Higher

Physics

HSN61100
Unit 1 Topic 1

Contents

Topic 1 – Vectors

Vectors and Scalars

It is likely that all quantities you have met in Physics until now have been 'scalars'.

A scalar is a quantity with magnitude (size) only. For example, the time taken to complete a task or the average speed of a car over a journey.

A vector is a quantity with both magnitude and direction. For example, the velocity of an aeroplane flying NE at a speed of 800 kmh^{-1}, or the force applied to a snooker ball.

The following table shows examples of vectors and scalars:

Vector	Scalar
Displacement	Distance
Velocity	Speed
Acceleration	Mass
Force	Energy

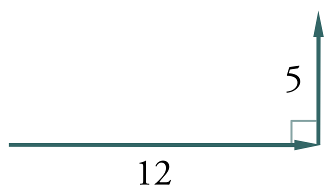
Vector Addition

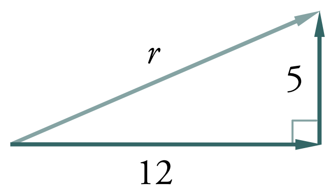
Adding vectors is not as simple as adding scalars because the directions of the vectors being added must be taken into consideration. The sum of two (or more) vectors is called the resultant vector.

Example

Draw the resultant of the two vectors shown below and calculate its magnitude.

In order to find the resultant of any two vectors, they must be added 'nose-to-tail' and not 'tail-to-tail' as shown above. We can draw these vectors 'nose-to-tail' by sliding one along the length of the other:





The resultant vector r can then be drawn by completing the triangle.

Since the triangle is right-angled, we can work out the length (magnitude) of the resultant vector using Pythagoras's Theorem:

$$\begin{aligned} r &= \sqrt{12^2 + 5^2} \\ &= \sqrt{169} \\ &= 13 \end{aligned}$$

The magnitude of the resultant vector is 13 units.

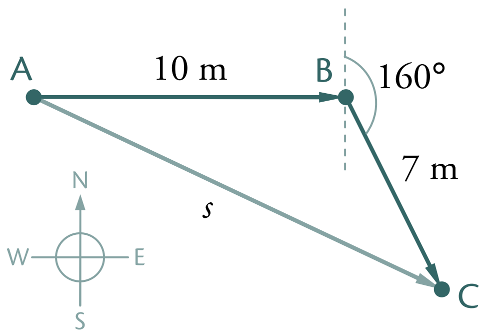
Distance and Displacement

The distance travelled by a moving object is exactly as you would expect. It is the sum of the distances of all the legs of the journey. Since each of these legs may be in a different direction, the total distance has no single direction and therefore distance is a scalar.

The magnitude (size) of the displacement is the distance from the starting point of the journey to the end point. It can be thought of as the distance "as the crow flies". Since this distance is measured in only one direction (ie a straight line) we can also specify its direction. Displacement is a vector.

Example

Imagine a journey which has two legs: one from points A to B and another from points B to C as shown below.



The total distance of this journey is given by adding the distance from A to B on to the distance from B to C.

$$\begin{aligned} \text{Total Distance} &= 10 + 7 \\ &= 17 \text{ m} \end{aligned}$$

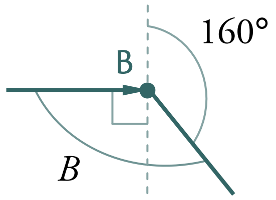
The magnitude of the displacement (shown as s above) completes a triangle and can therefore be calculated using the cosine rule:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

where a, b and c are the lengths of the sides of the triangle and A is the angle at point A. Note that this is the general rule and the letters do not correspond to the values in this problem.

To use the cosine rule, we need to know the angle between the vector from A to B and the vector from B to C (ie the angle inside the triangle at point B).

Consider a close-up view of point B from above:



$$B = 90 + (180 - 160) \\ = 110°$$

We can now use the cosine rule to work out the magnitude of s:

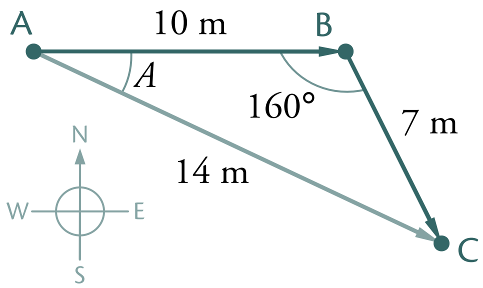
$$s^2 = 10^2 + 7^2 - 2 \times 10 \times 7 \cos(110°)$$
$$= 149 - 140 \cos(110°)$$
$$= 196.88$$
$$s = 14 \text{ m (to 2 s.f.)}$$

A common mistake would be to think that once we have found the magnitude of the displacement, then we have answered the question. We have not answered the question completely as we have still to consider the direction of the displacement vector.

We can calculate the angle at point A using the sine rule:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

where a, b and c are the sides opposite angles A, B and C in a triangle.



Given that we now know the values shown above, we can use the sine rule to calculate the angle A.

$$\frac{7}{\sin A} = \frac{14}{\sin 160°}$$
$$\sin A = \frac{7 \sin 160°}{14}$$
$$A = \sin^{-1}(0.171)$$
$$= 9.8° \text{ (to 2 s.f.)}$$

Therefore we can conclude:

$s = 14$ m (at an angle of 9.8° south of east) or $s = 14$ m (on a bearing of 099.8)

Note that the sine and cosine rules only need to be used when the triangle created by the vectors is non-right-angled. Calculations involving right-angled triangles are much more straightforward as Pythagoras's Theorem can be used.

Speed and Velocity

Average Speed
The average speed of an object over a journey is, as you would expect, given by the equation:

$$\text{average speed} = \frac{\text{distance travelled}}{\text{time}}$$

Notice that since the distance travelled does not have any one specific direction (ie it is not a vector), the average speed is also scalar quantity. Notice also that the equation involves the average speed, since the speed is unlikely to be constant over the entire journey. Speed is usually given in units of metres per second (ms^{-1}).

Example

A remote control car travels 15 metres due north, 7 metres due west, then 9 metres due south-west. The whole journey takes 17 seconds. Calculate the average speed of the remote control car over the whole journey.

$$\begin{aligned}\text{distance travelled} &= 15 + 7 + 9 \\ &= 31 \text{ m}\end{aligned}$$

$$\begin{aligned}\text{average speed} &= \frac{\text{distance travelled}}{\text{time}} \\ &= \tfrac{31}{17} \\ &= 1.8 \text{ ms}^{-1} \text{ (to 2 s.f.)}\end{aligned}$$

Therefore the average speed over the journey is 1.8 ms^{-1}.

Average Velocity
The magnitude (size) of the average velocity of an object over a journey is given by the equation:

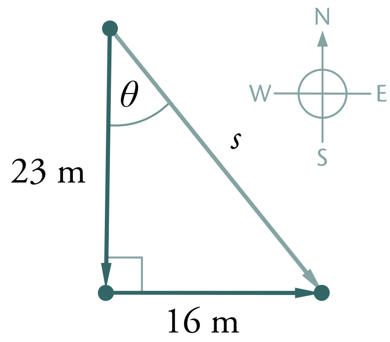
$$\text{average velocity} = \frac{\text{displacement}}{\text{time}} \text{ which is often shortened to } v = \frac{s}{t}$$

Notice that since the velocity depends on the displacement of a journey, velocity is also a vector quantity.

Example

A child on a bike travels 23 metres due south then turns immediately and travels a further 16 metres due east. The whole journey takes 11 seconds. Calculate the average velocity of the child over the whole journey.

The best way to tackle questions involving vectors is to draw a basic diagram:



First calculate the magnitude of the displacement:

$$s = \sqrt{23^2 + 16^2}$$
$$= 28 \text{ m (to 2 s.f.)}$$

Then the direction of the displacement:

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$
$$= \tfrac{16}{23}$$
$$\theta = \tan^{-1}\left(\tfrac{16}{23}\right)$$
$$= 35° \text{ east from south}$$

Now the magnitude of the velocity:

$$v = \frac{s}{t}$$
$$= \frac{28}{11}$$
$$= 2.5 \text{ ms}^{-1} \text{ (to 2 s.f.)}$$

The velocity must be in the same direction as the displacement.

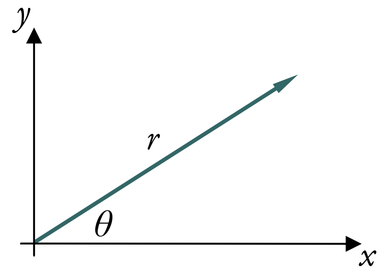
Therefore the average velocity of the child over the journey is 2.5 ms^{-1} at an angle of 35° east from south.

Components of Vectors

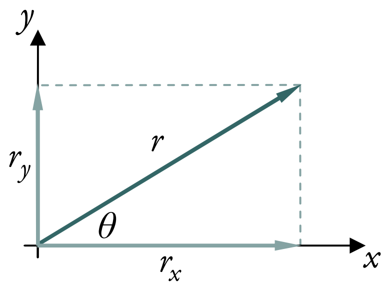
When considered with respect to a set of 2D Cartesian axes (ie the *x*-*y* axes you are probably familiar with), any vector can be expressed as the sum of two 'component' vectors, one in the *x* direction and one in the *y* direction.

This is sort of like finding the reverse of the resultant vector, since if you added the two components you would arrive back at the resultant vector.

Consider the vector shown below with respect to a set of Cartesian axes:



To travel from the beginning to the end of the vector *r*, we start at the origin and move in a straight line for *r* units at an angle of θ to the *x*-axis. However, to achieve the same result, we could travel a certain distance in the *x* direction, then move a certain distance in the *y* direction.



For example, if you were to move r_x units in the *x* direction from the origin, then r_y units in the *y* direction, you would end up at the end of *r*. So r_x and r_y are the *x*- and *y*-components of *r* respectively.

The magnitudes of r_x and r_y can be calculated using trigonometry:

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{r_x}{r}$$

$$r_x = r\cos\theta$$

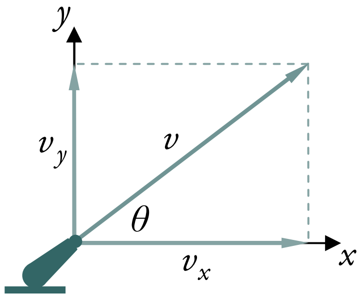
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin\theta = \frac{r_y}{r}$$

$$r_y = r\sin\theta$$

Example

Calculate the magnitude of the vertical and horizontal components of the velocity of a canon ball just after it has been fired at 25 ms^{-1} at an angle of 40° to the horizontal.



$$\begin{aligned} v_x &= v\cos\theta \\ &= 25\cos 40° \\ &= 19 \text{ ms}^{-1} \text{ (to 2 s.f.)} \end{aligned}$$

The magnitude of the horizontal component of the velocity is 19 ms^{-1}

$$\begin{aligned} v_y &= v\sin\theta \\ &= 25\sin 40° \\ &= 16 \text{ ms}^{-1} \text{ (to 2 s.f.)} \end{aligned}$$

The magnitude of the vertical component of the velocity is 16 ms^{-1}