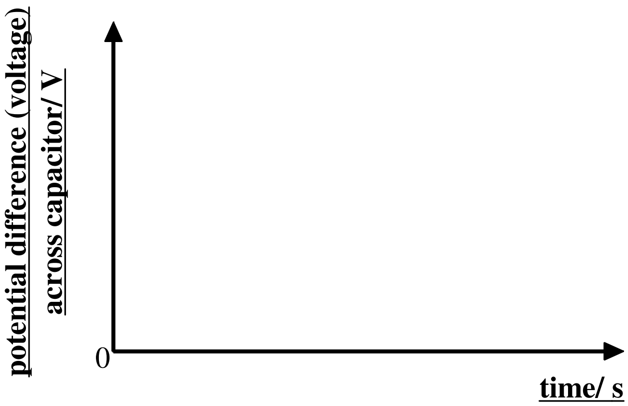
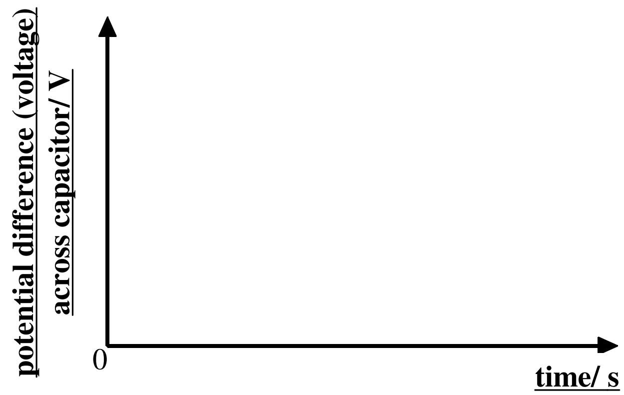
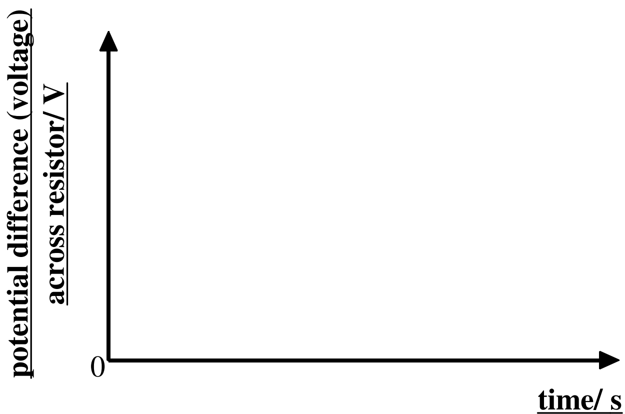
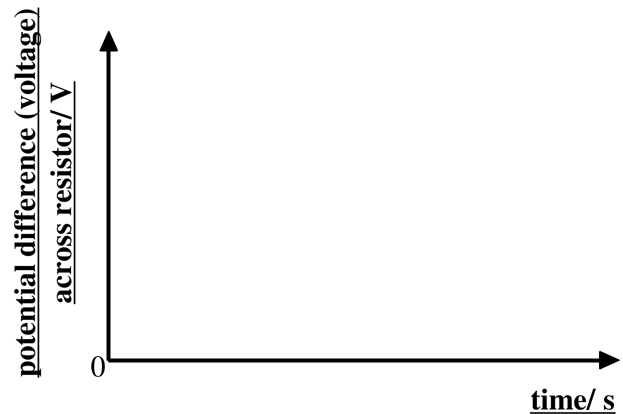
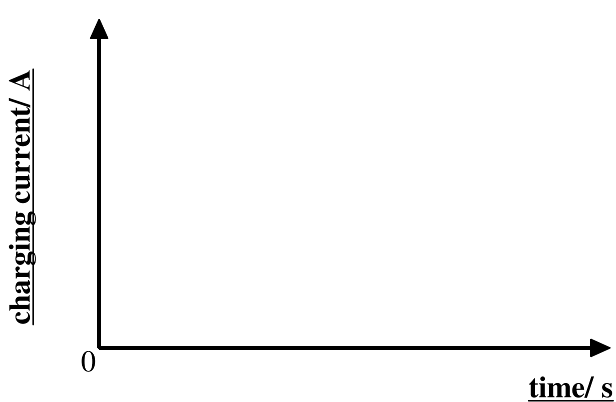


The **potential difference** across the **capacitor** always equals the **potential difference** across the **resistor**.

- The **potential difference** (**voltage**) across the resistor decreases from the **battery voltage** to **zero** (as the current in the circuit decreases).



Comparison of Graphs For Charging and Discharging Capacitors

Charging Capacitor	Discharging Capacitor
	
	
	

Time For A Capacitor to Charge and Discharge

The **time** taken for a capacitor to charge or discharge depends on the **capacitance of the capacitor** and the **resistance of the resistor** connected in series with it.

- Increasing the capacitance _______________ the charging and discharging time because ______ charge is stored on the capacitor.
- Increasing the resistance _______________ the charging and discharging time because ______ current flows at the start of the process.

(a) Calculate the **initial circuit current** when the switch is closed.

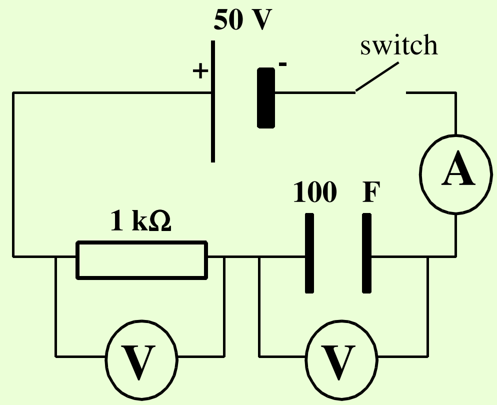
(b) (i) At the instant the circuit current reaches 3 mA, calculate the **potential difference** across the ***resistor***.

(ii) Calculate the **potential difference** across the ***capacitor*** at this same instant.

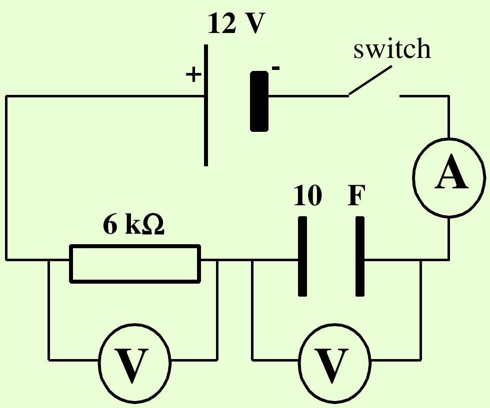
(c) (i) What will be the **potential difference** across the ***capacitor*** when it is fully charged?

(ii) What will be the **potential difference** across the ***resistor*** at this time?

(d) Calculate the **final charge** stored on the capacitor.



(e) Calculate the **energy** stored by the capacitor when itis fully charged.



(a) When the switch is closed, calculate the **initial circuit current**.

(b) (i) At the instant the circuit current reaches 0.5 mA, calculate the **potential difference** across the ***resistor***.

(ii) Calculate the **potential difference** across the ***capacitor*** at this same instant.

(c) (i) What will be the **potential difference** across the ***capacitor*** when it is fully charged?

(ii) What will be the **potential difference** across the ***resistor*** at this time?

(d) Calculate the **final charge** stored on the capacitor.

(e) Calculate the **energy** stored by the capacitor when it is fully charged.

(f) State and explain how the **time** taken to **fully charge** the capacitor be affected if:

(i) the 10 F capacitor was replaced with a capacitor of **higher capacitance**;

(ii) the 6 kΩ resistor was replaced with a resistor of **lower resistance**?

(a) Explain how you would use this circuit to **charge** and **discharge** the capacitor.

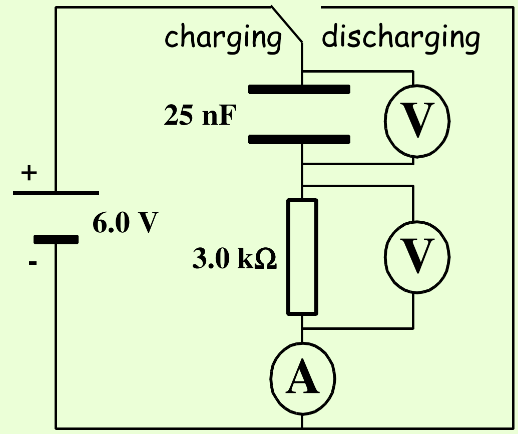
(b) Explain how the reading on the **voltmeter** connected across the **capacitor** indicates when the capacitor is **fully charged**.

(c) During the **charging process**, at the instant the **potential difference** across the ***capacitor*** reaches 4.5 V:

(i) Calculate the **potential difference** across the **resistor**?

(ii) Calculate the **circuit current**.

(d) (i) What will be the **potential difference** across the ***capacitor*** when it is fully charged?



(ii) Calculate the **final charge** stored on the capacitor.

(iii) Calculate the **energy** stored on the fully charged capacitor.

(e) Describe the **movement of electrons** when the capacitor ***discharges***.

(f) State and explain what happens to the **current** in the circuit as the capacitor ***discharges.***

(g) Calculate the size of the **maximum discharge current**.

(h) With the aid of ***sketch graphs***, describe what happens to the **potential difference** across the ***capacitor*** and ***resistor*** as the capacitor ***discharges***.