AH Physics homework.

1.1 Differentiation & the equations of motion.

1. (a) $\frac{20}{5} = 4\,ls^{-1}$ (b) $3 \times 4 = 12\,l$

(c) $V = 6 + 4t$

(d) $V = 36 - 1.2t$

'36' – initial volume of water is 36 litres.

"–" - the bucket is emptying

"1.2" – the bucket is emptying at a rate of 1.2 litres per second

2 (a) 40 $\longrightarrow$ initial velocity is $40\,ms^{-1}$

1.5 $\longrightarrow$ uniform acceleration of $1.5\,ms^{-2}$

(b) Yes. $a = 1.5\,ms^{-2}$ is constant (uniform) acceleration.

3. (a) $\frac{ds}{dt}$ is the rate of change of displacement

This is the velocity, usually measured in ms^{-1}.

3. (b) $\frac{ds}{dt} = 3$

$$\int \frac{ds}{dt} \cdot dt = \int 3\,dt$$

$$\int ds = 3\int dt$$

$s = 3t + c$ $\quad$ c = constant of integration.

at $t=0$, $s=0$

$0 = (3\times0) + c \rightarrow c=0.$

at $t = 7s$, $s = 3\times7$

$s = \underline{21m}.$

(c) necessary to evaluate the constant of integration.

4. (a) initial displacement of the bicycle is 20m.

(b) velocity is $0.8\ ms^{-1}$

(c) $\frac{ds}{dt} = \frac{d}{dt}(0.8t + 20)$

$= \underline{0.8}\ ms^{-1}$. The bike is travelling at constant speed.

5. (a) accelerating.

(b) $s = 0.2t^2 + 0.8t + 20$

$$\frac{ds}{dt} = \frac{d}{dt}\left(0.2t^2 + 0.8t + 20\right)$$

$$\underline{\frac{ds}{dt} = 0.4t + 0.8}$$

$$\frac{d^2s}{dt^2} = \frac{d}{dt}\left(0.4t + 0.8\right)$$

$$\underline{\frac{d^2s}{dt^2} = 0.4.}$$

(c) $0.2\,ms^{-2}$ — acceleration
$0.8\,ms^{-1}$ — velocity
$20\,m$ — initial displacement.

6. $s = 2.4t^2 + 5t + 30$

7. $s = ut + \frac{1}{2}at^2$

$96 = (u \times 4) + (\frac{1}{2} \times a \times 4^2)$

$96 = 4u + 8a$ — (1)

$480 = (u \times 10) + (\frac{1}{2} \times a \times 10^2)$

$480 = 10u + 50a$ — (2)

Simultaneous equations in u & a.

$(2 \times (2)) - (5 \times (1))$ — eliminates u, find a.

$2 \times (2) \rightarrow 960 = 20u + 100a$ — (3)

$5 \times (1) \rightarrow 480 = 20u + 40a$ — (4)

$(3) - (4) \rightarrow 480 = 60a$

$a = \frac{480}{60}$

$\underline{\underline{a = 8 \text{ ms}^{-2}}}$

substitute for a in (1)

$96 = 4u + 8a$

$96 = 4u + (8 \times 8)$

$96 = 4u + 64$

$4u = 96 - 64$

$4u = 32$

$\underline{\underline{u = 8 \text{ ms}^{-1}}}$

8. $V = (12s + 225)^{\frac{1}{2}}$

$V^2 = 12s + 225$

$V^2 = 225 + 12s$ is in format $V^2 = u^2 + 2as$

$u^2 = 225$

$\underline{u = 15 ms^{-1}}$

$2as = 12s$

$\underline{a = 6 ms^{-2}}$

9. $a = 0.06t + 3$

(a) $t = 2s$ $a = (0.06 \times 2) + 3$

$\underline{a = 3.12 ms^{-2}}$

$t = 40s$ $a = (0.06 \times 40) + 3$

$\underline{a = 5.4 ms^{-2}}$

(b) $a = \frac{dv}{dt}$

$\int a \, dt = \int \frac{dv}{dt} \cdot dt$

$\int (0.06t + 3) \, dt = \int dv$

$0.03t^2 + 3t + c = v$ c = constant of integration.

at $t = 0, v = 0 \rightarrow c = 0$

$\underline{\underline{v = 0.03t^2 + 3t}}$

9 (c) $\frac{ds}{dt} = v$

$$\int \frac{ds}{dt} \cdot dt = \int v \, dt$$

$$\int ds = \int (0.03t^2 + 3t) \, dt$$

constant of integration.

$$s = 0.01t^3 + 1.5t^2 + c$$

at $t = 0$, $s = 0$ $\rightarrow c = 0$

$$\underline{\underline{s = 0.01t^3 + 1.5t^2}}$$

(d) at $t = 40s$,

$v = 0.03t^2 + 3t$

$v = (0.03 \times 40^2) + (3 \times 40)$

$v = 48 + 120$

$\underline{v = 168 \, ms^{-1}}$

$s = 0.01t^3 + 1.5t^2$

$s = (0.01 \times 40^3) + (1.5 \times 40^2)$

$s = 640 + 2400$

$\underline{s = 3040 \, m.}$

1.1 Special Relativity.

1. $$E = M_0 c^2$$
$$= 400 \times (3 \times 10^8)^2$$
$$= \underline{3.6 \times 10^{19}\,J.}$$

2. $$E = M_0 c^2$$
$$50 = M_0 \times (3 \times 10^8)^2$$
$$M_0 = \frac{50}{9 \times 10^{16}}$$
$$\underline{M_0 = 5.6 \times 10^{-16}\,kg.}$$

3. (a) $$M = \frac{M_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$
$$= \frac{1.673 \times 10^{-27}}{\sqrt{1 - \left(\frac{1.2 \times 10^8}{3 \times 10^8}\right)^2}}$$
$$= \underline{1.825 \times 10^{-27}\,kg.}$$

(b) $$M = \frac{M_0}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$
$$\frac{M}{M_0} = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

3(b) $\frac{M_0}{M} = \sqrt{1-\left(\frac{v}{c}\right)^2}$

$\left(\frac{M_0}{M}\right)^2 = 1-\left(\frac{v}{c}\right)^2$

$\left(\frac{v}{c}\right)^2 = 1-\left(\frac{M_0}{M}\right)^2$

$v^2 = c^2\left(1-\left(\frac{M_0}{M}\right)^2\right)$

$v^2 = (3\times10^8)^2 \times \left(1-\left(\frac{1.673}{2.0}\right)^2\right)$

$v^2 = (9\times10^{16}) \times 0.3003$

$v^2 = 2.702\times10^{16}$

$\underline{\underline{v = 1.64\times10^8\ ms^{-1}}}$

4. (a) $M = \frac{M_0}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$

$M=2M_0$

$2M_0 = \frac{M_0}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$

$\frac{1}{2} = \sqrt{1-\left(\frac{v}{c}\right)^2}$

$\frac{1}{4} = 1-\left(\frac{v}{c}\right)^2$

$\left(\frac{v}{c}\right)^2 = 1-\frac{1}{4} = 0.75$

$v = \sqrt{0.75c^2} = \underline{2.6\times10^8\ ms^{-1}}$

4(b) $$M = \frac{M_0}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \qquad M = 2M_0$$

$$v = 2.6 \times 10^8 \, ms^{-1} \text{ (from 4(a).)}$$

5. $$M = \frac{M_0}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \qquad M = 10M_0$$

$$10 = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

$$\frac{1}{10} = \sqrt{1-\left(\frac{v}{c}\right)^2}$$

$$\frac{1}{100} = 1-\left(\frac{v}{c}\right)^2$$

$$\left(\frac{v}{c}\right)^2 = 0.99$$

$$v = \sqrt{0.99c^2}$$

$$\underline{\underline{v = 2.98 \times 10^8 \, ms^{-1}}}$$

We are using the ratio of relativistic mass to rest mass, actual value of rest mass does not matter.

6. (a) $\frac{v}{c} = 0.75 \rightarrow v = 2.25 \times 10^{8}\, ms^{-1}$

(b) $\frac{v}{c} = 0.2 \rightarrow v = 6 \times 10^{7}\, ms^{-1}$

(c) $\frac{v}{c} = 0.98 \rightarrow v = 2.94 \times 10^{8}\, ms^{-1}$

(d) $\frac{v}{c} = 1.5 \rightarrow v = 4.5 \times 10^{8}\, ms^{-1}$!!

7. For a photon in a vacuum, $v=c$

$\rightarrow \frac{v}{c} = 1.$

8. (a) $\frac{1}{\sqrt{1-0.8^2}} = 1.67$

(b) $\frac{1}{\sqrt{1-0.1^2}} = 1.005$

(c) $\frac{1}{\sqrt{1-0.995^2}} = 10.01$

if $v=c$ then $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \underline{\underline{\infty}}$.

9. (a) $M_o = 4kg.$

(b) $M = 5kg$

(c) $M = \infty kg$ at $v=c$

10. (a) $E_k = \frac{1}{2} M v^2$

$= \frac{1}{2} \times 1 \times (2\times10^8)^2$

$E_k = 2\times10^{16} J$

(b) $M = \frac{M_o}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$ $\frac{v}{c} = \frac{2}{3}$

$M = \frac{1}{\sqrt{1-\left(\frac{2}{3}\right)^2}}$

$M = 1.34kg.$

$E_k = Mc^2 - M_oc^2$

$= c^2(M - M_o)$

$= 9\times10^{16} \times (1.34 - 1)$

$E_k = 9\times10^{16} \times 0.34$

$E_k = \underline{\underline{3.06\times10^{16} J.}}$

11. No. In Special Relativity, E_K is not directly proportional to v^2.

12. At $25\,ms^{-1}$, ok to use Newtonian Physic

$E_K = \frac{1}{2}mv^2$

$E_K = \frac{1}{2} \times 1000 \times 25^2$

$E_K = 312500\,J.$

In SR, $E_K = mc^2 - m_0c^2$

$E_K = c^2(m - m_0)$

Equate Newtonian + SR kinetic energy

$312500 = c^2(m - m_0)$

$m - m_0 = \frac{312500}{c^2}$

$m - m_0 = \underline{3.47 \times 10^{-12}\,kg}$

change in mass.